University of California, Davis
Department of Economics

PRELIMINARY EXAMINATION FOR THE Ph.D. DEGREE

MACROECONOMICS

June 27, 2022

Directions: The exam consists of six questions. Questions 1,2 concern ECN 200D (Geromichalos), questions 3,4 concern ECN 200E (Cloyne), and questions 5,6 concern ECN 200F (Caramp). You only need to answer five out of the six questions. If you prefer (and have time), you can answer all six questions, and your grade will be based upon the best five scores. Feel free to impose additional structure on the problems below, but please state your assumptions clearly. You have 5 hours to complete the exam and an additional 20 minutes of reading time.

Question 1 (*20 points*)

Consider the standard neoclassical growth model in discrete time. There is a large number of identical households normalized to 1. Each household wants to maximize life-time discounted utility

$$U(\{c_t\}_{t=0}^{\infty}) = \sum_{t=0}^{\infty} \beta^t u(c_t), \;\; \beta \in (0,1).$$

Each household has an initial capital k_0 at time 0, and one unit of productive time in each period that can be devoted to work. Final output is produced using capital and labor, according to a CRS production function F. This technology is owned by firms (whose measure does not really matter because of the CRS assumption). Output can be consumed (c_t) or invested (i_t). Households own the capital (so they make the investment decision), and they rent it out to firms. Let $\delta \in (0,1)$ denote the depreciation rate of capital. Households own the firms, i.e., they are claimants to the firms' profits, but these profits will be zero in equilibrium.

The function u is twice continuously differentiable and bounded, with $u'(c) > 0$, $u''(c) < 0$, $u'(0) = \infty$, and $u'(\infty) = 0$. Regarding the production technology, we will introduce the useful function $f(x) \equiv F(x,1) + (1-\delta)x$, $\forall x \in \mathbb{R}_+$. The function f is twice continuously differentiable with $f'(x) > 0$, $f''(x) < 0$, $f(0) = 0$, $f'(0) = \infty$, and $f'(\infty) = 1 - \delta$.

In this model the government taxes households' **investment** at the constant rate $\tau \in [0,1]$. The government returns all the tax revenues, T, to the households in the form of lump-sum transfers. Throughout this question focus on recursive competitive equilibrium (RCE).

a) Write down the problem of the household recursively.[1] Carefully distinguish between aggregate and individual state variables. Then, define a RCE.

b) Write down the dynamic equation that the aggregate capital stock follows in this economy.[2]

c) Now focus on steady-states. Describe the steady-state equilibrium value of the aggregate capital stock in this economy, and denote it by $K^*(\tau)$.

d) Describe the value of K^* when $\tau = 0$ and when $\tau = 1$.

[1] Here firms face a static problem. I am not asking you to explicitly spell it out, but it will be critical for correctly defining a RCE.

[2] **Hint:** Obtain the Euler equation for the typical household and impose the RCE conditions. I recommend you express the equilibrium condition(s) in terms of the function f, rather than F. This will make life easier in the forthcoming parts.

e) In class, we studied the RCE steady state level of capital in an economy where the government taxed the income from renting capital (as opposed to investment, which is the case here). In that model, we saw that for $\tau = 1$ the equilibrium capital stock reached zero. Based on your answer to part (d), does this also happen here? Provide an intuitive explanation of why (or why not).

f) Focus on a Cobb-Douglas production function, i.e., let $F(K, N) = K^a N^{1-a}$, $a \in (0, 1)$. Provide a closed-form solution for $K^*(\tau)$.

g) Again using a Cobb-Douglas production function, calculate the government's total tax revenue, T, and plot it as a function of the tax rate τ (i.e., plot the Laffer curve). Which value of τ maximizes tax revenues?

Question 2 (*20 points*)

Consider the search-theoretic monetary model discussed in class. Time is discrete with an infinite horizon. Each period consists of two subperiods. In the day, trade is bilateral and anonymous as in Kiyotaki and Wright (1993) (call this the KW market). At night trade takes place in a Walrasian or centralized market (call this the CM). There are two types of agents, buyers and sellers, and the measure of both is normalized to 1. The per period utility for buyers is $u(q)+U(X)-H$, and for sellers it is $-q+U(X)-H$, where q is the quantity of the day good produced by the seller and consumed by the buyer, X is consumption of the night good (the numeraire), and H is hours worked in the CM. In the CM, all agents have access to a technology that turns one unit of work into a unit of good. The functions u, U satisfy the usual assumptions; I will only spell out the most crucial ones: There exists $X^* \in (0, \infty)$ such that $U'(X^*) = 1$, and we define the first-best quantity traded in the KW market as $q^* \equiv \{q : u'(q^*) = 1\}$.

Compared to the baseline model, there is one important difference: money is not the unique medium of exchange. Alongside money, there are also government bonds that can serve as means of payment in the decentralized (KW) market. These are one-period nominal bonds that agents can purchase in the CM of period t, and they will pay out 1 dollar in the CM of period $t+1$. To capture the (realistic) idea that bonds may not be as liquid as money, we will assume that a buyer who enters the KW market with (m, b) units of money and bonds, respectively, can use all of her money but only a fraction $\lambda \in [0, 1]$ of her bonds as means of payment. Thus, λ is a measure of the relative liquidity (sometimes also called "pledgeability") of bonds.

Let $\sigma \leq 1$ denote the probability with which a buyer meets a seller in the KW market. To simplify things, we assume that buyers make take-it-or-leave-it offers to sellers. The rest is standard. Goods are non storable. The supply of money is controlled by a monetary authority and evolves according to $M_{t+1} = (1+\mu)M_t$. New money is introduced, or withdrawn if $\mu < 0$, via lump-sum transfers to buyers in the CM.

a) Describe the value function of a buyer who enters the CM with a portfolio (m, b).

b) Describe the terms of trade in a typical KW meeting.

c) Describe the objective function of the typical buyer, $J(m', b')$. Obtain the first-order conditions that characterize the buyer's optimal money and bond holdings.

d) Describe the steady-state equilibrium value of the quantity of special good traded in the KW market, q.

e) Describe the interest rate on government bonds, i_b, as a function of the interest rate i that we derived through the Fisher equation. (So that i is the interest rate on a hypothetical perfectly illiquid bond; in class we referred to i as the 'Fisher rate'.)

f) How does i_b depend on the degree of bond liquidity λ? What happens to i_b in the extreme cases where λ is 0 or 1, and what is the intuition behind these results?

g) Take as given that in this economy the welfare can be sufficiently summarized by the function $\mathcal{W} = \sigma[u(q) - q]$. How does the equilibrium welfare depend on the rate i? How about the degree of bond liquidity λ?

Question 3 (*20 points*)

Consider the social planner's problem for a simple real business cycle model. The household values consumption (C) and leisure ($1-N$, where N is hours worked) and lifetime utility is given by:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\ln C_t - \Theta N_t\right) \tag{1}$$

Output, Y, is produced using capital K and labor (hours) N

$$Y_t = A_t K_t^{\alpha} N_t^{1-\alpha} \tag{2}$$

A_t is a TFP shock and is governed by a discrete state Markov chain. Capital evolves according to the following law of motion:

$$K_{t+1} = (1-\delta)K_t + S_t I_t \tag{3}$$

where I is investment. Note that one unit of investment does not necessarily translate into one unit of capital because of the term S_t. Assume that S_t is stochastic and S_t is also governed by a discrete state Markov chain. Let's refer to this as an "investment shock" since it affects the marginal efficiency of investment. Initially assume $0 < \delta \leq 1$.

There is no trend growth. Finally,

$$Y_t = C_t + I_t$$

a) Write down the recursive formulation of the planner's problem.

b) Derive all the first order conditions.

c) Assume full depreciation so $\delta = 1$. Using guess and verify, find the policy functions for consumption C_t, hours N_t, investment I_t and capital tomorrow K_{t+1} (**Hint:** as usual, start by guessing that consumption is a constant share of output. Also note that S_t complicates the implied guess for the policy function for K_{t+1}).

d) Compare the business cycle properties implied by the TFP shock, A_t, and the investment shock S_t. Explain the economic intuition for these results.

e) If $\delta < 1$ briefly explain how you would solve this model computationally using value function iteration. Give one advantage of this method.

Question 4 (*20 points*)

In recent years there has been much discussion about the role of firm market power in macroeconomics. This question considers the economic consequences of a temporary (but persistent) rise in market power in the New Keynesian model.

There are a continuum of identical households and the representative household makes consumption (C) and labor supply (N) decisions to maximize lifetime expected utility. In linearized form, the household's Euler equation is:

$$E_t\hat{c}_{t+1} - \hat{c}_t = \frac{1}{\sigma}(\hat{i}_t - E_t\hat{\pi}_{t+1}) \tag{4}$$

and their labor supply condition is given by:

$$\hat{w}_t = \sigma\hat{c}_t + \psi\hat{n}_t \tag{5}$$

The production structure of the model is similar to the standard New Keynesian environment. Monopolistically competitive intermediate goods firms produce an intermediate good of variety j using labor. Final goods firms purchase intermediate goods and transform them into a composite final good using a CES production function:

$$Y_t = \left(\int_0^1 y_t(j)^{\frac{\epsilon_t - 1}{\epsilon_t}} dj\right)^{\frac{\epsilon_t}{\epsilon_t - 1}} \tag{6}$$

The main difference from the baseline model is that there is now exogenous time variation in the degree of market power. Specifically, let's capture this by allowing the elasticity of substitution between varieties, ϵ_t, to be time varying. Recall, in the standard model the steady state price markup would be constant and equal to $\frac{\epsilon}{\epsilon-1}$. In this model, we are now allowing for some exogenous variation in this markup over time. Let's refer to this as a "markup shock", denoted by $\mu_t = \frac{\epsilon_t}{\epsilon_t - 1}$.

In linearized form, the equilibrium conditions for firms are:

$$\hat{y}_t = \hat{n}_t \tag{7}$$

$$\hat{w}_t = \hat{mc}_t \tag{8}$$

$$\hat{\pi}_t = \beta E_t(\hat{\pi}_{t+1}) + \lambda\hat{mc}_t + \lambda\hat{\mu}_t \tag{9}$$

The resource constraint is:

$$\hat{y}_t = \hat{c}_t \tag{10}$$

Monetary policy follows a simple Taylor Rule:

$$\hat{i}_t = \phi_\pi\hat{\pi}_t \tag{11}$$

The markup shock in percentage deviations from steady state, $\hat{\mu}_t$, evolves:

$$\hat{\mu}_t = \rho\hat{\mu}_{t-1} + e_t \tag{12}$$

where e_t is i.i.d. and $0 < \rho < 1$. In percentage deviations from steady state: $\hat{mc}_t$ is real marginal cost, $\hat{\mu}_t$ is the markup shock, $\hat{c}_t$ is consumption, $\hat{w}_t$ is the real wage, $\hat{n}_t$ is hours worked, $\hat{y}_t$ is output. In deviations from steady state: $\hat{i}_t$ is the nominal interest rate, $\hat{\pi}_t$ is inflation. λ is a function of model parameters, including the degree of price stickiness.[3] Assume that $\phi_\pi > 1$, $\sigma > 0$ and $\psi > 0$.

a) The efficient level of output (in percentage deviations from steady state) is defined as the level of output that would occur under flexible prices and with no time-varying distortions, i.e. when $\hat{\mu}_t = 0$. Show that the efficient level of output (in percentage deviations from steady state) is equal to zero:

$$\hat{y}_t^e = 0 \tag{13}$$

b) Now define the (welfare relevant) output gap, $\hat{x}_t$, as the deviation of output, $\hat{y}_t$, from the efficient level of output $\hat{y}_t^e$. Show that this model can be written in terms of three equations:

$$E_t\hat{x}_{t+1} - \hat{x}_t = \frac{1}{\sigma}(\hat{i}_t - E_t\hat{\pi}_{t+1}) \tag{14}$$

$$\hat{\pi}_t = \beta E_t(\hat{\pi}_{t+1}) + \lambda(\sigma + \psi)\hat{x}_t + \lambda\hat{\mu}_t \tag{15}$$

$$\hat{i}_t = \phi_\pi\hat{\pi}_t \tag{16}$$

together with the stochastic process for $\hat{\mu}_t$. (**Hint:** Note that since $\hat{y}_t^e = 0$, this also implies $\hat{x}_t = \hat{y}_t$ in this model).

c) Using the method of undetermined coefficients, find the response of the output gap, $\hat{x}_t$, and inflation, $\hat{\pi}_t$, to an exogenous increase in $\hat{\mu}_t$. To do this, guess that the solution for each variable is a linear function of the shock $\hat{\mu}_t$.

d) Find the solution for the response of nominal interest rates, $\hat{i}_t$, in this model. Explain the economic intuition for the response of inflation, output and interest rates to a positive markup shock in this model.

e) Now suppose that, instead of following the simple Taylor Rule above, we choose $\hat{x}_t$ and $\hat{\pi}_t$ to maximize welfare under discretionary policy. As usual, the period losses are given by $\frac{1}{2}(\hat{\pi}_t^2 + \vartheta\hat{x}_t^2)$ and assume the steady state is efficient. Is it possible to fully stabilize inflation and the output gap ($\hat{x}_t$) with optimal policy? Explain. You do not necessarily need to derive anything, answer using your economic intuition.

[3] $\lambda = \frac{(1-\theta)(1-\beta\theta)}{\theta}$ where θ is the probability that a firm cannot adjust its price and $0 \leq \theta < 1$.

Question 5 (*20 points*)

In this exercise, we will reconsider the problem of insurance and liquidity we studied in class. In particular, we will study whether a higher supply of liquid assets (e.g., government bonds) crowds in or crowds out private investment.

Consider an economy that lasts three periods, $t \in \{0, 1, 2\}$. There is a single good that can be used for investment and consumption. As usual, assume that the good is perishable, that is, it cannot be stored between periods. The economy is populated by two types of agents: entrepreneurs and financiers. For simplicity, assume that there is a unit mass of agents of each type. All agents in the economy have preferences that can be represented by $\mathcal{U} = c_0 + c_1 + c_2$.

Financiers are the agents with the "funds." They have a **large** endowment each period so that the equilibrium risk-free interest rate in the economy is equal to zero. Entrepreneurs are the agents with the "ideas." In period 0, entrepreneurs have a net worth of $N > 0$ and have access to a flexible scale investment opportunity. In particular, if they invest I units of the good, they get a project of size I. In period 1, a liquidity shock s is realized. The liquidity shock can take two values: $s_H > s_L > 0$. Let $\lambda \equiv Prob(s = s_H)$ and $\overline{s} \equiv \mathbb{E}_0[s]$. Entrepreneurs have to pay s units of the good for each unit of the project they choose to continue. That is, if the entrepreneur wants to continue a scale $i \in [0, I]$ of the project, she has to pay si units of the good in period 1. Finally, in period 2, the project pays off R units of the good per unit of the project remaining, that is, Ri. Importantly, the entrepreneurs face a limit to how much they can borrow. We denote by ρ the pledgeable output per unit invested. That is, if the entrepreneur has i units of the project in period 2, it can only commit to repaying a total of ρi to financiers.

We make the following assumptions: *i*) The project satisfies: $R > 1 + \overline{s}$; *ii*) The liquidity shock satisfies: $s_L < \rho < s_H < R$ and $(1 - \lambda)(R - s_L) > 1.$; *iii*) The pledgeable output satisfies: $(1 - \lambda)(\rho - s_L) < 1$.

Finally, we assume that there is a supply $L_S > 0$ of a fully pledgeable asset (outside liquidity), which pays off one unit of the good in period 1 per unit of the asset.

a) Briefly explain in words the implications of assumptions *i*) − *iii*).

b) Assume that the financiers' endowment is not pledgeable. Argue that if $L_S = 0$, the equilibrium is such that the entrpereneurs continue full scale if $s = s_L$ but abandon the project if $s = s_H$, that is $i(s_L) = I$ and $i(s_H) = 0$.

c) Suppose that $L_S > 0$ and $(1 - \lambda)(s_H - s_L) < 1$. Let q denote the price of the asset in period 0. Argue that the optimal contract solves:

$$\max_{I, z \geq 0, x \in [0,1]} (R - \rho)(1 - \lambda + \lambda x) I$$

subject to

$$(1-\lambda)\left[(\rho - s_L) I + z\right] + \lambda\left[z - x\left(s_H - \rho\right) I\right] \geq I + zq - N$$
$$xI\left(s_H - \rho\right) \leq z,$$

where z denotes the units of the asset purchased by the entrepreneur.

d) Show that in the optimal contract

$$x(q) = \begin{cases} 1 & \text{if } q \in [1, q^{\max}) \\ \in [0,1] & \text{if } q = q^{\max} \\ 0 & \text{if } q > q^{\max} \end{cases}$$

with $q^{\max} = 1 + \frac{\lambda}{1-\lambda}\frac{1-(1-\lambda)(s_H - s_L)}{s_H - \rho}$. Why can't we have $q < 1$ in equilibrium?

e) Given $L_S > 0$, let's compute the equilibrium price q. In particular, show that there exists $\underline{L}_S > 0$ such that

$$L_S > \underline{L}_S \implies q < q^{\max}$$

and

$$L_S < \underline{L}_S \implies q = q^{\max}.$$

Show that if $L_S > \underline{L}_S$, the level of investment I is increasing in L_S. **Hint:** Using the market clearing condition for the asset, first find conditions for an equilibrium with $q < q^{\max}$. Then study what happens if the conditions are not satisfied.

f) Suppose $L_S < \underline{L}_S$. Show that I is decreasing in L_S. Explain why an increase in the supply of liquid assets can reduce the level of investment. **Hint:** Note that I is decreasing in x but xI is increasing in x.

Question 6 (*20 points*)

In this exercise, we will revisit the Diamond & Dybvig model under alternative assumptions about the withdrawal process and in the presence of aggregate risk.

Consider an economy that lasts for three periods, denoted by $t = 0, 1, 2$. The economy is populated by a continuum of measure one of consumers with preferences over consumption given by

$$U(c_1, c_2) = \mathbb{E}[u((1-\theta)c_1 + \theta c_2)]$$

where $\theta \in \{0, 1\}$ is an idiosyncratic shock realized at $t = 1$, and $u'(\cdot) > 0$, $u''(\cdot) < 0$ and satisfies Inada conditions. Note that when $\theta = 0$, the consumer values consumption only in period 1, and if $\theta = 1$, she values consumption only in period 2. In what follows, we will call the former the "impatient" consumer and the latter the "patient" consumer. Let $\pi \equiv \Pr(\theta = 0)$. An appropriate version of the Law of Large Numbers implies that π is also the fraction of impatient consumers in the population.

Each consumer has one unit of endowment of the final good in $t = 0$. There are two technologies available in the economy:

- **Storage:** transforms x units the final good in t into x units of the final good in $t+1$, for $t \in \{0, 1\}$.
- **Long-term investment:** transforms I units of the final good in $t = 0$ into RI units of the final good in $t = 2$, where R is a **random variable** with cumulative distribution $F(R)$ in support $[0, R^{\max}]$ and that satsifies $\mathbb{E}[R] > 1$. The realization of R is observed in period 1.

We assume that if the long-term investment is liquidated in $t = 1$ it pays zero.

We will consider the role of banks in this economy. Banks offer deposit contracts that specify a payoff if withdrawn in period 1 and a payoff if withdrawn in period 2. Given the deposits received in period 0, they decide how much to invest in the different technologies. Let L_0 denote the investment in the storage technology in period 0, $1-L_0$ the investment in the long-term technology in period 0, and $L_1(R)$ the investment in the storage technology in period 1. Note the dependence of the investment in the storage technology in period 1 on R. We will denote the promises to the households in periods 1 and 2 by $c_1(R)$ and $c_2(R)$. An important difference relative to the model from class is that the bank does not service withdrawals on a *first-come, first-served* basis but by distributing the available funds equally among all households withdrawing (more on this below).

a) To warm up, suppose R is not stochastic. Solve the planner's problem. In particular, show that if the utility function satsifies $-\frac{u''(c)c}{u'(c)} > 1$, then $R > c_2^{FB} >$

$c_1^{FB} > 1$. **Hint:** *i)* Note that when R is not stochastic, it is optimal to set $L_1 = 0$; *ii)* Recall that for a differentiable function $f(x)$ we have $f(R) = f(1) + \int_1^R f'(c)dc$.

b) Now, consider the bank described above (but R is still not stochastic). Let x denote the fraction of patient households that withdraw in period 1. Show that if the bank promises $c_1 = \frac{\pi c_1^{FB}}{\pi+(1-\pi)x}$ and $c_2 = \frac{(1-\pi)c_2^{FB}}{(1-\pi)(1-x)}$, there is a unique equilibrium in the economy. In particular, show that patient agents never withdraw in period 1, that is, $x = 0$. **Hint:** First show that the deposit contract is feasible. Then, show that patient households never withdraw early.

c) Now, back to a stochastic R. Argue that the planner's problem is given by

$$\max_{L_0, L_1(R), X, c_1(R), c_2(R) \geq 0} \mathbb{E}[\pi u(c_1(R)) + (1-\pi)u(c_2(R))]$$

subject to

$$\pi c_1(R) + L_1(R) \leq L_0$$
$$(1-\pi)c_2(R) \leq L_1(R) + R(1-L_0).$$

d) Consider the economy in period 1, given L_0. Let $\overline{R}(L_0) \equiv \frac{1-\pi}{\pi}\frac{L_0}{1-L_0}$. Show that the optimal consumption levels are

$$c_1(R) = \begin{cases} L_0 + (1-L_0)R & \text{if } R < \overline{R}(L_0) \\ \frac{L_0}{\pi} & \text{if } R \geq \overline{R}(L_0), \end{cases} \quad \text{and} \quad c_2(R) = \begin{cases} L_0 + (1-L_0)R & \text{if } R < \overline{R}(L_0) \\ \frac{(1-L_0)R}{1-\pi} & \text{if } R \geq \overline{R}(L_0). \end{cases}$$

e) Consider the problem in period 0. Show that the optimal investment in the short-term technology solves

$$\int_{\overline{R}(L_0)}^{R^{\max}} u'\left(\frac{L_0}{\pi}\right) dF(R) = \int_{\overline{R}(L_0)}^{R^{\max}} u'\left(\frac{R(1-L_0)}{1-\pi}\right) dF(R).$$

Denote the solution L_0^*.

f) Consider the following deposit contract. Let $x(R)$ denote the fraction of patient households that withdraw in period 1. The bank offers

$$c_1(R) = \frac{L_0}{\pi + (1-\pi)x(R)}, \quad c_2 = \frac{R(1-L_0)}{(1-\pi)(1-x(R))}.$$

Show that if $L_0 = L_0^*$, then the resulting allocations are the solution to the planner's problem. **Hint:** Note that if $R < \overline{R}(L_0^*)$, $x(R) \neq 0$ since the patient households might prefer to withdraw early. Use an indifference condition to back out the equilibrium $x(R)$ and then plug into the expressions for $c_1(R)$ and $c_2(R)$ to obtain the equilibrium quantitities.