University of California, Davis
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Microeconomics

Date: June 24, 2019
Time: 5 hours
Reading Time: 20 minutes

PRELIMINARY EXAMINATION FOR THE Ph.D. DEGREE

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Please answer **any three** of the following four questions
[If you answer all four questions please indicate which three you want to be graded]

QUESTION 1

You have been arrested for allegedly committing a very serious crime. Three judges, Alice, Beth and Clara (from now on referred to as A, B and C) have heard all the evidence and are now going to vote on the verdict. There are three possible verdicts: Acquittal, Life sentence and Death penalty. During the trial it has become common knowledge among everybody (in particular, you and the three judges) that the judges' preferences over the possible verdicts are as follows:

	A's ranking:	**B's ranking:**	**C's ranking:**
Best	Death	Life	Acquittal
Middle	Life	Acquittal	Death
Worst	Acquittal	Death	Life

The legal system allows you to choose the procedure by which the judges reach a verdict. There are three different procedures. All of them involve two stages: in Stage 1 the judges simultaneously and secretly vote on a first issue; at the end of the first stage, it is made public what the first-stage votes were (i.e. which issue each judge voted for) and in Stage 2 there is a second simultaneous and secret vote on the second issue. In each stage the corresponding issue is decided by majority voting. The judges are strategic and the objective of each judge is to try to bring about an outcome which is best according to her ranking.

1. **Innocent/Guilty procedure (IG)**. In this procedure the first vote is on the issue of whether you are innocent or guilty. If a majority of the judges votes for innocence in the first stage then the outcome is that you are acquitted; otherwise the second vote is on which of the two punishments should be applied to you.
2. **Sequential Punishment procedure (SP).** In this procedure the first vote is on the issue of whether you should be sentenced to death; if a majority of the judges votes Yes, then the outcome is the death penalty. If a majority votes No in the first stage, then in the second stage the vote is on the issue of whether you should be given a life sentence; if a majority votes Yes, then the outcome is life in prison, otherwise the outcome is acquittal.
3. **The Punishment Assessment procedure (PA).** In this procedure the first vote is on which punishment is appropriate for the crime (whether you committed it or not): life sentence or death penalty. The second vote is whether you are innocent (so that the outcome is acquittal) or guilty (in which case the outcome is the punishment picked in the first vote by a majority of the judges).

(a) Represent the IG procedure as an extensive-form game-frame (a sufficiently informative sketch is enough; no need to write the payoffs).

(b) Represent the SP procedure as an extensive-form game-frame (a sufficiently informative sketch is enough; no need to write the payoffs).

(c) Represent the PA procedure as an extensive-form game-frame (a sufficiently informative sketch is enough; no need to write the payoffs).

(d) For the PA procedure write down one of the strategies of judge A. How many strategies does she have?

(e) For the IG procedure find a subgame-perfect equilibrium of the extensive-form game that represents it, by using – whenever possible – either the notion of dominant-strategy equilibrium (DSE) or the notion of Iterated Deletion of Weakly Dominated Strategies (IDWDS).

(f) For the SP procedure find a subgame-perfect equilibrium of the extensive-form game that represents it, by using – whenever possible – either the notion of DSE or the notion of IDWDS.

(g) For the PA procedure find two subgame-perfect equilibria of the extensive-form game that represents it, by using – whenever possible – either the notion of DSE or the notion of IDWDS.

(h) Which procedure will you ask the panel of judges to use?

Question 2

a. President Tony Dumb of the United States of Absurdistan is worried about his reelection prospects. He believes that his reelection prospects are strongly positively correlated with the stock market. That's why he announced on Twitter that he wants to *subsidize* returns on the stock market. When you went to heat your lunch in the microwave of the Stevens lounge across Professor Schipper's office, you overheard him mumbling to himself that this was a really good idea. You express surprise to Professor Schipper that you find him in agreement with President Dumb. Professor Schipper answers smilingly that you should check out the effect of the subsidy yourself.

   Let w denote the initial wealth of the voter. Consider an asset that yields a return r_g in the good state and a return r_b in the bad state, with $r_g > 0 > r_b$. That is, when the voter invests $x \geq 0$ into the asset, her wealth becomes $(w - x) + x(1 + r_g)$ in the good state and $(w - x) + x(1 + r_b)$ in the bad state. She assigns probability $\pi \in (0, 1)$ to the good state and the remaining probability to the bad state. We assume that r_g, r_b, and π are such that the expected return is strictly positive. We also assume that her twice continuously differentiable Bernoulli utility function $u(\cdot)$ is strictly increasing in wealth. Finally, assume that she is risk averse (and not risk neutral).

   aa. Show that her optimal investment x^0 in the absence of subsidies is strictly positive.

   ab. Assume now that the subsidy is $s > 0$ per unit of return. That is, in the good state the return after subsidy is $(1 + s)r_g$ and in the bad state the return after subsidy is $(1 + s)r_b$. Show how her optimal investment after installation of the subsidy differs from her optimal investment without the subsidy.

   ac. Discuss/interpret your results.

b. Suppose an agent faces two distributions of payoffs, F and G. Suppose that for any payoff x, the probability of x given that some payoff not below x is drawn is lower under F than G. Does there exist an expected utility maximizer with monotone increasing Bernoulli utility function who strictly prefers G over F?

   Hint: Assume that F and G have densities f and g, respectively. The probability of x conditional on a payoff being drawn not below x under F is $\frac{f(x)}{1-F(x)}$. We can now formalize the hypothesis as $\frac{f(x)}{1-F(x)} \leq \frac{g(x)}{1-G(x)}$ for all x. This is called the monotone hazard rate condition and used a lot in asymmetric information economics.

Question 3

In class we studied two types of commodity: what we called *private* and *public* goods. It's not difficult to think, however, that there are goods that are somewhere in between those two extremes: their consumption has direct private benefits, as well as aggregate social benefits.[1] Let us call these goods *mixed*. The goal of this exercise is to extend what we know of public economics to the case of mixed goods.

Consider a production economy with $L+1$ commodities, I individuals and one firm. Individual preferences are represented by functions of the form $u^i : \mathbb{R}_+^{L+2} \to \mathbb{R}$. Denoting by x^i a bundle of the first L commodities, and by $\vec{y} = (y^1, \ldots, y^I)$ the profile of individual demands for the last commodity, individual i's utility is given by

$$u^i(x^i, \textstyle\sum_{j=1}^I y^j, y^i).$$

(Note that the term y^i appears twice in the expression: as one of the summands in the second argument, and also explicitly as its third argument.[2])

The first L commodities are available in private endowments, $w^i \in \mathbb{R}_{++}^L$, while the last one has to be produced: there exists a firm that supplies $Y = f(X)$ units of that commodity if it uses a bundle X of the other commodities as input. Individual i is assumed to own a share s^i of the firm's equity.

All functions u^i are assumed to be of class C^2, differentiably strictly monotone, and differentiably strictly quasi-concave, and to satisfy the interiority property.[3] Technology $f : \mathbb{R}_+^L \to \mathbb{R}_+$ is assumed to be of class C^2, differentiably monotone and differentiably concave.[4]

(a) Extend the definition of Nash-Walras equilibrium to this economy.[5]

(b) Write the first-order conditions that characterize the solutions to the optimization problems in the definition of Nash-Walras equilibrium.

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[1] If I maintained my front garden, my house would look nicer and its value would certainly increase. But the fact that most of my neighbors maintain *their* front gardens makes my neighborhood look very nice, which improves the value of *my* house.

[2] To be sure: if the L-th commodity was private, we would only have $u^i(x^i, y^i)$; if it was public, we would only have $u^i(x^i, \sum_{j=1}^I y^j)$.

[3] These are the assumptions we used when studying *smooth* economies. Recall that they imply interiority of the individual demands, so that the latter can be characterized by the standard first-order conditions.

[4] Again, under these assumptions you can use the firm's first-order conditions to characterize its optimal production plan.

[5] That is, give a definition of competitive equilibrium where all the agents take as given the prices (à la Walras) and the demands of others (à la Nash).

(c) Extend the definition of Pareto efficient allocation to this economy.

(d) Write a constrained optimization problem that has to be solved by any Pareto efficient allocation in the economy.

(e) Argue that any Nash-Walras equilibrium allocation is Pareto efficient only in the trivial case when $I = 1$.

(f) The following is the extension of the concept of Lindahl equilibrium to this economy. Instead of one market for the mixed good, personalized markets for that good are introduced. Also, an extra market opens for the trade of *property rights* over the mixed good.[6] Each individual buys z^i units of the property rights, at a unit price r. The firm sells these property rights, Z, subject to the constraint that it cannot sell rights for more than the total amount of mixed good it produces. With the usual notation for everything else:

> *A* Lindahl equilibrium *is an array*
>
> $$(p, \vec{q}, r, \vec{x}, y, \vec{z}, X, Y, Z),$$
>
> *where* $\vec{x} = (x^1, \ldots, x^I)$, $\vec{q} = (q^1, \ldots, q^I)$, $y \in \mathbb{R}_+$ *and* $\vec{z} = (z^1, \ldots, z^I)$, *such that*
>
> i. *for each* i, (x^i, y, z^i) *solves the problem*
>
> $$\max_{\hat{x}, \hat{y}, \hat{z}} \left\{ u^i(\hat{x}, \hat{y}, \hat{z}) : p \cdot \hat{x} + q^i \hat{y} + r\hat{z} \leqslant p \cdot w^i + s^i[qY + rZ - p \cdot X] \right\}$$
>
> *where* $q = \sum_i q^i$;
>
> ii. *for the firm,* (X, Y, Z) *solves the problem*
>
> $$\max_{\hat{X}, \hat{Y}, \hat{Z}} \left\{ \sum_i q^i \hat{Y} + r\hat{Z} - p \cdot \hat{X} : \hat{Y} = f(\hat{X}) \text{ and } \hat{Z} \leqslant \hat{Y} \right\};$$
>
> iii. *markets clear:* $\sum_i x^i + X = \sum_i w^i$, $y = Y$ *and* $\sum_i z^i = Z$.

Without using the first-order conditions of the optimization problems, argue that the Lindahl solution restores Pareto efficiency: if

$$(p, \vec{q}, r, \vec{x}, y, \vec{z}, X, Y, Z),$$

is a Lindahl equilibrium, then the allocation where each i consumes bundle x^i of the private goods and z^i units of the mixed good is Pareto efficient.

(g) Now, use the first-order conditions of the optimization problems in the definition of Lindahl equilibrium to explain intuitively why this equilibrium restores Pareto efficiency

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[6] In the motivation, my H.O.A. would charge each homeowner i a price q^i per unit of overall garden maintenance in the neighborhood. In addition, I would have to pay r per unit of gardening done on my house.

QUESTION 4

Two generals need to coordinate their attacks on an enemy position. The position is either lightly fortified (L) or heavily fortified (H). Each general has two choices: Attack (A) or Not Attack (N) and the decisions will be made "simultaneously" by the two generals (more precisely, each general makes her decision without knowing the decision of the other general). Today is Day 0 and the two generals are in the same location and can discuss matters, but tomorrow they will take up positions in different locations. It is common knowledge between the two generals that from her location General 2 (= Player 2) will not be able to determine if the enemy position is L or H, while General 1 (= Player 1) will be able to correctly determine if the enemy position is L or H. They will be able to communicate by e-mail via a satellite link that is not completely reliable, in the sense that every message that is sent has a probability ε ($0 < \varepsilon < 1$) of not getting delivered. On Day 0 the players agree that Player 1 will send an email to Player 2 at precisely 6:00am *if and only if* the enemy position is L. Communication is done using computers; each computer is programmed to **automatically and instantaneously** send an acknowledgment whenever it receives a message and at each moment the computer screen displays the number of messages received from the other player. Communication is almost instantaneous so that if one minute has elapsed since the last message was received by Computer i ($i = 1,2$) then it means that the last acknowledgment sent by Computer i was either not received or it was received but the automatically generated acknowledgment by Computer j ($j \neq i$) was lost; thus, if no message is received within one minute of the reception of the last message, the computer's screen flashes the message "End of communication" and, on the next line, "total number of messages received by this computer: n". If Computer 2 (Player 2's computer) has not received any messages by 6:01am the screen displays the message "No messages received – End of communication".

On Day 0 both players agree that the probability that the enemy position is L (and thus that Player 1 will send the first e-mail) is p, with $0 < p < 1$.

(a) Draw a tree where at the first node there are two edges, one representing the possibility that the first message is sent and the other the possibility that it is not sent, and at every successive node preceded by a sent message, there are two edges, one representing the possibility that the automatically generated acknowledgment is not delivered and the other the possibility that the automatically generated acknowledgment is delivered. Label each terminal node with the total number of messages **sent** over the channel and the prior probability (that is, the probability as assessed on Day 0) that that node is reached.

(b) Using as states the terminal nodes of the tree of part (a), draw the information partition of Player 1 (that is, the possible future states of knowledge of Player 1 as assessed on Day 0).

(c) Using as states the terminal nodes of the tree of part (a), draw the information partition of Player 2 (that is, the possible future states of knowledge of Player 2 as assessed on Day 0).

(d) Suppose that, as a matter of fact, the enemy position is L so that Player 1 at 6:00am of Day 1 sends the notification to Player 2. How many messages need to be successfully exchanged between the two players for it to become common knowledge that the enemy position is L?

[continues on the next page]

Let the von Neumann-Morgenstern payoffs be as follows (A means "Attack" and N means "Do Not attack"), where $c > 0$.

If enemy position is L

		Player 2: A		Player 2: N	
Player 1	A	1+c	1+c	0	c
	N	c	0	c	c

If enemy position is H

		Player 2: A		Player 2: N	
Player 1	A	1+c	0	0	c
	N	c	0	c	c

For the following questions, suppose that on Day 0 the players agree to follow this strategy, call it strategy $\hat{s}_k$: if a player knows that a total of **at least** k messages were sent (with $k > 0$), then that player will attack, otherwise she/he will not attack.

(e) Suppose that $c = 2$. Are there values of k such that it is in the interest of **each** player to follow strategy $\hat{s}_k$ if she trusts that the other player will follow strategy $\hat{s}_k$? Explain [Hints: (1) you need to use Bayes' rule to obtain posterior beliefs; (2) consider first the case where k is odd and then the case where k is even.]

(f) Suppose now that $0 < c < 1$ (and $0 < \varepsilon < 1$). Find an inequality involving c and ε which is necessary and sufficient for each player to be willing to follow strategy $\hat{s}_k$ if the player trusts the other player to follow strategy $\hat{s}_k$.

(g) Let $c = 0.3$ and $\varepsilon = 0.2$. If the players want to maximize the ex ante probability that both players will attack (by rationally following strategy $\hat{s}_k$, expecting the other player to follow strategy $\hat{s}_k$), what value of k should they agree on and what will that probability be?