MACROECONOMICS PRELIM, JUNE 2022

ANSWER KEY FOR QUESTIONS 1 AND 2 (ECN 200D)

Question 1

a) Since getting the budget constraint right here is very important let's write that separately first. The household has an income which it can allocate to either c or i. However, as I clearly implied in the hint, for any extra unit that goes into i the household's income (or resources) will go down by τ. Hence, we have

$$c + i = w + rk - \tau i + T,$$

which can be re-written as

$$c + i(1+\tau) = w + rk + T,$$

and since we know that $k' = (1-\delta)k + i$ (or, if you prefer $i = k' - (1-\delta)k$), the budget constraint becomes

$$c = w + [r + (1-\delta)(1+\tau)]k - (1+\tau)k' + T.$$

Hence, the typical household's problem can be written recursively as

$$V(k, K) = \max_{c,k'} \Big\{ u(c) + \beta V(k', K') \Big\}$$

$$s.t.\ \ c = w + [r + (1-\delta)(1+\tau)]k - (1+\tau)k' + T, \qquad (1)$$

$$K' = H(K), \qquad (2)$$

$$w = w(K) = F_2(K, 1), \qquad (3)$$

$$r = r(K) = F_1(K, 1), \qquad (4)$$

$$T = T(K) = \tau[H(K) - (1-\delta)K], \qquad (5)$$

where k is the individual capital, and K is the aggregate capital. Moreover, (1) is the household's budget constraint, (2) is the aggregate law of motion of capital, (3) and (4) follow directly from market clearing, and (5) is the government revenue as a function of the aggregate state.

The definition of a RCE is standard, and can be found in the lecture notes (or in problem 4 of PS 6). In the definition of RCE, the most important part is to clarify that consistency requires $g(K, K) = H(K)$, where H was defined above, and g is the typical household's policy function.

b) The Euler equation for the typical household is given by

$$u'(c)(1+\tau) = \beta [F_1(K', 1) + (1-\delta)(1+\tau)]\, u'(c').$$

But the objective here was to express everything as a function of the aggregate capital stock only. To that end, notice that we can use the budget constraint to write consumption in a more useful form. First impose consistency (i.e., $k = K$) to obtain

$$c = w + [r + (1-\delta)(1+\tau)]K - (1+\tau)K' + T.$$

Next, replace the prices and the total revenue T with the terms that include the aggregate capital stock (i.e., $r = F_1(K,1)$, $w = F_2(K,1)$, and $T = \tau[K' - (1-\delta)K]$). We obtain:

$$\begin{aligned} c &= F_2(K,1) + [F_1(K,1) + (1-\delta)(1+\tau)]K - (1+\tau)K' + \tau[K' - (1-\delta)K] = \\ &= F_2(K,1) + F_1(K,1)K + (1-\delta)K - K'. \end{aligned}$$

Finally, exploiting Euler's Theorem and the definition of $f(K)$, we find that

$$c = f(K) - K'.$$

Using this expression back into the Euler equation, and noticing that $f'(K) = F_1(K,1) + 1 - \delta$, we obtain the second-order difference equation, which describes the law of motion of aggregate capital:

$$u'(f(K) - K')(1+\tau) = \beta[f'(K) + \tau(1-\delta)]\, u'(f(K') - K'').$$

c) In the steady-state equilibrium $c = c'$ and $K = K' = K''$. Imposing this condition on the Euler equation, it is easy to show that the steady state capital stock is given by:

$$K^*(\tau) \equiv \left\{ K : f'(K) = \frac{1 + \tau[1 - \beta(1-\delta)]}{\beta} \right\}. \tag{6}$$

d) When $\tau = 0$, it is easy to check that $K^*(0)$ solves $f'(K^*(0)) = 1/\beta$, which, of course, is exactly the same as the steady-state level of capital in the baseline model with no government and taxes (or, if you prefer, the same as in the social planner's problem). If $\tau = 1$, we have $f'(K^*) = [2 - \beta(1-\delta)]/\beta$.

e) No. As we just saw, for $\tau = 1$ we have $f'(K^*) = [2 - \beta(1-\delta)]/\beta$, which means that $K^*(1) > 0$. The reason why a proportional tax (equal to 100%) on capital income and on investment do not lead to the same result is simple. The tax on capital income is a tax on ALL the household's capital. The tax on investment is just a tax on new capital that comes to replace the depreciated one, so clearly it is not as severe.

f) With $F(K,N) = K^a N^{1-a}$, we have $f(K) = K^a + (1-\delta)K$ and $f'(K) = aK^{a-1} + 1 - \delta$. Using these functional forms in (6), we get

$$K^*(\tau) = \left[\frac{a\beta}{(1+\tau)[1 - \beta(1-\delta)]} \right]^{\frac{1}{1-a}}. \tag{7}$$

g) In the steady state, $T = T(K^*) = \tau\delta K^*$, i.e., investment is just enough to replace the depreciated capital. Replacing for K^* from (7), yields

$$T = \tau\delta K^*(\tau) = B\tau(1+\tau)^{\frac{1}{a-1}}, \tag{8}$$

where we have defined

$$B \equiv \delta\left[\frac{a\beta}{1-\beta(1-\delta)}\right]^{\frac{1}{1-a}},$$

a positive constant. It is easy to show that

$$\frac{\partial T}{\partial \tau} = B\frac{1}{(1+\tau)^{\frac{1}{1-a}+1}}\frac{1-a(1+\tau)}{1-a}.$$

.Both B and the first fraction in this expression are positive. Thus, the sign of $\partial T/\partial\tau$ will coincide with the sign of the numerator of the second fraction. We have two cases:

- Case 1: If $a \leq 1/2$, then $\partial T/\partial\tau > 0$ for all $\tau \in [0,1]$, and the maximization of T has a "corner solution", i.e., $\tau = 1$.

- Case 2: If $a > 1/2$, then T obtains an interior maximum at

$$\tau^* = \frac{1-a}{a}.$$

Question 2

a) The VF of the typical buyer is given by

$$W(m,b) = \max_{X,H,m',b'} \Big\{ U(X) - H + \beta V(m',b') \Big\}$$

subject to

$$X + \phi m + \phi p b' = H = T + \phi(m+b).$$

Notice that here I have defined p as the **nominal** price of bonds. That is not the only way to do it, but it is the easiest one.

b) Suppose the buyer enters the KW market with m, b units of money and bonds. If $\phi(m+\lambda b) \geq q^*$, then we know that $q = q^*$ and the buyer will spend an amount of money and bonds, say d_m, d_b, just enough to buyer her the first best, i.e., $\phi(d_m + \lambda d_b) = q^*$.

On the other hand, if $\phi(m+\lambda b) < q^*$, the buyer will give up all her money and bonds ($d_m = m, d_b = b$), and will purchase the amount $q = \phi(m + \lambda b)$, which, of course, is less than the first best.

c) After some work, or a careful conjecture, we can see that the objective function is given by:

$$J(m',b') = \phi m' - \phi p b' + \beta\sigma \left[u(\phi'(m' + \lambda b')) + \phi'(1-\lambda)b' \right] + \beta(1-\sigma)\phi'(m'+b').$$

The first tow terms are the cots of purchasing these two assets today. Then, tomorrow, with probability σ the buyer will meet a seller and will use all her money, and only a fraction λ of her bonds to purchase goods (as discussed in class, here we assume that we are in the binding region of the bargaining solution, i.e., the agent never brings more liquid assets than she needs to purchase q^*). With , probability $1-\sigma$ the buyer will not meet a seller and she will get to keep all her money and all her bonds.

The first order conditions are as follows.

With respect to money:

$$\phi = \beta\phi'(1-\sigma) + \beta\sigma\phi' u'(\phi'(m' + \lambda b'))$$

With respect to bonds:

$$\phi p = \beta\phi'(1 - \sigma + \sigma(1-\lambda)) + \beta\sigma\phi'\lambda u'(\phi'(m' + \lambda b'))$$

d) For the equilibrium q we just need to evaluate the FOC with respect to money at steady state. Following the usual procedure, we get

$$i = \sigma[u'(q) - 1],$$

where i is the Fisher rate, or the interest rate on a (hypothetical) perfectly illiquid bond. Here in the steady state we have $\phi'(m' + \lambda b') = q$ (the amount purchased in a typical KW meeting).

e) To obtain i_b as a function of i, we need to use the FOC with respect to bonds and combine it with our earlier result in part (d). Start by dividing the FOC with respect to bonds by $\beta\phi'$. After some some algebra, we obtain

$$\frac{\phi p}{\beta \phi'} - 1 = \sigma\lambda[u'(\phi'(m' + \lambda b')) - 1].$$

Now, we will do three things to simplify this expression.

1) Focus on steady state so that $\phi'(m' + \lambda b') = q$ (the amount purchased in a typical DM meeting).

2) We will use the standard trick of multiplying the numerator of the LHS by $(1 + \mu)M$, and the denominator by M' (these two things are equal). This will allow us to replace the term $\frac{\phi}{\phi'}$ with $1 + \mu$.

3) We will use the standard relationship between the price and the interest rate of an asset (in this case the partially liquid bond) to write $p = 1/(1 + i_b)$.

With these three facts, we can write the last expression as

$$\frac{1+\mu}{\beta} \frac{1}{1+i_b} - 1 = \sigma\lambda[u'(q) - 1].$$

Last step, use the standard Fisher equation (for an illiquid asset) to replace $(1 + \mu)/\beta$ with $1 + i$, do a little algebra and arrive at

$$\frac{i - i_b}{1 + i_b} = \sigma\lambda[u'(q) - 1].$$

The last step is to replace $\sigma[u'(q) - 1]$ with i (since the two are equal from part d). Doing this, and after some algebra, gives us the final result:

$$i_b = \frac{i(1-\lambda)}{1+\lambda i}$$

f) It is easy to show that

$$\frac{di_b}{d\lambda} = \frac{-i(1+i)^2}{(1+\lambda i)^2},$$

which is negative. Why is this intuitive? A higher λ means the bonds are more liquid, hence, more desirable, hence more expensive. Thus, a higher λ should increase the price, and, by default, decrease the interest rate on liquid bonds.

At the two extremes, when $\lambda = 0$, the bonds are completely illiquid, and their interest rate should be the Fisher rate (which, by definition is the rate on a perfectly illiquid bond). If $\lambda = 1$, bonds are as liquid as money, and they should have the same interest rate as money, i.e., zero.

g) It is easy to show that

$$\frac{dW}{di} = \sigma[u'(q) - 1]\,\frac{dq}{di},$$

and since

$$\frac{dq}{di} = \frac{1}{\sigma u''(q)} < 0,$$

welfare is decreasing in the Fisher rate.

Similarly,

$$\frac{dW}{d\lambda} = \sigma[u'(q) - 1]\,\frac{dq}{d\lambda},$$

and we can show that

$$\frac{dq}{d\lambda} = \frac{-[u'(q) - 1]}{\lambda u''(q)} > 0,$$

which implies that welfare is increasing in λ. Again this is intuitive, since having more liquid bonds allows agents to carry out more transactions and should improve welfare. (Although there are some important counter-examples in the literature!)

Question 3 (*20 points*)

Consider the social planner's problem for a simple real business cycle model. The household values consumption (C) and leisure ($1 - N$, where N is hours worked) and lifetime utility is given by:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\ln C_t - \Theta N_t\right) \tag{1}$$

Output, Y, is produced using capital K and labor (hours) N

$$Y_t = A_t K_t^{\alpha} N_t^{1-\alpha} \tag{2}$$

A_t is a TFP shock and is governed by a discrete state Markov chain. Capital evolves according to the following law of motion:

$$K_{t+1} = (1-\delta)K_t + S_t I_t \tag{3}$$

where I is investment. Note that one unit of investment does not necessarily translate into one unit of capital because of the term S_t. Assume that S_t is stochastic and S_t is also governed by a discrete state Markov chain. Let's refer to this as an "investment shock" since it affects the marginal efficiency of investment. Initially assume $0 < \delta \leq 1$.

There is no trend growth. Finally,

$$Y_t = C_t + I_t$$

a) Write down the recursive formulation of the planner's problem.

Answer:

$$V(K_t, A_t, S_t) = \max_{K_{t+1}, C_t, N_t} \Bigg\{ \quad \ln(C_t) - \Theta N_t + \beta E_t V(K_{t+1}, A_{t+1}, S_{t+1}) \tag{4}$$

$$\lambda_t \left(A_t K_t^{\alpha} N_t^{1-\alpha} - C_t - \frac{K_{t+1}}{S_t} + (1-\delta)\frac{K_t}{S_t} \right) \Bigg\} \tag{5}$$

b) Derive all the first order conditions.

Answer: FOCs:

$$[K_{t+1}] \quad -\frac{\lambda_t}{S_t} + \beta E_t \frac{\partial V(K_{t+1}, A_{t+1}, S_{t+1})}{\partial K_{t+1}} = 0 \tag{6}$$

$$[N_t] \quad \Theta = \frac{(1-\alpha)Y_t}{N_t}\lambda_t \tag{7}$$

$$[C_t] \quad \lambda_t = \frac{1}{C_t} \tag{8}$$

$$\frac{\partial V(K_t, A_t, S_t)}{\partial K_t} = \lambda_t \left(\alpha Y_t / K_t + \frac{1-\delta}{S_t} \right) \tag{9}$$

Which implies

$$\frac{1}{C_t S_t} = \beta E_t \left[\frac{\alpha Y_{t+1}/K_{t+1} + \frac{1-\delta}{S_{t+1}}}{C_{t+1}} \right] \tag{10}$$

c) Assume full depreciation so $\delta = 1$. Using guess and verify, find the policy functions for consumption C_t, hours N_t, investment I_t and capital tomorrow K_{t+1} (**Hint:** as usual, start by guessing that consumption is a constant share of output. Also note that S_t complicates the implied guess for the policy function for K_{t+1}).

Answer

Let's use the guess $C_t = (1-B)Y_t$.

First, inspect the first order condition for hours worked and use the first order condition for consumption. The labor supply condition can be written as

$$N_t = \frac{(1-\alpha)Y_t}{\Theta C_t} = \frac{(1-\alpha)Y_t}{\Theta(1-B)Y_t} = \frac{(1-\alpha)}{\Theta(1-B)}$$

Hours worked are constant.

Note that the guess implies

$$I_t = BY_t$$

from the resource constraint and

$$K_{t+1} = S_t B Y_t$$

from the capital law of motion.

Substituting these conditions into the consumption Euler equation and solving yields:

$$B = \beta\alpha$$

This leads to:

$$\bar{N} = \frac{(1-\alpha)}{\Theta(1-\alpha\beta)}$$

$$C_t = (1-\alpha\beta)A_t K_t^{\alpha} \bar{N}^{1-\alpha}$$
$$I_t = \alpha\beta A_t K_t^{\alpha} \bar{N}^{1-\alpha}$$
$$K_{t+1} = \alpha\beta S_t A_t K_t^{\alpha} \bar{N}^{1-\alpha}$$

Output, consumption and investment are affected by S_t in the next period, but do not respond to S_t shocks on impact. To see this you can use the policy function for capital again:

$$Y_t = A_t \left(\alpha\beta S_{t-1} Y_{t-1}\right)^{\alpha} \bar{N}^{1-\alpha}$$
$$C_t = (1-\alpha\beta)A_t \left(\alpha\beta S_{t-1} Y_{t-1}\right)^{\alpha} \bar{N}^{1-\alpha}$$
$$I_t = \alpha\beta A_t \left(\alpha\beta S_{t-1} Y_{t-1}\right)^{\alpha} \bar{N}^{1-\alpha}$$

d) Compare the business cycle properties implied by the TFP shock, A_t, and the investment shock S_t. Explain the economic intuition for these results.

Key points

- TFP shocks lead to an increase in output, consumption and investment, as in the data. In this model, there is no movement in hours worked. Note that when $\delta = 1$, the income and substitution effects cancel out and labor supply is constant.
- For the TFP shock intuition, the answer should briefly touch on the main mechanisms outlined in the lecture notes, the answer keys to problem sets and the class exams.
- The investment shock makes investment more productive, so one unit of investment (savings) generates a larger amount of capital. This generates an incentive to accumulate more capital. That said, with $\delta = 1$ there is a constant investment rate, $\alpha\beta$. Still, for a given amount of savings/investment, more capital is created (if $\delta < 1$ investment would increase at the expense of consumption, and labor supply would rise). In the *next period* this leads to more production. Because $\delta = 1$ implies a constant investment rate, higher GDP translates into higher consumption and investment. Unlike the TFP shock, the investment shock with $\delta = 1$ cannot generate contemporaneous co-movement of output, consumption and investment. With a one period lag, however, it can generate these co-movements.

e) If $\delta < 1$ briefly explain how you would solve this model computationally using value function iteration. Give one advantage of this method.

Sketch Answer

Provide a brief summary of the steps we studied in class using Matlab: setting up the problem recursively, discretizing the state space (for the state variables (K, A, S)), centering things around the deterministic steady state, choosing functional forms and calibrating the parameters model. Set up the value function as a vector of optimal utility values for each realization of the state variables (e.g. each (K, A, S)). Make an initial guess of the value function e.g. all zeros. Plug this into the Bellman equation and find a new value function. Check convergence. Once we have the value function we can find the optimal choices given the states today, this calculation gives us the policy functions for the control variables. Advantages include: we can find the global non-linear solution, so we can handle model non-linearities. The method is also very reliable and relatively general conditions ensure convergence.

Question 4 (*20 points*)

In recent years there has been much discussion about the role of firm market power in macroeconomics. This question considers the economic consequences of a temporary (but persistent) rise in market power in the New Keynesian model.

There are a continuum of identical households and the representative household makes consumption (C) and labor supply (N) decisions to maximize lifetime expected utility. In linearized form, the household's Euler equation is:

$$E_t\hat{c}_{t+1} - \hat{c}_t = \frac{1}{\sigma}(\hat{i}_t - E_t\hat{\pi}_{t+1}) \tag{11}$$

and their labor supply condition is given by:

$$\hat{w}_t = \sigma\hat{c}_t + \psi\hat{n}_t \tag{12}$$

The production structure of the model is similar to the standard New Keynesian environment. Monopolistically competitive intermediate goods firms produce an intermediate good of variety j using labor. Final goods firms purchase intermediate goods and transform them into a composite final good using a CES production function:

$$Y_t = \left(\int_0^1 y_t(j)^{\frac{\epsilon_t-1}{\epsilon_t}}dj\right)^{\frac{\epsilon_t}{\epsilon_t-1}} \tag{13}$$

The main difference from the baseline model is that there is now exogenous time variation in the degree of market power. Specifically, let's capture this by allowing the elasticity of substitution between varieties, ϵ_t, to be time varying. Recall, in the standard model the steady state price markup would be constant and equal to $\frac{\epsilon}{\epsilon-1}$. In this model, we are now allowing for some exogenous variation in this markup over time. Let's refer to this as a "markup shock", denoted by $\mu_t = \frac{\epsilon_t}{\epsilon_t-1}$.

In linearized form, the equilibrium conditions for firms are:

$$\hat{y}_t = \hat{n}_t \tag{14}$$

$$\hat{w}_t = \hat{mc}_t \tag{15}$$

$$\hat{\pi}_t = \beta E_t(\hat{\pi}_{t+1}) + \lambda\hat{mc}_t + \lambda\hat{\mu}_t \tag{16}$$

The resource constraint is:

$$\hat{y}_t = \hat{c}_t \tag{17}$$

Monetary policy follows a simple Taylor Rule:

$$\hat{i}_t = \phi_\pi\hat{\pi}_t \tag{18}$$

The markup shock in percentage deviations from steady state, $\hat{\mu}_t$, evolves:

$$\hat{\mu}_t = \rho\hat{\mu}_{t-1} + e_t \tag{19}$$

where e_t is i.i.d. and $0 < \rho < 1$. In percentage deviations from steady state: $\hat{mc}_t$ is real marginal cost, $\hat{\mu}_t$ is the markup shock, $\hat{c}_t$ is consumption, $\hat{w}_t$ is the real wage, $\hat{n}_t$ is hours worked, $\hat{y}_t$ is output. In deviations from steady state: $\hat{i}_t$ is the nominal interest rate, $\hat{\pi}_t$ is inflation. λ is a function of model parameters, including the degree of price stickiness.[1] Assume that $\phi_\pi > 1$, $\sigma > 0$ and $\psi > 0$.

a) The efficient level of output (in percentage deviations from steady state) is defined as the level of output that would occur under flexible prices and with no time-varying distortions, i.e. when $\hat{\mu}_t = 0$. Show that the efficient level of output (in percentage deviations from steady state) is equal to zero:

$$\hat{y}_t^e = 0 \tag{20}$$

Answer

The key is to note that the efficient level of output occurs under flexible prices and no distortionary shocks. This means $\hat{mc}_t = 0$ and $\hat{\mu}_t = 0$. Using these assumptions, $\hat{w}_t = 0$. Also use the production function and the resource constraint in the labor supply condition:

$$0 = \sigma\hat{y}_t^e + \psi\hat{y}_t^e$$

which implies:

$$\hat{y}_t^e = 0$$

b) Now define the (welfare relevant) output gap, $\hat{x}_t$, as the deviation of output, $\hat{y}_t$, from the efficient level of output $\hat{y}_t^e$. Show that this model can be written in terms of three equations:

$$E_t\hat{x}_{t+1} - \hat{x}_t = \frac{1}{\sigma}(\hat{i}_t - E_t\hat{\pi}_{t+1}) \tag{21}$$

$$\hat{\pi}_t = \beta E_t(\hat{\pi}_{t+1}) + \lambda(\sigma + \psi)\hat{x}_t + \lambda\hat{\mu}_t \tag{22}$$

$$\hat{i}_t = \phi_\pi\hat{\pi}_t \tag{23}$$

together with the stochastic process for $\hat{\mu}_t$. (**Hint:** Note that since $\hat{y}_t^e = 0$, this also implies $\hat{x}_t = \hat{y}_t$ in this model).

Answer

[1] $\lambda = \frac{(1-\theta)(1-\beta\theta)}{\theta}$ where θ is the probability that a firm cannot adjust its price and $0 \leq \theta < 1$.

Substitute the resource constraint, equation 17, into the Euler equation, equation 11, and make use of the result that $\hat{x}_t = \hat{y}_t$. This produces the first equation above.

Next, equate the labor supply equation, equation 12, with equation 15 and use equation 17 and equation 14. This produces:

$$\hat{mc}_t = (\sigma + \psi)\hat{y}_t$$

Finally note that $\hat{x}_t = \hat{y}_t$. Substitute this back into the Phillips Curve, equation 16. This produces the second equation above. The third equation is already the Taylor Rule given.

c) Using the method of undetermined coefficients, find the response of the output gap, $\hat{x}_t$, and inflation to an exogenous increase in $\hat{\mu}_t$. To do this, guess that the solution for each variable is a linear function of the shock $\hat{\mu}_t$.

Answer

Note that $\lambda\hat{\mu}_t$ looks just like a standard cost-push shock, what we called u_t in the lectures. Let's guess that the solution is of the form:

$$\hat{x}_t = \Lambda_x \hat{\mu}_t$$
$$\hat{\pi}_t = \Lambda_\pi \hat{\mu}_t$$

The steps are exactly the same as in the technology shock example in problem set 8. We substitute the Taylor Rule into the IS curve (equation 21) and substitute the guesses into the IS curve and the New Keynesian Phillips Curve (equation 22). We also make use of the stochastic process for $\hat{\mu}_t$.

This yields the following results for the unknown coefficients:

$$\Lambda_\pi = \lambda \frac{(1-\rho)\sigma}{\sigma(1-\rho)(1-\beta\rho) + \kappa(\phi_\pi - \rho)} > 0 \tag{24}$$

$$\Lambda_x = \lambda \frac{\rho - \phi_\pi}{\sigma(1-\rho)(1-\beta\rho) + \kappa(\phi_\pi - \rho)} < 0 \tag{25}$$

d) Find the solution for the response of interest rates in this model. Explain the economic intuition for the response of inflation, output and interest rates to a positive markup shock in this model.

Answer

Substitute the solution for inflation into the policy rule:

$$\hat{i}_t = \phi_\pi \lambda \frac{(1-\rho)\sigma}{\sigma(1-\rho)(1-\beta\rho) + \kappa(\phi_\pi - \rho)} > 0$$

Interest rates have to rise to control inflation. The welfare relevant output gap, $\hat{x}_t$ falls, whereas inflation rises.

The intuition can be summarized as follows. The markup shock raises the degree of monopolistic distortions. Firms would like to increase price and lower output. If prices were flexible, the natural rate of output would fall (and fall below the efficient level). With sticky prices, some firms raise price and inflation rises. Firms that can't raise price won't see output fall by as much as would be the case under flexible prices. Actual output is therefore above the natural rate of output (this is the source of the trade-off because this concept of the output gap is actually positive, so if the policymaker tried to set $\hat{x}_t$ to zero the inflation outcome would be even worse). The central bank has to raise interest rates to deal with higher inflation, as dictated by the policy rule. Higher nominal interest rates will lead to higher real interest rates which discourage consumption and lowers demand further. This disinflationary effect partially offsets the inflationary forces. With a very aggressive policy response, the effect on inflation could be offset, and output would fall by more. This, of course, would be accompanied by a much more negative value for $\hat{x}_t$.

e) Now suppose that, instead of following the simple Taylor Rule above, we choose $\hat{x}_t$ and $\hat{\pi}_t$ to maximize welfare under discretionary policy. As usual, the period losses are given by $\hat{\pi}_t^2 + \vartheta \hat{x}_t^2$ and assume the steady state is efficient. Is it possible to fully stabilize inflation and the output gap ($\hat{x}_t$) with optimal policy? Explain. You do not necessarily need to derive anything, answer using your economic intuition.

Answer

Discretionary policy means you can only choose outcomes for output and inflation today. Notice that the loss function would be minimized each period if $\hat{x}_t = \hat{\pi}_t = 0$. But when $\hat{\mu}_t > 0$ the Phillips Curve shows that $\hat{x}_t = 0$ and $\hat{\mu}_t > 0$ is inconsistent with $\hat{\pi}_t = 0$. This markup shock prevents the policymaker from simultaneously stabilizing the output gap and inflation. There is a "trade-off". The policymaker dislikes deviations in both variables, so the optimal policy will generate a balanced outcome with $\hat{x}_t < 0$ and $\hat{\pi}_t > 0$, where the precise weights depend on ϑ (and the other parameters of the model). More detail can be found in the slides for Topic 10.