University of California, Davis
Department of Economics
Advanced Economic Theory

Date: July 14, 2006
Time: 3 hours
Reading Time: 20 minutes

# PRELIMINARY EXAMINATION FOR THE Ph.D. DEGREE

You need to answer four questions, at least one from each part.

## Part 1.

**1.** Assume that the qualification of job candidates is assessed in terms of three criteria measured in terms of real-valued scores. For the purposes of this question, a hypothetical candidate can be identified with a vector of scores, i.e. with an element in $\mathbb{R}^3$. *Of course, here and throughout the exam, you need to justify/explain your answers in appropriate detail.*

Let $\succ$ on $\mathbb{R}^3$ be defined by $x \succ y$ if and only if $x_1 + x_2 + x_3 > y_1 + y_2 + y_3$ and $x_i > y_i + 1$ for at least one $i \in \{1, 2, 3\}$.

i) Is $\succ$ acyclic?

ii) Is $\succ$ transitive?

iii) Consider $\mathcal{A}(., \succ)$ on the set of finite choice-sets $\mathcal{F}(\mathbb{R}^3)$. Which of the important choice-consistency conditions ($\alpha, \beta, \gamma, \eta$ or $\eta^*$) are satisfied by $\mathcal{A}(., \succ)$?.

iv) Determine the transitive hull of $\succ$.

**2.** (A simple model of Loss Aversion).

Let $\mathbb{R}$ be the set of outcomes interpreted as changes of wealth relative to the "status-quo" level of wealth 0.

i) Appealing to the vNM theorem, characterize the preferences of decision maker over the set of lotteries with a finite number of outcomes $\mathcal{L}$ who is an EU maximizer and has a vNM utility function $u : \mathbb{R} \to \mathbb{R}$ of the following type:

$$u(x) = x \text{ if } x \geq 0, \text{ and } u(x) = \lambda x \text{ if } x \leq 0, \text{ for some } \lambda > 1. \qquad (*)$$

ii) How can you identify the coefficient $\lambda$ from choice behavior?

iii) Consider the induced preferences over the set $\mathcal{L}_2$ of lotteries with at most one positive and and at most one negative outcome. Identify an interesting testable property of these preferences that distinguishes a loss-averse decision-maker represented by $(*)$ from a usual EU maximizer with a smooth, strictly concave vNM utility function.

iv) Building on i) and iii), *state* a characterization of preferences over $\mathcal{L}$ represented by $(*)$ that does not appeal to the Independence axiom or the vNM theorem; you do not need to prove the characterization to get full credit.

## Part 2

3. Consider a (real) exchange economy $\mathcal{E}(u, \omega, V)$ with two periods ($t = 0, 1$) and uncertainty described by $S$ states of nature at date 1. There is only one good. Agent $i$ ($i = 1, \ldots, I$) has initial endowment $\omega^i \in \mathcal{R}^{S+1}_{++}$ and a utility function $u^i$ continuous, strongly monotonic and strictly quasi-concave on $\mathcal{R}^{S+1}_{+}$, and such that indifference surfaces through strictly positive consumption bundles do not intersect the boundary of $\mathcal{R}^{S+1}_{+}$. Agents obtain consumption streams different from their endowments by trading $J$ financial securities on competitive markets: security $j$ has price $q_j$ at date zero and promises to deliver $V^j_s$ unit of the good if state $s$ occurs at date 1.

(a) Define a financial market equilibrium for this economy

(b) Assuming differentiability of the utility functions, write the first-order conditions that must be satisfied in order that the consumption-portfolio choice of agent $i$ be optimal at equilibrium, and interpret these first-order conditions.

(c) Using (b), prove that, if the rank of the matrix $V$ is equal to $S$, a financial market equilibrium allocation is Pareto optimal.

(d) Explain why, typically, if $J < S$, a financial market equilibrium allocation is not Pareto optimal.

(e) Explain how you can formalize mathematically and prove that "typically" a financial market equilibrium is not Pareto optimal when financial markets are incomplete. The sketch of a proof is sufficient.

4. Let $\mathcal{E}(u, \omega, N, M)$ be a one-good, two-period monetary economy in which the utility functions of the agents are

$$u^i(x^i) = a_0^i x_0^i + \sum_{s=1}^{S} \beta_s \left( \alpha^i x_s^i - \frac{1}{2}(x_s^i)^2 \right), \quad i = 1, \ldots, I$$

where $a_0^i > 0$, $\alpha^i > 0$, and where $\beta_s > 0$ is the probability that state $s$ ($s = 1, \ldots, S$) occurs at date 1 (which of course implies that $\sum_{s=1}^{S} \beta_s = 1$). The coefficient $\alpha^i$ is "large" when compared to the initial resources $\omega^i = (\omega_0^i, \omega_1^i, \ldots, \omega_S^i)$ of agent i, (for example $\alpha^i > \max\{\omega_0^i, \omega_1^i, \ldots, \omega_S^i\}$). The money price of the (only) good in state $s$ is denoted by $p_s$, $s = 0, \ldots, S$. There are two nominal securities traded at date 0: the first one is an indexed bond whose nominal payoff at date 1 is $N_s^1 = p_s/p_0$, $s = 1, \ldots, S$. The second security is the default-free nominal bond with payoff $N_s^2 = 1$. The prices $\hat{q}$ of the indexed bond and $q$ of the nominal bond define the *real* ($\hat{r}$) and the *nominal* ($r$) interest rates $\left( \hat{q} = \frac{1}{1+\hat{r}},\ q = \frac{1}{1+r} \right)$. Let $w_s$ denote the aggregate resources in state $s$, $s = 0, \ldots, S$ ($w_s = \sum_{i=1}^{I} \omega_s^i$) and let $w_{\mathbf{1}}$ denote the random variable $(w_1, \ldots, w_S)$.

(a) Write out agent $i$ 's maximum problem in an equilibrium of the economy $\mathcal{E}(u, \omega, N, M)$ and derive the first-order conditions for this maximum problem.

(b) Derive from these first-order conditions and from the market clearing condition for the indexed bond market that the equilibrium price $\hat{q}$ of the indexed bond satisfies

$$\hat{q} = \frac{1}{a_0}(\alpha - E(w_{\mathbf{1}}))$$

where $a_0 = \sum_{i=1}^{I} a_0^i$, $\alpha = \sum_{i=1}^{I} \alpha^i$.

(c) Explain intuitively the results of comparative statics that can be derived from the pricing relation derived in (b).

(d) Show that the equilibrium price of the nominal bond is given by

$$q = \frac{1}{1+\hat{r}} E\left( \frac{1}{1+i} \right) - \frac{1}{a_0} cov \left( w_{\mathbf{1}}, \frac{1}{1+i} \right)$$

where $1 + i = (1 + i_1, \ldots, 1 + i_S) = (p_1/p_0, \ldots p_S/p_0)$ is the (random) rate of inflation.

(e) Suppose that the aggregate resources do not fluctuate at date 1. What is the approximate relation between $r, \hat{r}$ and $E(i)$? (Use the approximation $\frac{1}{1+x} \approx 1 - x$ if $x$ is small).

(f) Suppose output (aggregate resources) and inflation are random (remember $(w_s, M_s)$ are exogenous data) and satisfy a Phillips curve relation, i.e. inflation and output are positively correlated. Use (d) to show that the nominal interest rate does not compensate for the full amount of expected inflation. Explain intuitively where this result comes from.

**Part 3**

5. This problem set is written in honor of Robert Aumann, Nobel laureate in economics in 2005. One of the nice game theoretic concepts due to Aumann is the notion of Correlated Equilibrium.

(a) Prove that for any 2x2 game, there does not exist a Correlated Equilibrium in which both players receive a strictly higher payoff than in any Nash equilibrium of the game.

(b) Show a counterexample for 2-player games with a larger action space.

(c) Show a counterexample for 2x2 games with non-common priors. I.e., show that there exists a 2x2 game in which both players' payoffs in a Subjective Correlated Equilibrium strictly exceeds the payoff in any Nash equilibrium.

6. In this problem set we stick to correlation. Let's have a look what a public randomization device can do in extensive form games. Consider the following game:

In stage 1, players $A$ and $B$ simultaneously choose $a \in [-1, 1]$ and $b \in \{L, R\}$ respectively. In state 2, players $C$ and $D$ are informed of the choices made by players $A$ and $B$ in stage 1. They then simultaneously choose $c \in \{L, R\}$ and $d \in \{L, R\}$ respectively and the game ends.

Player $D'$s payoff depends only on his own action and that of player $A$: if he chooses $L$ he gets a payoff of $-a$, and if he chooses $R$ he gets a payoff of $a$. Player $C$ likewise obtains $-a$ if he chooses $L$ and $a$ if he chooses $R$. Player $B$'s payoff depends on his own action and that of player $C$: if he chooses $L$ then he gets 1 if player $C$ chooses $L$ and $-1$ if $C$ chooses $R$; and if he chooses $R$ then he gets 2 if $C$ chooses $R$ and $-2$ if $C$ chooses $L$. Finally there are three contributions to player $A$'s payoff. First, if $B$ and $C$ make the same choice he gets $-|a|$, whereas if they make different choices he gets $|a|$. Secondly, if $C$ and $D$ make the same choice he gets 0, whereas if they make different choices he gets $-10$. Thirdly, he gets $-\frac{1}{2}|a|^2$. $A$'s overall payoff is the sum of these three contributions.

(a) In above game, does there exist a subgame perfect equilibrium? If yes, what is it? If not, argue why not.

(b) Consider following extension of above game: Suppose that at the outset of stage 2, $C$ and $D$ observe not only the actions $a$ and $b$ taken by $A$ and $B$ in stage 1 but also the realization of a public signal distributed uniformly on $[0, 1]$.

Does there exist a subgame perfect equilibrium? If yes, what is it? If not, argue why not. (Hint: If $a \neq 0$, let $C$ and $D$ ignore the public signal.)