University of California, Davis
Department of Economics

PRELIMINARY EXAMINATION FOR THE Ph.D. DEGREE

MACROECONOMICS

July 7, 2023

Directions: The exam consists of six questions. Questions 1,2 concern ECN 200D (Geromichalos), questions 3,4 concern ECN 200E (Cloyne), and questions 5,6 concern ECN 200F (Caramp). You only need to answer five out of the six questions. If you prefer (and have time), you can answer all six questions, and your grade will be based upon the best five scores. Feel free to impose additional structure on the problems below, but please state your assumptions clearly. You have 5 hours to complete the exam and an additional 20 minutes of reading time.

Question 1 (*20 points*)

Consider a neoclassical growth model in discrete time. There is a large number of identical households normalized to 1. Each household wants to maximize life-time discounted utility

$$U(\{c_t\}_{t=0}^{\infty}) = \sum_{t=0}^{\infty} \beta^t u(c_t), \;\; \beta \in (0,1),$$

where u is twice continuously differentiable and bounded, with $u'(c) > 0$, $u''(c) < 0$, $u'(0) = \infty$, and $u'(\infty) = 0$. As is standard, households own the capital stock (they make the investment decision), and they rent it to firms. Each household has one unit of productive time in each period, and they rent these "labor services" to firms.

The main difference of this model compared to the baseline model studied in class concerns production. There is a large number of identical firms each of which has the following production function:

$$F(k, l, \bar{k}) = Ak^a l^{1-a} \bar{k}^{\gamma},$$

where k, l are the amounts of capital and labor rented by the firm, and $\bar{k}$ is the aggregate capital stock in the whole wide economy. Assume that $a \in (0,1)$ and $\gamma \in (0, 1-a)$. Thus, here there is a *productive externality* from the rest of the economy: a higher aggregate capital stock increases each firm's productivity. Naturally, a typical (small) firm takes the aggregate capital stock as given when choosing its inputs.

The rest of the model is standard. Output can be consumed (c_t) or invested (i_t), and we will let $\delta \in (0,1)$ denote the depreciation rate of capital. Thus, **if we set** $\gamma = 0$ **this would be identical to the growth model studied in class.**

a) Define a recursive competitive equilibrium for this economy. Be clear about which variables households and firms take as given when they solve their optimization problems. Find the competitive equilibrium steady-state aggregate capital stock as a function of the parameters of the model.

b) Write the social planner's problem for this economy in recursive form, assuming that the social planner internalizes the externality in production.[1] Characterize the steady-state aggregate capital stock implied by the planner's problem. Discuss whether that capital stock is higher or lower than the one obtained in the competitive equilibrium (in part (a)) and provide economic intuition for your result.

[1] That is, in the social planner's problem, the production function is

$$F(\bar{k}, l, \bar{k}) = A\bar{k}^{a+\gamma} l^{1-a}.$$

c) Now introduce a government into the competitive equilibrium that you defined in part (a). The government subsidizes investment expenditures at a proportional rate τ and finances these subsidies by imposing a lump-sum tax T on each household. (Assume that the government's budget has to be balanced in every period.) Define a recursive competitive equilibrium for this economy.

d) For what subsidy rate τ is the competitive equilibrium steady-state aggregate capital stock the same as the steady-state aggregate capital stock in the social planner's problem?

Question 2 (*20 points*)

Consider the Mortensen-Pissarides model in continuous time. Labor force is normalized to 1, but, unlike the baseline model seen in class, here workers do not live forever. All workers, *regardless of the state they are in*, retire at Poisson rate δ and exit the labor market. Every retired worker gets replaced by a young worker, so that the steady state mass of workers is constant at 1. New workers who just entered the labor market must enter as unemployed and have no previous work experience. Let us think of them as workers who just graduated and refer to them as "type-0" workers. Once a type-0 worker finds her first job, she becomes an employed worker of type-0. After that worker *loses her first job* she returns to unemployment, but she is thereafter a type-1 worker. (Because she has held at least one job in the past.) Thus, at any point in time, workers can be of type-0 or type-1, and they can be employed or unemployed. Let u_i (e_i) denote the measure of unemployed (employed) workers of type $i = \{0, 1\}$.

The key assumption we will make here is that type-0 workers are less productive when employed. (Notice, again, that this is only the group of workers who are holding *their very first job.*) One justification could be that due to their complete lack of experience, these workers need to be trained on how to perform the tasks that are necessary for the job. As a result, while type-1 workers produce p units of the numeraire good (per unit of time) when employed, type-0 workers produce $p - \kappa$, $\kappa > 0$. And, as discussed above, one can think of κ as the training cost a firm must incur when they hire a worker with no working experience.

The rest of the model is standard. Let the total measure of unemployed workers be denoted by $u = u_0 + u_1$, and let the measure of vacancies be denoted by ν. Then, the matching technology is given by $m(u, \nu)$, where m is a CRS function increasing in both arguments. Define the market tightness $\theta \equiv \nu/u$. Notice that firms cannot discriminate between type-0 and type-1 workers (even though they would like to).

A large measure of firms decide whether to enter the labor market with exactly one vacancy. When a firm meets an unemployed worker a job is formed, and the output, which depends on the worker type, has already been discussed in detail. While the vacancy is unfilled, firms have to pay a search/recruiting cost c per unit of time. In an active match (job), the firm pays the worker a wage which is determined through Nash bargaining when the two parties first meet. Let β represent the worker's bargaining power. (Notice that the parameters c and β are unrelated to the worker's type.)

The destruction rate of existing jobs is given by the Poisson rate λ. (Again, this is independent of the match type.) Once this shock arrives, the firm shuts the job down. Subsequently, the worker moves to the pool of unemployed workers of type-1, and the firm exits the labor market. Unemployed workers of all types enjoy an unemployment benefit of z per unit of time. Throughout this question focus on steady state equilibria and denote the discount rate of agents by r.

a) Describe the two Beveridge curves (BC_0,BC_1) for this economy. More precisely, describe the variables u_0 and u_1 as functions of parameters and the market tightness θ, defined above.

b) Describe the value functions of workers in the various states.

c) Describe the value functions of firms in the various states. What does free entry imply here?

d) Describe the solution to the bargaining problem between a firm and a type-1 worker, and derive the wage curve for workers of type-1 (WC_1); this should be an equation that links the wage of workers of type-1, w_1, to the market tightness θ.

e) Describe the solution to the bargaining problem between a firm and a type-0 worker, and derive the wage curve for workers of type-0 (WC_0).[2] Provide intuition for this wage curve, in particular, the (unusual) fact that w_0 depends on w_1.

f) Shortly define a steady state equilibrium for this model. (Summarize how many equilibrium variables we have, which conditions they satisfy, and list the basic steps that you would follow to solve for these variables. Your answer does not need to be more than 4-5 lines.)

g) Is it possible that w_0 could be negative in this model and why?

[2] You should find that w_0 depends on the term $U_1 - U_0$. Do some manipulations on the value functions described in part (b) to replace that term. Eventually, you should be left with an equilibrium condition that links w_0 with *two* other equilibrium variables: θ, which is typical in this model, and w_1, which is unusual but quite intuitive.

Question 3 (*20 points*)

This question examines two different potential drivers of the business cycle — shocks to productivity and shocks to the supply of labor. Consider the following decentralized business cycle model. There are a continuum of identical households and the representative household maximizes lifetime expected utility:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\ln(C_t) - \frac{1}{\gamma_t} N_t \right) \tag{1}$$

subject to their budget constraint:

$$C_t + I_t = w_t N_t + r_t^k K_t + \Pi_t \tag{2}$$

where w_t is the real wage, N_t is hours worked, I_t is investment, r_t^k is the rental price of capital and Π_t are profits from firms. As usual, K_t is the capital stock available at time t (i.e. the closing stock of capital at the end of $t-1$). γ_t affects the dis-utility of supplying labor and will be discussed below. Assume $0 < \beta < 1$.

Capital is related to investment I as follows:

$$K_{t+1} = (1-\delta)K_t + I_t \tag{3}$$

but assume full depreciation so $\delta = 1$.

There are a continuum of identical firms who produce under perfect competition. Firms produce output, Y_t, using capital and labor. The production function is given by:

$$Y_t = A_t K_t^{\alpha} (N_t)^{1-\alpha} \tag{4}$$

where $0 < \alpha < 1$. A_t is total factor productivity.

This model features two exogenous shocks. The first is a shock that affects the dis-utility of supplying labor, γ_t. The second is a shock to productivity, A_t. Both γ_t and A_t are stochastic and follow a stationary Markov process. There is no trend growth.

a) Write down the household's problem in recursive form and write down the firm's maximization problem. Derive the household's first order conditions and the firm's optimal hiring rules.

b) Carefully define a recursive competitive equilibrium.

c) Using your answers to parts (a) and (b) show that the first order conditions for this decentralized economy coincide with the equilibrium conditions from the social planner's problem. Explain why this is the case. In the course of your answer, make sure you show that firms make zero profit and that the aggregate resource constraint is satisfied. (**Hints:** you do not need to set up the social planner's problem for this question. The goal is to combine your results from parts (a) and (b) to find the equivalent equilibrium conditions).

d) Using guess and verify, find the solved policy functions for output, consumption, investment and hours worked. Guess that investment is a constant share of output, and that hours worked are a constant share of γ_t.

e) Compare and contrast the business cycle properties implied by these two shocks. How well does each type of shock account for the business cycle dynamics we see in the data? Explain. In the course of your answer, make sure you discuss how, and why, these two shocks affect output, consumption, investment and hours worked in this model. (The model is very stylized. You can therefore focus on the general predictions rather than precise quantitative magnitudes.).

Question 4 (*20 points*)

This question uses the New Keynesian model to examine whether higher labor income taxes could be used to reduce inflation, as sometimes suggested by policymakers. For simplicity, let's assume the economy starts in steady state and then consider a surprise, temporary, increase in taxes.

There are a continuum of identical households and the representative household's (period) utility function is:

$$\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\psi}}{1+\psi} \tag{5}$$

In nominal terms, the household's budget constraint is:

$$Q_t B_{t+1} + P_t C_t = B_t + (1-\tau_t) P_t w_t N_t + \Pi_t + T_t \tag{6}$$

C_t is consumption, N_t is hours worked, w_t is the real wage, τ_t is the distortionary labor income tax rate, Π_t are firm profits distributed lump sum, T_t are lump-sum transfers from the government. B_t are bonds in period t (i.e. the closing stock of bonds in $t-1$). P_t is the aggregate price level. Q_t is the bond price. $\sigma > 0$ and $\psi > 0$.

Aside from the introduction of taxes, this model is the same as we saw in class.

In linearized form, the household's Euler equation and labor supply conditions are:

$$E_t \hat{c}_{t+1} - \hat{c}_t = \frac{1}{\sigma}(\hat{i}_t - E_t \hat{\pi}_{t+1}) \tag{7}$$

$$\hat{w}_t = \sigma \hat{c}_t + \psi \hat{n}_t + \hat{\tau}_t \tag{8}$$

The linearized equilibrium conditions for firms are:

$$\hat{y}_t = \hat{n}_t \tag{9}$$

$$\hat{w}_t = \hat{mc}_t \tag{10}$$

$$\hat{\pi}_t = \beta E_t(\hat{\pi}_{t+1}) + \lambda \hat{mc}_t \tag{11}$$

The resource constraint is:

$$\hat{y}_t = \hat{c}_t \tag{12}$$

Monetary policy follows a simple Taylor Rule:

$$\hat{i}_t = \phi_\pi \hat{\pi}_t \tag{13}$$

The (linearized) labor income tax rate follows a Markov process:

$$\hat{\tau}_t = \rho \hat{\tau}_{t-1} + e_t \tag{14}$$

where $0 < \rho < 1$ and e_t is an i.i.d. disturbance. Tax revenues generated by the income tax are redistributed lump-sum to households via T. The government does not issue any debt and there is no government spending in this model.

In percentage deviations from steady state: $\hat{mc}_t$ is real marginal cost, $\hat{c}_t$ is consumption, $\hat{w}_t$ is the real wage, $\hat{n}_t$ is hours worked, $\hat{y}_t$ is output. In deviations from steady state: $\hat{i}_t$ is the nominal interest rate, $\hat{\pi}_t$ is inflation and $\hat{\tau}_t$ is the labor income tax rate. λ is a function of model parameters, including the degree of price stickiness.[3] Assume that $\phi_\pi > 1$ and β is the household's discount factor, where $0 < \beta < 1$.

a) Using the relevant linearized equations above, show that, under flexible prices, the natural rate of output $\hat{y}_t^n$ (in % deviations from steady state) in this model depends on the labor income tax rate. In particular, show that:

$$\hat{y}_t^n = -\frac{1}{\sigma + \psi}\hat{\tau}_t \tag{15}$$

Provide some economic intuition for this result.

b) Now assume prices are sticky. Show that the Phillips Curve can be written as:

$$\hat{\pi}_t = \beta E_t(\hat{\pi}_{t+1}) + \lambda(\sigma + \psi)\hat{x}_t + \lambda\hat{\tau}_t \tag{16}$$

where $\hat{x}_t$ is a measure of the output gap. In this question, define the output gap as $\hat{x}_t = \hat{y}_t - \hat{y}_t^e$, where $\hat{y}_t^e$ is the *efficient* level of output (in % deviations from steady state). The efficient level of output is defined as the level of output that would be obtained under flexible prices and in an economy without distortions. Note that, under flexible prices, the labor income tax is the only time-varying distortion in this model. You can therefore assume that $\hat{y}_t^e = 0$, $\forall t$.

c) Using the method of undetermined coefficients, find the response of the output gap, $\hat{x}_t$, and inflation, $\hat{\pi}_t$, to an exogenous increase in the labor income tax rate when prices are sticky and monetary policy follows the Taylor Rule above. To do this, guess that the solution for each variable is a linear function of the shock $\hat{\tau}_t$. (**Hint:** you will need to rewrite the consumption Euler equation in terms of the output gap $\hat{x}_t$).

d) If policymakers increase the tax rate $\hat{\tau}_t$, will this reduce inflation? Explain how and why an increase in the labor income tax affects the output gap $\hat{x}_t$ and inflation $\hat{\pi}_t$ in this model.

e) Find the response of the nominal interest rate. How does monetary policy respond to a tax increase in this model? Do you find this surprising? Explain.

[3] $\lambda = \frac{(1-\theta)(1-\beta\theta)}{\theta}$ where θ is the probability that a firm cannot adjust its price and $0 \le \theta < 1$.

Question 5 (*20 points*)

Consider the fixed-investment model with two alternative projects: the two projects have the same investment cost I and the same payoffs, R in the case of success and 0 in the case of failure. The entrepreneur has initial wealth $N < I$ and must collect $I - N$ from risk-neutral financiers who demand a rate of return equal to 0. Project 1 has probability of success p_H if the entrepreneur works and $p_L = p_H - \Delta p$ if she shirks. Similarly, the probabilities of success are q_H and $q_L = q_H - \Delta q$ for project 2, where

$$\Delta q = \Delta p.$$

Project 1 (respectively, 2) delivers private benefit B (respectively, b) when the entrepreneur shirks; no private benefit accrues in either project if the entrepreneur works. We make the following assumptions:

1. project 1 has a higher probability of success

$$p_H > q_H,$$

2. both projects have positive NPV if the entrepreneur works, but negative if she shirks

$$p_H R > q_H R > I > p_L R + B > q_L R + b$$

3. pledgeable income is higher for project 2 but lower than the cost of investment

$$p_H \left(R - \frac{B}{\Delta p} \right) < q_H \left(R - \frac{b}{\Delta q} \right) < I.$$

Assume that *at most one project* can be implemented (because, say, the entrepreneur has limited attention), and (except in question d)) that the financiers can verify which project, if any, is implemented.

a) Suppose effort is observable. Show that only project 1 is financed, independently of N.

b) From now on, effort is not observable. Characterize the financing problem between the entrepreneur and the financiers. In particular, clearly state: i) the resource constraint; ii) the financiers' participation constraint; iii) the entrepreneur's incentive compatibility constraints.

c) Divide the set of possible net worths N, $[0, I)$, into three regions, $[0, \overline{N}_q)$, $[\overline{N}_q, \overline{N}_p)$, and $[\overline{N}_p, I)$ and show that the equilibrium investment policies in these regions are "not invest," "invest in project 2," and "invest in project 1." Find the expressions for $\overline{N}_q$ and $\overline{N}_p$. Why is project 2 sometimes financed?

d) In this question *only*, suppose that the financiers cannot verify which project the entrepreneur is choosing (they only observe success/failure). Argue that nothing is altered if $N \geq \overline{N}_p$. Formally show that if $N \in [\overline{N}_q, \overline{N}_p)$, the entrepreneur doesn't get financing.

e) Suppose now that the private benefit of shirking on project 1 can be reduced from B to b by using a monitoring technology. This technology has an implementation cost of c. Assume that

$$p_H R - c > q_H R > I$$

and

$$p_H \left(R - \frac{b}{\Delta p} \right) < I + c$$

Show that monitoring is useful if and only if

$$c < p_H \frac{B - b}{\Delta p}$$

Is there any level of N and c such that project 2 is implemented?

Question 6 (*20 points*)

We will study a way of detecting financial constraints through the behavior of cash holdings.

There are 3 periods $t = 0, 1, 2$. The firms has some initial installed capital that produces a cash flow process $\{c_0, c_1\}_{t\in\{0,1\}}$. The firm has the option to invest in a long-term project I_0 in $t = 0$ that pays off $F(I_0)$ at time 2. Additionally, the firm expects to have another investment opportunity at time $t = 1$. If the firm invests at time 1 I_1, the technology produces $G(I_1)$ at time $t = 2$. You can think of this problem as if the firm has access to three production technologies: 1) installed capital produces $\{c_t\}_{t\in\{0,1\}}$, 2) investment in $t = 0$ produces $F(I_0)$ in $t = 2$ if she invest I_0, 3) investment in $t = 1$ produces $G(I_1)$ in $t = 2$ if she invests I_1. We make the usual assumptions on F, G: they are increasing, concave, continuously differentiable, and marginal product when $I \to 0$ is infinity.

Assume $\beta = 1$ so that the equilibrium interest rate is zero (the economy is populated by financiers with very large endowments). We will assume that the firm has access to:

- Long term debt B_0 at time $t = 0$. The firm can get B_0 at $t = 0$ and repay B_0 at $t = 2$
- Short term debt B_1 at time $t = 1$. The firm can get B_1 at time $t = 1$ and repay B_1 at $t = 2$.
- Cash C at time $t = 0$. The firm saves in cash in $t = 0$ and uses it in $t = 1$.
- The firm **cannot** issue equity, this means that dividends need to be weakly positive.

Investment can be liquidated at the final date $t = 2$ generating a payoff equal to $q(I_0 + I_1)$ where $q \leq 1$ and $I_0, I_1 > 0$. You can think of q as the (exogenous) resale price of capital in the final period of the firm's life. Denote the total cash flows from investment by $f(I_0) = F(I_0) + qI_0$ and $g(I_1) = G(I_1) + qI_1$. We assume that the cash flows $F(I_0)$ and $G(I_1)$ are non-verifiable and cannot be contracted upon. This means that the firm cannot pledge these funds to outside investors, but it can raise external finance by pledging the underlying productive assets as collateral. The liquidation value of assets that can be pledged to creditors is given by $(1 - \tau)qI$ where the parameter $\tau \in [0, 1)$ captures the quality of legal institutions that govern creditors' rights. For high enough τ firms might pass up on positive NPV projects due to financial constraints.

a) Let C denote the amount of cash the firm carries from period 0 to time 1, $B = (B_0, B_1)$ the debt policy, $I = (I_0, I_1)$ the investment policy. Let d_t denote the firm's dividend in period t, so that

$$d_0 = c_0 + B_0 - I_0 - C$$

$$d_1 = c_1 + B_1 - I_1 + C$$

$$d_2 = f(I_0) + g(I_1) - B_0 - B_1$$

State the firm's problem. **Hint:** Remember that dividends cannot be negative.

First, we will characterize the investment level chosen by unconstrained firms.

b) State the first-order conditions of the firm's problem. Characterize the level of investment chosen by an unconstrained firm, that is, the case in which the borrowing constraints in both periods are not binding. **Hint:** Note that d_2 is always strictly positive.

A firm is financially constrained if its investment policy is distorted from the unconstrained level because of capital market frictions. From now on, focus on the case where the borrowing constraints are binding in both periods.

c) Show that the optimal cash flow policy C^* is characterized by $f'\left(\frac{c_0 - C^*}{1-q+\tau q}\right) = g'\left(\frac{c_1 + C^*}{1-q+\tau q}\right)$. Explain the trade-offs involved in the decision to hold cash.

d) How much of its current cash flow c_0 will a constrained firm save? In other words, find an expression for $\frac{\partial C^*}{\partial c_0}$ and interpret this derivative. When will the firm save more? When will it save less?

f) Let $F(I) = A\ln(I)$ and $G(I) = B\ln(I)$. Derive an explicit formula for C^*. How does C^* depend on τ? Explain the intuition.

g) Consider the following empirical model.

$$\Delta CashHoldings_{i,t} = \alpha_0 + \alpha_1 Cashflow_{i,t} + X'_{i,t}\beta + \epsilon_{i,t}$$

where $X_{j,t}$ denote other firm-level characteristics. Through the lens of our model:

- Should we expect a systematic relationship between $Cashflow_{i,t}$ and $CashHoldings_{i,t}$ for unconstrained firms?
- Should we expect a systematic relationship between $Cashflow_{i,t}$ and $CashHoldings_{i,t}$ for constrained firms?
- Empirical estimates find α_1 significant and positive for constrained firms and α_1 insignificant and positive or negative depending on details of the empirical specification for unconstrained firms. Is this in line with our theory?