MACROECONOMICS PRELIM, JUNE 2020

ANSWER KEY FOR QUESTION 1

a) As I point out in the question, the probability with which a firm will meet a type i depends only on the relative measure of this types in the aggregate pool of unemployed. But also, the destruction rate of jobs does not depend on the type of worker employed. Then the various value functions for the firm are given by:

$$rV = -pc + q(\theta)\left[\pi J_H + (1-\pi)J_L - V\right], \tag{1}$$
$$rJ_i = p - w_i - \lambda J_i. \tag{2}$$

Similarly, for the workers we have:

$$rU_i = z_i + \theta q(\theta)(W_i - U_i), \tag{3}$$
$$rW_i = w_i + \lambda(U_i - W_i). \tag{4}$$

b) Since in equilibrium $V = 0$, (1) implies that

$$\pi J_H + (1-\pi)J_L = \frac{pc}{q(\theta)}. \tag{5}$$

Next, solve (2) with respect to J_i and plug what you found into (5). After some algebra you should arrive at the following equation, which is the model's **JC curve**:

$$\pi w_H + (1-\pi)w_L = p - \frac{pc(r+\lambda)}{q(\theta)}.$$

c) Solving the bargaining problem for a typical match between a firm and a worker of type i will yield (as usual)

$$w_i = \beta p + (1-\beta)rU_i = \beta p + (1-\beta)z_i + (1-\beta)\theta q(\theta)(W_i - U_i),$$

where the second equality follows from (3). Replacing $(1-\beta)(W_i - U_i)$ with βJ_i (from the standard solution to the bargaining problem), will yield

$$w_i = \beta p + (1-\beta)z_i + \theta q(\theta)\beta J_i. \tag{6}$$

Now the trick here is to multiply (6) evaluated at $i = H$ by π, and the same equation evaluated at $i = L$ by $1-\pi$, and sum those two expressions up to obtain

$$\pi w_H + (1-\pi)w_L = \beta p + (1-\beta)[\pi z_H + (1-\pi)z_L] + \theta q(\theta)\beta\left[\pi J_H + (1-\pi)J_L\right].$$

Finally, let's define the average unemployment benefit in the economy as $\bar{z} \equiv \pi z_H + (1-\pi) z_L$. Using this definition and equation (5) allows us to rewrite the last expression as

$$\pi w_H + (1-\pi) w_L = \beta p + (1-\beta)\bar{z} + \theta\beta p c,$$

which is our model's **wage curve**.

d) Just replace the LHS of the WC from the JC curve to obtain one equation in one unknown:

$$p - \frac{pc(r+\lambda)}{q(\theta)} = \beta p + (1-\beta)\bar{z} + \theta\beta p c.$$

Notice that after a little algebra we can write this as:

$$(1-\beta)(p-\bar{z}) = pc\frac{r+\lambda+\beta\theta q(\theta)}{q(\theta)}.$$

This is identical to the equilibrium condition we saw in class, after one replaces z with $\bar{z}$.

e) In class (in the model of endogenous job destruction), we made the guess that there will exist $R \in [0,1]$, such that the job will continue if and only if $x \geq R$. Here we will make an analogous assumption: For a worker of type $i = \{L, H\}$, there will exist $R_i \in [0,1]$ such that a match will stay alive if and only if $x \geq R_i$. Of course (just like in our lecture notes), these reservation values satisfy $J_i(R_i) = 0$. Moreover, and again in accordance to the methodology used in class, we conjecture that $J_i(x)$ is an increasing function. Finally, we know that $J_L(x)$ lies above $J_H(x)$, for all x, because the two types are equally productive, but the H-type gets a higher wage because of her higher outside option (when she is unemployed). All these points taken together imply that $R_L < R_H$.

In steady state, $\dot{u}_L = 0$ and $\dot{u}_H = 0$, so that the measure of u_L and u_H are given by

$$u_L = \frac{\lambda G(R_L)}{\theta q(\theta) + \lambda G(R_L)}(1-\pi), \quad u_H = \frac{\lambda G(R_H)}{\theta q(\theta) + \lambda G(R_H)}\pi.$$

The unemployment rate of workers of type i within the type-i population is

$$\gamma_L \equiv \frac{u_L}{1-\pi} = \frac{\lambda G(R_L)}{\theta q(\theta) + \lambda G(R_L)}, \quad \gamma_H \equiv \frac{u_H}{\pi} = \frac{\lambda G(R_H)}{\theta q(\theta) + \lambda G(R_H)}.$$

While in the model of exogenous job destruction it was $\gamma_L = \gamma_H = \lambda/(\theta q(\theta) + \lambda)$, here $\gamma_L \neq \gamma_H$ because $R_L \neq R_H$. And since $R_L < R_H$, it will be $\gamma_L < \gamma_H$.

f) As already discussed, we have

$$\frac{u_L}{u} \neq 1-\pi, \quad \frac{u_H}{u} \neq \pi.$$

There are three possible states for a firm: vacant, matched with a worker of type L, or matched with a worker of type H. Let V, J_L and J_H be the value functions for each of the states:

$$rV = -c + q(\theta)\left[\frac{u_L}{u}J_L(1) + \frac{u_H}{u}J_H(1) - V\right],$$
$$\forall x \geq R_i,\;\; rJ_i(x) = px - w_i(x) + \lambda\left[\int_{R_i}^{1} J_i(s)dG(s) - J_i(x)\right],\;\; i = L, H.$$

g) There are two possible states for a worker of each type: unemployed or employed. Let U_i and W_i be the value functions of a worker of type $i = \{L, H\}$ for each of the states:

$$rU_i = z_i + \theta q(\theta)(W_i(1) - U_i),$$
$$\forall x \geq R_i,\;\; rW_i(x) = w_i(x) + \lambda\left[G(R_i)U_i + \int_{R_i}^{1} W_i(s)dG(s) - W_i(x)\right],\;\; i = L, H.$$

h) Notice the similarity of this part (and of the whole question) with one of the questions in your midterm. There, if you were a H productivity type, you were well-off in the model with exogenous destruction, because your wage was higher, and you were EVEN BETTER OFF in the model with endogenous destruction, because (not only your wage was higher but also) you spent more time employed and less time unemployed.

But here things are different. H types still make a higher wage, because they have a higher outside option, but precisely because of that high outside option, they end up losing their jobs to productivity shocks more often, i.e., $R_H > R_L$. As we saw in part e, this means that the unemployment rate among high types is higher. So if I was an L type I would prefer to live in a world of endogenous job destruction. Why? In a world of exogenous destruction my wage is lower than the H type and I get to be employed for the same amount of time as that type! In the world of endogenous job destruction, my wage is (still) lower than the H type's wage, but at least I get to be employed for longer periods of time, on average. So at least I am better off than the H type in one dimension.

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PRELIMINARY EXAMINATION FOR THE Ph.D. DEGREE

MACROECONOMICS: 200E Question Answer Key

June, 2020

Question X (*YY points*)

This question considers the macroeconomic effects of a time-varying sales tax in the New Keynesian model.

There are a continuum of identical households. The representative household makes consumption (C) and labor supply (N) decisions to maximize lifetime expected utility:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\ln C_t - \chi \frac{N_t^{1+\psi}}{1+\psi} \right) \tag{1}$$

subject to their budget constraint:

$$(1+\tau_t^s)C_t + B_t = w_t N_t + (1+i_{t-1}) \frac{P_{t-1}}{P_t} B_{t-1} + D_t + T_t \tag{2}$$

where w_t is the real wage, N_t is hours worked, B_t are real bond holdings at the end of period t, i_{t-1} is the nominal interest rate paid between $t-1$ and t, P_t is the price of the final consumption good and D_t are real profits from firms that are distributed lump sum. T_t are real lump sum transfers from the government. As usual, $0 < \beta < 1$ and $\psi > 0$. τ_t^s is a sales tax charged on the purchases of consumption goods.

The production side of the model is the standard New Keynesian environment. Monopolistically competitive intermediate goods firms produce an intermediate good using labor. Intermediate goods firms face a probability that they cannot adjust their price each period (the Calvo pricing mechanism). Intermediate goods are aggregated into a final (homogenous) consumption good by final goods firms. The production side of the economy, when aggregated and linearized, can be described by the following set of linearized equilibrium conditions (the production function, the optimal hiring condition for labor and the dynamic evolution of prices):

$$\hat{y}_t = \hat{n}_t \tag{3}$$

$$\hat{w}_t = \hat{mc}_t \tag{4}$$

$$\hat{\pi}_t = \beta E_t(\hat{\pi}_{t+1}) + \lambda \hat{mc}_t \tag{5}$$

The resource constraint is:

$$\hat{y}_t = \hat{c}_t \tag{6}$$

Monetary policy follows a simple Taylor Rule:

$$\hat{i}_t = \phi_\pi \hat{\pi}_t \tag{7}$$

The (linearized) sales tax rate follows an AR(1) process

$$\hat{\tau}_t^s = \rho \hat{\tau}_{t-1}^s + e_t \tag{8}$$

e_t is i.i.d. and tax revenues are redistributed lump-sum to households.

In percentage deviations from steady state: $\hat{mc}_t$ is real marginal cost, $\hat{c}_t$ is consumption, $\hat{w}_t$ is the real wage, $\hat{n}_t$ is hours worked, $\hat{y}_t$ is output. In deviations from steady state: $\hat{i}_t$ is the nominal interest rate, $\hat{\pi}_t$ is inflation and $\hat{\tau}^s_t$ is the sales tax rate. λ is a function of model parameters, including the degree of price stickiness.[1] Assume that $\phi_\pi > 1$, $0 < \rho < 1$.

a) First consider the representative household's problem. Write down the household's problem in *recursive form* and derive the household's first order conditions.

Answer

$$V(B_{t-1}, \tau^s_t, \mathcal{B}_{t-1}) = \max_{C_t, B_t, N_t} \left\{ \left(\ln C_t - \chi \frac{N_t^{1+\psi}}{1+\psi} \right) + \beta E_t V(B_t, \tau^s_{t+1}, \mathcal{B}_t) \right\} \tag{9}$$

subject to

$$(1+\tau^s_t) C_t + B_t = w_t N_t + (1+i_{t-1}) \frac{P_{t-1}}{P_t} B_{t-1} + D_t + T_t \tag{10}$$

where $\mathcal{B}$ are aggregate bond holdings. Setting this up as a Lagrangian (using λ_t as the multiplier), we can derive the first order conditions:

$$\frac{1}{C_t(1+\tau^s_t)} = \lambda_t \tag{11}$$

$$\chi N_t^\psi = \lambda_t w_t \tag{12}$$

$$\lambda_t = \beta E_t \frac{\partial V}{\partial B_t} \tag{13}$$

Using the envelope condition (take partial derivative of the value function today wrt B_{t-1} and shift forward one period):

$$\lambda_t = \beta E_t \lambda_{t+1} (1+i_t) \frac{P_t}{P_{t+1}} \tag{14}$$

b) Show that the linearized first order condition for labor supply from part (a) is:

$$\hat{w}_t = \hat{c}_t + \psi \hat{n}_t + \hat{\tau}^s_t \tag{15}$$

and that the flexible price natural rate of output (in linearized form) is given by:

$$\hat{y}^n_t = -\frac{1}{1+\psi} \hat{\tau}^s_t \tag{16}$$

[1] $\lambda = \frac{(1-\theta)(1-\beta\theta)}{\theta}$ where θ is the probability that a firm cannot adjust its price.

Hints: you will need to use equations (3), (4), (6) and (18). To simplify the algebra, define $\hat{\tau}_t^s = \ln\left(\frac{1+\tau_t^s}{1+\tau_{ss}^s}\right)$ and assume a zero steady state tax rate, $\tau_{ss}^s = 0$.[2]

Answer

After combining equations 11 and 12 the easiest way to linearize this is to take logs and then subtract the same expression evaluated at the steady state:

$$\ln w_t - \ln w = \psi(\ln N_t - \ln N) + (\ln C_t - \ln C) + \ln(1 + \tau_t^s) \tag{17}$$

where the final term reflects the fact that $\tau_{ss}^s = 0$ in steady state. Using hat notation to denote percentage deviations from steady state yields the equation in the question:

$$\hat{w}_t = \hat{c}_t + \psi\hat{n}_t + \hat{\tau}_t^s \tag{18}$$

Next, note that

$$\hat{w}_t = \hat{mc}_t \tag{19}$$

Furthermore, the natural rate of output occurs under flexible prices, so $\hat{mc}_t = 0$. Because all firms are free to set the same price, there is no markup dispersion and $\hat{mc}_t = 0$. Also making use of the production function and the resource constraint yields:

$$0 = \hat{y}_t + \psi\hat{y}_t + \hat{\tau}_t^s \tag{20}$$

Solving for $\hat{y}_t$ and putting a superscript n to denote the level of output under the assumption of flexible prices yields the result in the question:

$$\hat{y}_t^n = -\frac{1}{1+\psi}\hat{\tau}_t^s \tag{21}$$

c) This model can be reduced to two equations:

$$E_t\tilde{y}_{t+1} - \tilde{y}_t = (\phi_\pi\hat{\pi}_t - E_t\hat{\pi}_{t+1}) + \frac{\psi}{1+\psi}(1-\rho)\hat{\tau}_t^s \tag{22}$$

$$\hat{\pi}_t = \beta E_t(\hat{\pi}_{t+1}) + \kappa\tilde{y}_t \tag{23}$$

plus the stochastic process for the sales tax. $\tilde{y}_t = \hat{y}_t - \hat{y}_t^n$ is the output gap. $\kappa = (1+\psi)\lambda$.

Using the method of undetermined coefficients, find the response of the output gap and inflation to an exogenous cut in sales taxes when prices are sticky and monetary

[2] If the tax rate is sufficiently small, this implies $\hat{\tau}_t^s = \tau_t^s$ (approximately). Another way to think about this is that the policy choice variable is $(1+\tau_t^s)$, so $\hat{\tau}_t^s = \frac{(1+\tau_t^s)-(1+\tau_{ss}^s)}{(1+\tau_{ss}^s)}$ but with a steady state tax rate of zero $\tau_{ss}^s = 0$.

policy follows the Taylor Rule above. To do this, guess that the solution for each variable is a linear function of the tax shock $\hat{\tau}_t^s$.

Answer:

Let's guess:

$$\hat{\pi}_t = \Lambda_\pi \tau_t^s$$
$$\tilde{y}_t = \Lambda_y \tau_t^s$$

Substitute these guesses into the New Keynesian Phillips and making use of the stochastic process for taxes to remove the expectations term yields:

$$\Lambda_y = \frac{\Lambda_\pi(1-\beta\rho)}{\kappa}$$

Next, substitute the guesses into the dynamic IS curve and, again, make use of the stochastic process for taxes to remove the expectations terms. Then make use of the expression for Λ_y that we just derived. Solve for Λ_π:

$$\Lambda_\pi = \kappa\eta\left((1-\beta\rho) + \frac{\kappa(\phi_\pi - \rho)}{1-\rho}\right)^{-1} < 0$$

Combining this with the solution for Λ_y we found above:

$$\Lambda_y = \eta(1-\beta\rho)\left((1-\beta\rho) + \frac{\kappa(\phi_\pi - \rho)}{1-\rho}\right)^{-1} < 0$$

where $\eta = -\frac{\psi}{1+\psi} < 0$

d) Discuss how, and why, a sales tax cut affects the natural rate of output, the output gap and inflation in this model.

Answer:

This shock works a bit like a shock to demand (note that it shows up in the same place in the Euler equation as a preference shock). As can be seen above, a cut in sales taxes leads to a positive output gap and positive inflation. Why? Lower sales taxes raise demand for consumption goods (from the consumer's point of view this is like a fall in the price). As consumer demand increases some firms cannot adjust their price (sticky prices come from the Calvo pricing mechanism). Some firms raise prices and some firms raise output. As a result both prices and output increase. This accords with common views about the demand effects of tax cuts — a boost to demand leads to an increase in output and inflation.

The Λ terms also make sense. A higher κ — more flexible prices — raises the effect on inflation and lowers the effect on the output gap. As ϕ_π rises the response of output and inflation gets smaller. This makes sense because ϕ_π is the coefficient in the monetary policy rule, a higher coefficient implies more aggressive policy.

There is an important difference between this shock and a standard preference shock. Unlike the preference shock, the cut in sales taxes raises the natural rate of output. This means tax cuts boost GDP even under flexible prices. From the labor supply condition, we can see that a cut in sales taxes raises the marginal utility of consumption and this increases labor supply. With sticky prices the effect on GDP is even larger (a combination of a higher natural rate and a demand effect).

e) Now suppose the monetary policymaker attempts to target the *natural* real interest rate. Is this policy optimal from a welfare perspective in this model? Explain. You do not need to derive anything, answer using your knowledge of this model. (**Hints:** To answer this question, think about what the first best allocation, $\hat{y}_t^e$, would look like and whether the policymaker can close the welfare relevant output gap $\hat{x}_t = \hat{y}_t - \hat{y}_t^e$. Assume the steady state is efficient).

Answer:

Consider the dynamic IS curve:

$$E_t\tilde{y}_{t+1} - \tilde{y}_t = (\hat{i}_t - E_t\hat{\pi}_{t+1}) + \frac{\psi}{1+\psi}(1-\rho)\hat{\tau}_t^s \tag{24}$$

The natural rate of interest occurs when $\tilde{y}_t = 0$ for all t, i.e. when output equals the natural rate of output. We can see that the natural real interest rate is entirely driven by the tax term on the right hand side. This means if the real interest rate were set equal to the natural rate, the output gap would indeed be zero. A zero output gap is then consistent with zero inflation. This could be implemented with a policy rule such as $\hat{i}_t = \hat{r}_t^n + \phi_\pi\hat{\pi}_t$

Although this looks like the Divine Coincidence, this outcome would not be welfare maximizing. The reason is that the social planner would set $\tau_t^s = 0$ always. This means there should be no variation in GDP in the first best. In other words, the *welfare relevant* output gap should be zero $\hat{x}_t = \hat{y}_t = 0$. In contrast, the equilibrium characterized by $\hat{r}_t = \hat{r}_t^n$ produces zero inflation but $\hat{y}_t = \hat{y}_t^n$.

Prelim 2020: Answer Key

Nicolas Caramp

Question 3

a) *State the conditions an optimal contract must satisfy. Let* $\rho_i \equiv R - \frac{B_i}{p_H - p_L}$*. Show that* $\rho_i I$ *is the maximum amount an entrepreneur of type* i *can commit to repaying at* $t = 1$ *(for this reason we will call* ρ_i *as "pledgeability").*

An optimal contract between Es and Fs must satisfy the following conditions:

1. Resource constraint:
$$R_F + R_E = R$$

2. Participation constraint:
$$L_i \leq \frac{p_H R_F}{1+r}$$

3. Incentive compatibility constraint
$$p_H R_E \geq p_L R_E + B_i$$

where L_i is the size of the loan made to entrepreneur i.

Taking the incentive compatibility constraint, replacing $R_E = R - R_F$ and doing some algebra, we get
$$R_F \leq R - \frac{B_i}{p_H - p_L} \equiv \rho_i$$
That is, ρ_i is maximum that Es can promise Fs so that the contract incentivizes the Es to put effort into the project. Fs would not accept a promise higher than ρ_i (per unit of the project) since they anticipate that such a promise would imply a low probably of success (because the E would shirk).

b) *State the entrepreneurs' problem. Show that Es invests up to the maximum possible scale, i.e.,*
$$I = \frac{N_i}{1 - \frac{p_H \rho_i}{1+r}}.$$
How does the project scale depend on B_i*? Explain.* ***Hint:*** *Since Fs have a large endowment, the Es keep all the surplus from the project.*

The entrepreneurs' problem is given by

$$\max_{C_1,C_2,I,R_F,R_E\geq 0} C_1+\beta p_H C_2$$

subject to

$$\begin{gathered} C_1+I\leq N_i+L_i \\ C_2\leq (R-R_F)I \\ R_E+R_F=R \\ L_i=\frac{p_H R_F}{1+r}I \\ R_F\leq \rho_i \end{gathered}$$

We can simplify this problem as

$$\max_{I,R_F\geq 0} \underbrace{(\beta p_H R-1)}_{>0} I+N_i$$

subject to

$$\begin{gathered} N_i+L_i-I\geq 0 \\ L_i=\frac{p_H R_F}{1+r}I \\ R_F\leq \rho_i \end{gathered}$$

where we used that $1+r=\frac{1}{\beta}$. Since the objective function is increasing in I, the entrepreneur chooses $R_F=\rho_i$ to maximize the size of the project, which is given by

$$I=\frac{N_i}{1-\frac{p_H\rho_i}{1+r}}$$

Note that we need to impose that $0<\frac{p_H\rho_i}{1+r}<1$ for all i to guarantee that the investment level is well defined. It is immediate to see that I is decreasing in B_i.

c) *Let $I\equiv\int_{\underline{B}}^{\overline{B}}\int_{\underline{N}}^{\overline{N}} I(N_i,B_i)dF(N_i)dG(B_i)$ be the total investment in this economy. Argue that if the financiers' endowment is sufficiently large, then $1+r=\frac{1}{\beta}$. Let e_0 denote the Fs' endowment in period 0. What is the minimum value of e_0 such that $1+r=\frac{1}{\beta}$ in equilibrium?*

If Fs have sufficiently high endowment in period 0, then they will make loans to Es and they will also consume part of it. Thus, they have to be indifferent between lending and consuming (at the margin), which implies that

$$\underbrace{1+r}_{\text{return in the market}} = \underbrace{\frac{1}{\beta}}_{\text{MRS}}$$

Let

$$I^* \equiv \int_{\underline{B}}^{\overline{B}} \frac{N}{1-\beta p_H \rho_i} dG(B_i)$$

The condition is $e_0 \geq I^* - N$.

d) *Start from a situation in which there is no heterogeneity so that $\underline{B} = \overline{B} = B$ and $\underline{N} = \overline{N} = N$. Does an increase in heterogeneity in N_i (but keeping the average constant) increase, decrease or not change aggregate investment I? How about an increase in heterogeneity in B_i (keeping the average constant)?* ***Hint:*** *If X is a random variable and $h(x)$ is a convex function of x, then $E[h(X)] > h(E[X])$.*

Since $I(N_i, B_i)$ is linear in N_i, heterogeneity in N_i has no impact on aggregate investment as long as the mean N doesn't change.

Let's switch to the effect of heterogeneity in B_i. Note that

$$\frac{\partial I(N, B_i)}{\partial B_i} = -\frac{N_i}{\left(1 - \frac{p_H \rho_i}{1+r}\right)^2} \frac{1}{(p_H - p_L)(1+r)}$$

and

$$\frac{\partial^2 I(N, B_i)}{\partial B_i^2} = 2\frac{N_i}{\left(1 - \frac{p_H \rho_i}{1+r}\right)^3} \left(\frac{1}{(p_H - p_L)(1+r)}\right)^2 > 0$$

That is, $I(N, B_i)$ is convex in B_i. Thus, by Jensen's inequality we know that

$$\int_{\underline{B}}^{\overline{B}} I(N, B_i) dG(B_i) = E[I(N, B_i)] > I(N, E[B_i])$$

so that aggregate investment increases with the heterogeneity in B_i.

e) *Suppose that the cost of monitoring is cI and an entrepreneur i hires the service. How does contract with Fs change? Show that the scale of the project is larger with monitoring if and only if*

$$c \leq (1-\phi) p_H \frac{R - \rho_i}{1+r}$$

Hint: *Carefully state the conditions a contract must satisfy.*

The optimal contract now satisfies

1. Resource constraint
$$R_E + R_F = R$$

2. Participation constraint
$$(1+c)I - N_i \leq \frac{p_H R_F I}{1+r}$$

3. Incentive compatibility constraint

$$R_F \leq R - \frac{b_i}{p_H - p_L}$$

Following similar steps as before, we get that

$$\tilde{I} = \frac{N_i}{1 + c - \frac{p_H \tilde{\rho}_i}{1+r}}$$

where

$$\tilde{\rho}_i \equiv R - \frac{b_i}{p_H - p_L}$$

The scale of the project is larger if and only if

$$\frac{N_i}{1 + c - \frac{p_H\left(R - \frac{b_i}{p_H - p_L}\right)}{1+r}} \geq \frac{N_i}{1 - \frac{p_H\left(R - \frac{B_i}{p_H - p_L}\right)}{1+r}}$$

or

$$c \leq p_H \frac{(1-\phi)B_i}{(p_H - p_L)(1+r)} = (1-\phi)p_H \frac{R - \rho_i}{1+r}$$

f) *What is the maximum cost such that E chooses monitoring? How does the maximum cost depend on i? Explain the intuition of why E chooses to pay to be monitored.*

The maximum amount Es are willing to pay for monitoring is the level such that the profit without monitoring is equal to the profit with monitoring:

$$p_H \frac{b_i}{(p_H - p_L)(1+r)} \frac{N_i}{1 + c - \frac{p_H\left(R - \frac{b_i}{p_H - p_L}\right)}{1+r}} \geq p_H \frac{B_i}{(p_H - p_L)(1+r)} \frac{N_i}{1 - \frac{p_H\left(R - \frac{B_i}{p_H - p_L}\right)}{1+r}}$$

or

$$c \leq (1-\phi)\left(\frac{p_H R}{1+r} - 1\right) \equiv c^{\max}$$

Note that it is independent of i.

The entrepreneur might choose monitoring because it increases the pledgeability of the project. Note that if $c < c^{\max}$, the size of the project is larger with monitoring since

$$(1-\phi)\left(\frac{p_H R}{1+r} - 1\right) < (1-\phi)p_H \frac{R - \rho_i}{1+r} \iff \frac{p_H \rho_i}{1+r} < 1$$

which holds by assumption.

g) *Suppose now that the cost of monitoring a project is c, independent of the scale. Calculate the maximum cost that an entrepreneur i is willing to pay. How does the maximum cost depend on* N_i *and* B_i*?*

With a cost c independent of the scale, the project scale in the optimal contract is

$$I + c - N_i = \frac{p_H \tilde{\rho}_i}{1+r} I$$

or

$$I = \frac{N_i - c}{1 - \frac{p_H \tilde{\rho}_i}{1+r}}$$

The maximum an entrepreneur is willing to pay for monitor as long as

$$p_H \frac{b_i}{1+r} \frac{N_i - c}{1 - \frac{p_H \tilde{\rho}_i}{1+r}} \geq p_H \frac{B_i}{1+r} \frac{N_i}{1 - \frac{p_H \rho_i}{1+r}}$$

or

$$c \leq (1-\phi) \frac{\frac{p_H (R - \rho_i)}{1+r}}{1 - \frac{p_H \rho_i}{1+r}} N_i \equiv \tilde{c}_i^{\max}$$

which is increasing in N_i and B_i.

h) *Given your answer to g), under what conditions are entrepreneurs more likely to pay for monitoring? Explain the intuition.*

Es with a higher private B_i and higher net worth are more likely to pay. The reason is the following:

1. The higher the B_i, the higher the benefit of monitoring (remember that the benefit from monitoring is proportional to B_i), thus, the higher the benefit of paying c
2. The higher N_i, the larger the scale of the project. Since the cost of monitoring doesn't scale, the benefits of monitoring increase with size.