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Microeconomics

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PRELIMINARY EXAMINATION FOR THE Ph.D. DEGREE

Answer four questions (out of five)

Question 1. Complements and substitutes.

A consumer consumes L *consumption goods*, with quantities denoted $(x_1, \ldots, x_L)$, and her preferences are represented by a twice differentiable, strictly quasiconcave and locally nonsatiated utility function $\tilde{u} : \Re^L_+ \to \Re : x \equiv (x_1, ..., x_L) \mapsto \tilde{u}(x)$. But these consumption goods are not sold in the market. Instead, consumption good j ($j = 1,\ldots, L$) is home-produced by using $N[j]$ *marketed goods*, with quantities denoted $(z_{1j},\ldots, z_{N[j], j})$, according to the Leontief technology

$$x_j = \min\left\{\frac{z_{1j}}{a_{1j}}, \frac{z_{2j}}{a_{2j}}, ..., \frac{z_{N[j],j}}{a_{N[j],j}}\right\},$$

where all denominators are positive. (The first subscript indicates the marketed good, and the second one the consumption good.) A double index (k, j) labels the kth marketed good in the list of marketed goods used in the production of consumption good j, $j = 1,\ldots, L$, $k = 1,\ldots, N[j]$. Hence, there are altogether $N \equiv \sum_{j=1}^{L} N[j]$ marketed goods, $N[j]$ of which are used in the home production of consumption good j. Denote by π_{kj} the price of the (k, j) marketed good, $j = 1,\ldots, L$, $k = 1,\ldots, N[j]$.

1(a). A vector $\pi \equiv (\pi_{11}, ..., \pi_{N[1],1}, \pi_{12}, ..., \pi_{N[2],2}, ..., \pi_{1L}, ..., \pi_{N[L],L})$ of prices of marketed goods induces, via the home production technology, a vector $p \equiv (p_1,\ldots, p_L)$ of (implicit) prices for the consumption goods. For $j = 1,\ldots, L$, define the function $\tilde{p}$ that expresses the price of consumption good j in terms of the prices of the marketed goods.

1(b). The *EMIN*$[p, u]$ *Problem* is defined by $\min_x \; p\bullet x$ subject to $\tilde{u}(x) \geq u$, with solution function the Hicksian demand function h for consumer goods. Show that $S(p, u)p = 0$, where $S(p, u)$ is the Slutsky matrix.

1(c). Define the *Hicksian demand for marketed good* (k, j) by

$$\xi_{kj} : \Re^N_{++} \times U \to \Re : \xi_{kj}(\pi, u) = a_{kj} h_j(\tilde{p}(\pi), u),$$

where U is the relevant domain of utility levels.

When can we say that marketed good (m, i) is a (net) complement of marketed good (k, j) at (π, u)?

When can we say that marketed good (m, i) is a (net) substitute of marketed good (k, j) at (π, u)?

1(d). Coffee and cream are popular textbook examples of complements. But Paul Samuelson suggested that, when a consumer uses cream both in her coffee and in her tea, cream and coffee may actually behave as substitutes. In order to analyze this somewhat paradoxical result, we specialize the previous model to the case of two home-produced consumer goods: coffee (good 1) and tea (good 2). Coffee requires coffee beans (marketed good (1, 1)) and cream for coffee (marketed good (2, 1)), whereas tea requires tea leaves (marketed good (1, 2)) and cream for tea (marketed good (2, 2)). (The prices of marketed goods (2, 1) and (2, 2) may well be the same, but this does not play any role here.)

1(d)(i). Is marketed good (1, 1) (coffee beans) a complement or a substitute for marketed good (2, 1) (cream for coffee)? Argue your answer.

1(d)(ii). Is marketed good (1, 1) a complement or a substitute for marketed good (2, 2) (cream for tea)? Argue your answer.

1(d)(iii). Define the demand for "cream" as the sum of the demand for marketed goods (2, 1) and (2, 2). What can you say about the complementarity or substitutability of cream and marketed good (1, 1)? Argue and discuss your answer.

Question 2. Capping emissions.

A firm that behaves in a perfectly competitive manner in all markets produces one output by using L-1 inputs according to a direct production function

$$f : \Re_{+}^{L-1} \to \Re : z \equiv (z_1, \ldots, z_{L-1}) \mapsto f(z) .$$

assumed to be differentiable with a strictly positive gradient on $\Re_{++}^{L-1}$ and concave.

Denote by $p > 0$ the price of the output, and by $w \equiv (w_1, \ldots, w_{L-1}) \in \Re_{++}^{L-1}$ the vector of input prices. Assume that the cost function $c : \Re_{++}^{L-1} \times \Re_{+} \to \Re : (w_1, \ldots, w_{L-1}; q) \mapsto c(w_1, \ldots, w_{L-1}; q)$ is differentiable and convex in q, and that at the profit maximizing solution the quantities of the inputs and of output are positive.

2(a). What is the relation between the output price and the marginal cost at a profit-maximizing solution? Prove your answer.

2(b). How does an increase in an input price affect the marginal cost at a profit-maximizing solution? Prove your answer.

2(c). In order to limit greenhouse gas emissions, the public authority imposes a fixed cap or quota k on the CO_2 emissions of the firm. All inputs may contribute to emissions: more precisely, the firm's emissions are a convex, differentiable function $\eta : \Re_{+}^{L-1} \to \Re : (z_1, \ldots, z_{L-1}) \mapsto \eta(z_1, \ldots, z_{L-1})$ with nonnegative partial derivatives.

2(c)(i). Is it necessarily true that the production of a larger amount of output requires emissions to increase? Discuss it in the simpler two-dimensional case.

2(c)(ii). Write the Kuhn-Tucker conditions of the profit-maximizing problem of the firm that faces an emission cap, and discuss the implications of the size of the emission cap on the Lagrange multiplier.

2(c)(iii). Compare the Kuhn-Tucker conditions of the profit-maximizing problem of the firm under the emissions constraint with those of the standard profit-maximizing problem (without the emissions constraint).

2(c)(iv). Argue that the profit-maximizing output of the firm cannot be higher under the emissions constraint than in the absence of such a constraint.

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Question 3. Consider a community of I agents in which each agent's utility function is given by

$$u_i(x_i, h_i, y) = \alpha \ln(x_i) + \beta \ln(h_i) + \gamma_i \ln(y), \qquad \alpha + \beta = 1$$

where x_i denote the amount of consumption good, h_i the number of units of living space and y the services provided to the community. The coefficients $\alpha > 0$ and $\beta > 0$ are the same for every agent but the coefficients $\gamma_i = \gamma(\omega_i) > 0$ are inversely related to income, i.e. the function γ is decreasing: rich agents value public services less than poorer agents do. Each agent has an endowment $(\omega_i, \bar{h}_i)$ in consumption good and housing. The agents are labeled in such a way that $i' < i \Longleftrightarrow \omega_{i'} \leq \omega_i$. Only the agents with high endowments in consumption good own houses: $\omega_i < \omega_I \Longleftrightarrow \bar{h}_i = 0$ and $\bar{h}_i$ is the same for all agents in the highest income group. Although γ_i decreases with income, the products $\gamma_i \omega_i$ increase with income: $\omega_{i'} < \omega_i \Longleftrightarrow \gamma_{i'} \omega_{i'} < \gamma_i \omega_i$. City services can be produced from the consumption good with constant returns, one unit of consumption good producing one unit of services.

The mayor of this small community is not allowed to tax income so she is prodding the citizens to voluntarily contribute to the financing of the city services, with a pitch to rich citizens to be responsible and contribute according to their means. Let's study how successful she can be. The price of the consumption good is normalized to 1 and the price of living space is denoted r (for rent). Let z_i denote the contribution of agent i to city services. We only study "symmetric" equilibria in which all agents of the same type contribute the same amount to the provision of public services: $(\omega_{i'}, \bar{h}_{i'}, \gamma_{i'}) = (\omega_i, \bar{h}_i, \gamma_i) \Longleftrightarrow z_{i'} = z_i$.

3(a) Solve agent i's optimization problem and find agent i's demand for consumption good, housing space and contribution to city services as a function of (r, Z^{-i}), where Z^{-i} denotes the contribution of the other citizens: $Z^{-i} = \sum_{i' \neq i} z_{i'}$.

3(b) Show that if $z_i > 0$, then $y < \gamma_i(\omega_i + r\bar{h}_i)$. Deduce that if agent i does not contribute, and $i' < i$, then agent i' does not contribute either. [If you do not succeed to prove this, accept it as a fact.]

3(c) Let $N = \{i \in I \,|\, z_i = 0\}$ denote the set of agents who do not contribute to city services and $C = \{i \in I \,|\, z_i > 0\}$ denote the set of agents who contribute. Let γ_C be defined by

$$\frac{1}{\gamma_C} = \sum_{i \in C} \frac{1}{\gamma_i}$$

Suppose all the citizens take the price r as given (we will calculate its equilibrium value later). Calculate y as a function of (γ_C, R_C, H, r), where

$$R_C = \sum_{i \in C} \omega_i, \qquad H = \sum_{i=1}^{I} \bar{h}_i = \sum_{i \in C} \bar{h}_i$$

where the second equality in the definition of H is justified by the result of question (b) and the symmetry assumption.

[Hint: Remembering that $y = z_i + Z^{-i}$ calculate z_i as a function of $(\omega_i, \bar{h}_i, y)$ and sum over $i \in C$].

3(d) For each $i \in C$ calculate z_i and Z^{-i} as a function of (γ_c, R_C, H, r) and the income of agent i.

3(e) Assuming that the market for housing space is perfectly competitive, calculate the equilibrium price r (do not forget that citizens in N also buy housing space; the notation $R_N = \sum_{i \in N} \omega_i$ may be convenient to use).

3(f) Substituting the value of r in (c) find the level of public services when the set of contributors is C.

Question 4

Consider a two-stage game played between two firms. In **Stage 1** the firms play a perfect information game where Firm 1 decides the quality of its product: H (high) or L (low); then, after observing firm 1's choice, firm 2 also decides the quality of its own product: H or L. In **Stage 2** the qualities of the two products become common knowledge and the two firms play a **simultaneous** game where each firm chooses the price of its product. High-quality goods are produced at a constant marginal cost of 2, while low-quality goods are produced at zero cost. There are no fixed costs. When both firms choose H, the demand function is given by $Q = 10 - P$ (where P is the lowest price, consumers buy only from the lowest-price firm and, if both firms charge the same price, then consumers split themselves equally between the two firms). When both firms choose L, the demand function is given by $Q = 4 - P$ (where P is the lowest price, consumers buy only from the lowest-price firm and, if both firms charge the same price, then consumers split themselves equally between the two firms). When one firm chooses H and the other chooses L then the demand functions are given as follows (p_H is the price charged by the H firm and p_L is the price charged by the L firm): for the H firm $q_H = 10 - \frac{p_H}{4} + \frac{p_L}{4}$ and for the L firm $q_L = \frac{p_H}{4} - \frac{p_L}{4}$. There is no discounting.

4(a) For the case where each firm can only choose one of two prices, p_a and p_b, draw the extensive form of this game (there is **no need to write the payoffs**).

4(b) Assuming that any non-negative price is possible, find the pure-strategy subgame-perfect equilibrium of this game. [Note: give the entire strategy profile.]

Suppose now that the industry does not exist yet and the government decides to run an auction involving only two bidders and what is being auctioned is the right to be Firm 1 in the above game. Thus whoever wins the auction will be Firm 1 in the above two-stage game and whoever loses the auction will be Firm 2. Call the participants in the auction Players A and B. In all of the auctions of Parts (c)-(e) the following applies: (1) the auction is a simultaneous sealed-bid auction, (2) the winner is the player who submits the higher bid (the other player is called the loser), (3) if the two bids are the same, then Player A (being the nephew of the mother-in-law of the sister of the government official who is in charge of the auction) will be declared the winner, (4) bids can be **any non-negative** real numbers. All of this is common knowledge between the bidders. There is no discounting.

4(c) Case 1. The auction is a first-price auction (the winner pays her own bid and the loser pays nothing). Find all the pure-strategy subgame-perfect equilibria (SPEs) of the three-stage game just described (the first stage is the auction and the other two stages are the same as in the game described above). Prove that what you claim to be SPEs are indeed a SPEs and that there are no other SPEs. If your claim is that there are no SPEs, then prove it.

4(d) Case 2. The auction is a second-price auction (the winner pays the bid of the loser and the loser pays nothing). Find all the pure-strategy subgame-perfect equilibria (SPEs) of the three-stage game. [No proofs necessary here: a brief explanation will suffice.]

4(e) Case 3. The auction is an all-pay first-price auction (each player pays her own bid, including the loser). Find all the pure-strategy subgame-perfect equilibria (SPEs) of the three-stage game. Prove that what you claim to be SPEs are indeed SPEs and that there are no other SPEs. If your claim is that there are no SPEs, then prove it.

Question 5

Consider the following situation. It is common knowledge between Players 1 and 2 that tomorrow one of three states will occur: *a*, *b* or *c*. It is also common knowledge between them that (1) if state *a* realizes, then Player 1 will only know that either *a* or *b* occurred and Player 2 will only know that either *a* or *c* occurred, (2) if state *b* realizes, then Player 1 will only know that either *a* or *b* occurred and Player 2 will know that *b* occurred, (3) if state *c* realizes, then Player 1 will know that *c* occurred while Player 2 will only know that either *a* or *c* occurred. Tomorrow they will play the following *simultaneous* game: each will report, confidentially, one of her two possible states of information to a third party (for example, Player 1 can only report $\{a,b\}$ or $\{c\}$). Note that lying is a possibility: for example, if the state is a Player 1 can report $\{c\}$. Let R_1 be the report of Player 1 and R_2 the report of Player 2. The third party, who knows the true state, then proceeds as follows: (1) if the reports are compatible, in the sense that $R_1 \cap R_2 \neq \varnothing$, then he gives the players the following sums of money:

If the true state is *a*

$R_1 \cap R_2 =$	a	b	c
1 gets	5	4	6
2 gets	5	6	4

If the true state is b

$R_1 \cap R_2 =$	a	b	c
1 gets	5	4	4
2 gets	0	1	1

If the true state is c

$R_1 \cap R_2 =$	a	b	c
1 gets	0	1	1
2 gets	5	4	4

(2) if the reports are incompatible ($R_1 \cap R_2 = \varnothing$) then he gives the players the following sums of money:

The true state is	a	b	c
1 gets	5	4	1
2 gets	5	1	4

5(a) Represent the simultaneous two-player game as an extensive-form game, making Player 1 move before Player 2 and having "Nature" select the state (for the moment do not associate probabilities to Nature's moves).

5(b) Suppose first that both players have no idea what the probabilities of the states are and are not willing to form subjective probabilities. It is common knowledge that each player is selfish (i.e. only cares about how much money she herself gets) and greedy (i.e. prefers more money to less) and ranks sets of outcomes according to the worst outcome, in the sense that she is indifferent between sets *X* and *Y* if and only if the worst outcome in *X* is equal to the worst outcome in *Y* and prefers *X* to *Y* if and only if the worst outcome in *X* is better than the worst outcome in *Y*.

5(b.1) Write the normal-form (or strategic-form) of the game of Part (a).

5(b.2) Find all the pure-strategy Nash equilibria of this game.

5(b.3) Among the Nash equilibria, is there one where each player tells the truth?

5(c) Suppose now that it is common knowledge between the players that the probabilities of the states are as follows: $\begin{pmatrix} a & b & c \\ 2/5 & 1/5 & 2/5 \end{pmatrix}$. It is still common knowledge that each player is selfish and greedy. This time assume that it is common knowledge that both players are risk-neutral.

5(c.1) Suppose that Player 2 expects Player 1 to report truthfully. Is it rational for Player 2 to also report truthfully?

5(c.2) Is "always lying" for each player part of a pure-strategy weak sequential equilibrium? Prove your claim.