University of California, Davis
Department of Economics
Microeconomics

Date: August 20, 2018
Time: 5 hours
Reading Time: 20 minutes

PRELIMINARY EXAMINATION FOR THE Ph.D. DEGREE

Please answer any three of the following four questions

[If you answer all four questions please indicate which three you want to be graded]

Question 1

Consider an economy with I consumers and L goods. For each consumer $i \in \{1, ..., I\}$, the consumption set is $\mathbb{R}^L_+$. Her utility function is given by

$$u^i(x^i) = -\sum_{k=1}^{L} \alpha_k e^{-\beta^i_k x^i_k},$$

where $\alpha_k, \beta^i_k > 0$ for $k \in \{1, ..., L\}$. As usual we denote $x^i = (x^i_1, ..., x^i_L)$. Consider price vectors $(p_1, ..., p_L)$ and wealth levels $(w^1, ..., w^I)$ for which the solution to the utility maximization problem is interior for every consumer $i \in \{1, ..., I\}$.

a.) Derive the Walrasian demand function for good j by consumer i. (Be careful with your calculations. Double-check! It is easy to make mistakes.)

b.) What is the slope of consumer i's Engel curve for good j at (p, w^i)?

c.) Find a condition as general as possible on parameters α_k, β^i_k, $i \in \{1, ..., I\}$, $k \in \{1, ..., L\}$ guaranteeing the existence of a positive representative consumer. Do we need restrictions on parameters α_k, $k \in \{1, ..., L\}$?

d.) Consider now the special case with just a single consumer and two goods. The consumer's utility function is given by

$$u(x_1, x_2) = -\alpha_1 e^{-\beta_1 x_1} - \alpha_2 e^{-\beta_2 x_2}.$$

Derive the wealth-expansion path for a given price vector (p_1, p_2).

e.) In problem d.), when does the wealth-expansion path intersect the x_1-axis and when does it intersect the x_2-axis?

Question 2

It is intuitive to think that the presence of more agents in the economy "shrinks" its core, since there are more coalitions that can object a given allocation. You will understand in this question why this is indeed the case.[1]

Fix a standard, two-person exchange economy $\mathcal{E} = \{(u^1, w^1), (u^2, w^2)\}$. Define its *replica* as the four-person exchange economy

$$\mathcal{E}^2 = \{(u^1, w^1), (u^2, w^2), (u^3, w^3), (u^4, w^4)\},$$

where $(u^3, w^3) = (u^1, w^1)$ and $(u^4, w^4) = (u^2, w^2)$.

1. Argue that if (p, x^1, x^2) is a competitive equilibrium for $\mathcal{E}$, then (p, x^1, x^2, x^3, x^4) with $x^3 = x^1$ and $x^4 = x^2$, is an equilibrium for $\mathcal{E}^2$.
2. Argue that if both utility functions are strictly quasi-concave, and (p, x^1, x^2, x^3, x^4) is a competitive equilibrium for $\mathcal{E}^2$, then, $x^1 = x^3$ and $x^2 = x^4$.
3. Argue that if both utility functions are strictly quasi-concave, and (x^1, x^2, x^3, x^4) is in the core of $\mathcal{E}^2$, then, $x^1 = x^3$ and $x^2 = x^4$.
4. Argue that if both utility functions are monotone and strictly quasi-concave, and (p, x^1, x^2) is a competitive equilibrium for $\mathcal{E}$, then (x^1, x^2, x^1, x^2) is in the core of $\mathcal{E}^2$.
5. Suppose that
$$u^1(x) = u^2(x) = x^1 x^2,$$
$w^1 = (1, 0)$ and $w^2 = (0, 1)$. Argue that allocation $((0, 0), (1, 1))$ is in the core of $\mathcal{E}$, yet allocation
$$((0, 0), (1, 1), (0, 0), (1, 1))$$
is *not* in the core of $\mathcal{E}^2$.
6. Use these results to argue, informally, that the replication of agents does not affect the set of equilibrium allocations of the economy but shrinks its core.[2]

[1] We are using the term "shrink" loosely, since the presence of more agents changes the dimension of the allocation space, so comparing the sizes of the cores will require some refinement of the argument.

[2] In the limit, one can show that replication *ad infinitum* reduces the core to just the set of equilibrium allocations.

QUESTION 3

You won \$200M (\$200,000,000) playing the lottery. Two charities have approached you for a donation. You summoned the directors of the two charities and informed them that they will play the following sequential game: First, the director of Charity 1 (call her D_1) will make a public request, which can be any integer dollar amount, starting from \$1 and up to \$200M. Then the director of Charity 2 (call him D_2) − having heard D_1's request − will announce his own request, which, again, can be any integer dollar amount starting from \$1 and up to \$200M.

You inform them that one of your objectives is to punish greed and thus you will proceed as follows: Let d_1 denote the amount requested by D_1 and d_2 the amount requested by D_2. If $d_1 > d_2$ then D_1 − being the greedy one − will get nothing, while D_2 will be rewarded with two times what he asked for, up to \$200M; that is, D_2 will get $\min\{2d_2, 200M\}$. Symmetrically, if $d_1 < d_2$ then D_2 will get nothing, while D_1 will get $\min\{2d_1, 200M\}$.

Finally, if $d_1 = d_2$ then each charity will get what it requested, up to \$100M, that is, each charity will get $\min\{d_1, 100M\}$.

- **(a)** Suppose that it is common knowledge that each director is selfish and greedy, in the sense that he/she only cares about how much money his/her own charity gets, and prefers more money to less. Find **all** the backward induction solutions.
- **(b)** Suppose now that it is common knowledge that each director ranks the outcomes exclusively on the basis of the **total** amount of money that goes to the two charities (the more the better). Find **all** the backward induction solutions.
- **(c)** Now let us change the game. It is no longer a sequential game; instead, each director submits a written request, not knowing how much money the other director is requesting (that is, the game is simultaneous). Assume that the preferences are as described in part (a).
  (c.1) Write the payoff functions of the two players.
  (c.2) Find **all** the pure-strategy Nash equilibria. Prove that what you propose are Nash equilibria and that there are no other Nash equilibria.
- **(d)** For the simultaneous game of part (c) find **all** the pure-strategy Nash equilibria when the preferences are as described in part (b). Prove that what you propose are Nash equilibria and that there are no other Nash equilibria.
- **(e)** For the simultaneous game of part (c) find **all** the pure-strategy Nash equilibria when each director has the following lexicographic preferences, that is, outcomes are ranked first on the basis of the total amount of money that goes to the two charities (the more the better) and then on the basis of how much money his/her own charity gets (the more the better). Prove that what you propose are Nash equilibria and that there are no other Nash equilibria.

QUESTION 4

Consider an individual whose von Neumann-Morgenstern utility-of-wealth function is

$$U(m) = \begin{cases} \sqrt{m} & \text{if she exerts no effort} \\ \sqrt{m} - c & \text{if she exerts effort} \end{cases} \quad \text{with } c > 0.$$

The individual has an initial wealth of W and faces a potential loss of ℓ. The probability of her incurring a loss is p_e if she exerts effort and p_n if she chooses no effort, with $0 < p_e < p_n < 1$.

(a) Draw a diagram where on the horizontal axis you measure wealth (W_1) in the "bad" state (where she suffers a loss) and on the vertical axis wealth (W_2) in the "good" state (where she does not suffer a loss); sketch the indifference curves that go through the no-insurance (*NI*) point (one corresponding to effort and the other to no effort).

For parts (b)-(e) assume that $W = 2{,}500$, $\ell = 1{,}600$, $p_e = \frac{1}{20}$, $p_n = \frac{1}{10}$.

(b) Calculate the slopes of the two curves of part (a) at the *NI* point.

(c) Suppose that $c = \frac{15}{16}$ and the individual decides **not** to insure. Will she exert effort or not?

(d) Suppose that $c = \frac{3}{2}$ and the individual decides **not** to insure. Will she exert effort or not?

In what follows, assume that (1) effort is **observable and verifiable,** (2) if indifferent between not insuring and insuring the individual will choose to insure and (3) $c = \frac{15}{16}$.

(e) Denote by *E* the contract given by the intersection of the 45° line and the indifference curve that goes through the no-insurance point (*NI*) corresponding to effort and *F* the contract given by the intersection of the 45° line and the indifference curve that goes through *NI* corresponding to no effort.

(e.1) Find the premium and deductible of contract *E*.

(e.2) Find the premium and deductible of contract *F*.

(e.3) Suppose that the insurance industry is a monopoly and the monopolist offers contract *E*, provided that the customer can prove that she chose effort (otherwise the contract will not be offered). What will its expected profits be?

(e.4) Suppose that the monopolist offers contract *F*, without any restrictions on effort (that is, the contract is offered no matter whether the customer chose effort or no effort). What will its expected profits be?

(e.5) Let *N* be the full-insurance contract that makes the consumer indifferent between (1) signing contract *N* and choosing no effort and (2) choosing *NI* and exerting effort. Show contract *N* in the wealth diagram and compute its premium and deductible.

(e.6) Draw the "reservation indifference manifold" of the consumer, that is, the set of points in the wealth diagram containing all the contracts that leave the consumer just as well off as she is if she does not insure.