University of California, Davis
Department of Economics
Microeconomic Theory

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PRELIMINARY EXAMINATION FOR THE Ph.D. DEGREE
Please answer four questions (out of five)

Question 1.

Two goods, good 1 and good 2, which is the numeraire good. There are I consumers. For $i = 1,\ldots, I$, Consumer i's utility function is $u^i : \Re_+ \times \Re \to \Re : u^i(x_1^i, x_2^i) = b^i(x_1^i) + x_2^i$, where b^i is differentiable, increasing and strictly concave, with $b^i(0) = 0$. Let $w^i > 0$ be Consumer i's wealth, understood to be sufficiently large.. All consumers are price takers (and face the same price). We assume that the price of the numeraire good is equal to 1, and we denote by $p > 0$ the price of good 1. Denote by $\hat{x}_1^i(p)$ Consumer i's Walrasian demand for good 1, and by $X(p) := \sum_{i=1}^{I} \hat{x}_1^i$ the market aggregate demand for good 1.

There is a single firm which produces good 1 by using the numeraire as an input, with a differentiable cost function $C(y)$, where y is the total amount of good 1 produced. We assume in what follows that first order equalities characterize the solution to every optimization problem.

Given a price $p > 0$, we define the *markup* as $\dfrac{p - C'(X(p))}{p}$.

1.1. Given the market price p**,** write the Consumer i's optimization problem that yields her Walrasian demand for good 1. Obtain the first-order equation for this problem.

1.2. Given a price p, what is Consumer i's consumer surplus? What is the aggregate consumer surplus in the economy? If prices are regulated with the objective of maximizing aggregate consumer surplus, what would be the price policy? What can you say about the resulting markup?

1.3. Write the profit maximization problem. What can you say about the resulting markup?

1.4. Suppose now that prices are regulated in order to maximize the sum of total consumer surplus and profits. What would be the price policy? What can you say about the resulting markup?

1.5. Let Consumer i own a share $\theta^i \in (0,1]$ in the profits of the firm. As a consumer, she buys the good in the market, where she is a price taker, but she can vote at the shareholders' meeting on the price that the firm will charge. What is the best price for Consumer-Shareholder i?

1.6. Assume now that consumers indexed 1 to N, $N \leq I$, are the shareholders of the firm (i. e., they own positive shares in the firm's profits, with $\sum_{h=1}^{N} \theta^h = 1$), whereas consumers with indices higher than N are not shareholders. Suppose that, at the shareholders' meeting, all shareholders unanimously agree on a price $\overline{p}$. What can you say about the resulting markup? Does it depend on the relative size of the set of shareholders? What can you say about the share θ^h of Consumer-Shareholder h when all shareholders agree on the price $\overline{p}$? Interpret.

1.7. We now relax the assumption that b^i is increasing and differentiable on $\Re_+$, and specialize the model to a very simple case, where for $i = 1,\ldots, I$,

$$b^i(x_1^i) = \begin{cases} ax_1^i - \frac{1}{2}[x_1^i]^2, \text{ if } x_1^i \leq a, \\ \\ \frac{1}{2}a^2, \text{ if } x_1^i \leq a, \end{cases}$$

and $C(y) = cy$, with $a > c$. We assume that only a fraction $\sigma = N/I$ of the population of I consumers are shareholders in the firm, each owning a share $\theta = \frac{1}{\sigma I}$ in the firm's profits.

1.7.1 Compute the profit-maximizing price p^M.

1.7.2. Show that all shareholders agree on a price $p^*(\sigma)$, and compute it. How does $p^*(\sigma)$ vary with σ? What are the limits of $p^*(\sigma)$ as $\sigma \to 0$? As $\sigma \to 1$? Comment.

Question 2.

2.1. We consider the behavior of agents that can invest in a risky asset. Agents 1 and 2 have identical (nonrandom) initial wealth of ω units of a safe asset, which asset yields one unit of the single consumption good in any state of the world. There is also a risky asset, which yields different amounts of consumption good in the various states of the world: an agent can *ex ante* exchange safe and risky asset on a one-to-one basis in the financial markets. An action by Agent i (i = 1, 2) is now a *portfolio*, defined by amount γ^i of the risky asset, which leaves her with amount $\omega - \gamma^i$ of the safe asset.

There are two states concerning the returns of the risky asset, the bad one and the good one. The bad state occurs with probability $\pi \in (0,1)$: in it the risky asset (gross) return rate is $v_1 < 1$ (i. e., the net return rate is then $v_1 - 1 < 0$). The good state occurs with probability $(1 - \pi)$: in it the risky asset (gross) return rate is $v_2 > 1$ (i. e., the net return rate is then $v_2 - 1 > 0$). We assume in this section that $\frac{v_2 - 1}{1 - v_1} > \frac{\pi}{1 - \pi}$.

Agents 1 and 2 are expected utility maximizers, and their vNMB utility functions, defined on an outcome space X which is an interval of the real line, are denoted u^1 and u^2, respectively. For i = 1, 2, u^i is twice continuously differentiable, with positive first-order derivative, denoted $(u^i)'$, and negative second order derivative, denoted $(u^i)''$.

Here and in the remaining of Question 2 we assume that the solution to any agent's optimization problem is interior to its constraint set. We also assume that there are no constraints on short sales of any asset.

2.1.1. Write Agent i's optimization problem and obtain its first-order equality.

2.1.2. Let $u^2(x) = \psi(u^1(x)), \forall x \in X,$ for some twice continuously differentiable function ψ with positive first-order derivative ($\psi' > 0$). Recalling that $(u^i)' > 0$ and $(u^i)'' < 0$, what is the relation between the sign of its second-order derivative ψ'' and the quotient of the coefficients of absolute aversion of Agents 1 and 2, denoted $r_A(x, u^1)$ and $r_A(x, u^2)$, respectively? Prove your answer.

2.1.3. Postulate that $u^2(x) = \psi(u^1(x)), \forall x \in X,$ for a twice-continuously differentiable function ψ with $\psi' > 0$ and $\psi'' < 0$. How do the investments γ^1 and γ^2 in the risky asset chosen by Agents 1 and 2, respectively, compare? Prove your answer.

2.2. Assume now that the initial wealth ω is random, and can take the values $\varpi+\delta_1$ or $\varpi+\delta_2$, where $\delta_1<\delta_2$. No market exists for contracting insurance to cover the loss $\delta_2-\delta_1$.
But the market for the risky asset is open, as in Section 2.1 above: one unit of the risky asset yields the gross return of $v_1 < 1$ with probability π, and the gross return of $v_2 > 1$ with probability $1-\pi$.

We postulate two agents with preferences as in Section 2.1.3 above, i. e., $u^2(x)=\psi(u^1(x)),\forall x\in X$, for a twice-continuously differentiable function ψ with $\psi'>0$ and $\psi''<0$. Agent *i* chooses *ex ante* (i. e., before any uncertainty is resolved) her investment γ^i in the risky asset.

In this section we assume that $\frac{v_2-1}{1-v_1}<\frac{\pi}{1-\pi}$. We postulate that the (random) initial wealth and the (random) returns to the risky asset are perfectly *negatively* correlated: with probability π, the return to the risky asset is v_1 *and* the wealth of the agent is $\varpi+\delta_2$, whereas with probability $1-\pi$ the return to the risky asset is v_2 *and* the wealth of the agent is $\varpi+\delta_1$.

How do the investments $\gamma^1$and γ^2 in the risky asset chosen by Agents 1 and 2, respectively, now compare? Argue your answer and give an intuitive explanation.

2.3. Assume now that the (random) returns to the risky asset are distributed independently from the (random) initial wealth. In particular, assume that the initial wealth is low ($\varpi+\delta_1$) with probability $\rho\in(0,1)$ and high ($\varpi+\delta_2$) with probability $1-\rho$. These probabilities are independent from those of the returns of the risky asset, which are as before, $\pi\in(0,1)$ for the low return ($v_1<1$) and $1-\pi$ for the high return ($v_2>1$). We assume in this section that $\frac{v_2-1}{1-v_1}>\frac{\pi}{1-\pi}$. Again, Agent *i* (*i* = 1, 2) chooses *ex ante* (i. e., before any uncertainty is resolved) her investment γ^i in the risky asset.

2.3.1. Write Agent *i*'s optimization problem.

2.3.2. Consider now a Fictional Agent who can choose the variable $\gamma\in\Re$ which defines the uncertain prospect

$$z=\left\langle\begin{array}{l}\gamma[v_1-1]\text{ with probability }\pi,\\ \gamma[v_2-1]\text{ with probability 1- }\pi,\end{array}\right.$$

and has expected utility preferences with vNMB utility function V (also denoted $V(z)$ when making explicit its argument $z \in \Re$). What is the optimization problem of this fictional agent?

2.3.3. We consider now two such fictional agents, named Fictional Agent 1 and Fictional Agent 2, with vNMB utility functions $V^1(z)$ and $V^2(z)$. Assume that the coefficient of absolute risk aversion of $V^1(z)$, $r_A(z, V^1)$, is higher than that of $V^2(z)$, $r_A(z, V^2)$. How would their investments γ^1 and γ^2 in the risky asset compare?

2.3.4. Let the vNMB utility functions of Fictional Agent i, $i = 1, 2$, be defined by:

$$V^i(z) = \rho u^i(z + \varpi + \delta_1) + [1-\rho]u^i(z + \varpi + \delta_2).$$

where for $i = 1, 2$, the utility function u^i is the one of 2.3.1.

What is the relationship between the optimization problem of Agent i of 2.3.1 above and Fictional Agent i of this section?

2.3.5. It turns out that examples can be constructed (involving the utility functions u^1 and u^2 of two agents, as well as the parameters $\pi, v_1, v_2, \omega, \rho, \varpi, \delta_1, \delta_2$) where $r_A(x,u^1) < r_A(x,u^2)$, yet $r_A(z,V^1) > r_A(z,V^2)$, for V^i ($i = 1, 2$) defined as in 2.3.4. In such an example:

(i) Which agent invests a higher amount of the risky asset when initial wealth is certain?

(ii) Which agent invests a higher amount of the risky asset when initial wealth is uncertain?

Explain.

Question 3

3. We want to compare the level of production and the welfare in an economy depending whether income redistribution is made by voluntary giving or by taxation. To discuss this, let us consider a simple model with three classes of agents: the 'rich' agents who own capital, the 'workers' who provide labor, and the 'poor' agents who live off gifts or subsidies. There are n_r identical rich, n_w identical workers and n_p identical poor.

There are two dates, 0 and 1. At date 0 each rich agent receives an endowment of e units of date 0 good, some of which is consumed, some of which is saved to form capital. One unit saved at date 0 gives one unit of capital at date 1 (capital formation takes time) which can then be sold to firms. We do not consider date 0 consumption for the workers and poor which result from decisions in previous (un-modeled) periods.

At date 1 the rich sell capital to firms at price r, and buy consumption good at price $p_1 = 1$. In an economy with gifts they give some of the good to charities to be distributed to the poor (all poor are treated the same); in an economy with taxation they are taxed $\tau r k$ on the amount of k of capital they sell, i.e. capital is taxed at rate τ; the proceed of the tax is distributed equally to the poor. Each worker has 1 unit of time, some of which is sold to a firm for a wage w, some of which is consumed as leisure. Workers use the income for labor to buy consumption good (that they buy also at the price $p_1 = 1$). They are not taxed and they do not give to charities. Finally the poor do not have any initial endowment: they consume either the gifts from the rich or the subsidies from the government.

Firms operating at date 1 are owned by the rich agents and produce the consumption good from capital and labor. They all have the same constant return technology, so that without loss of generality we can assume that there is only one firm with the production function

$$Y = K^{\alpha} L^{1-\alpha}, \qquad 0 < \alpha < 1$$

where K is the total amount of capital and L the total amount of labor used.

The utility functions of the agents are given by

$$u_r(x_0, x_1, m) = \log(x_0) + \gamma \log(x_1) + \delta \log(m), \qquad \gamma > 0, \delta > 0$$
$$u_w(x_1, \ell) = x_1^{\beta} \ell^{1-\beta}, \quad 0 < \beta < 1, \qquad u_p(x_1) = x_1$$

where the indices r, w, p refer to the rich, workers and poor respectively, x denotes amount of consumption good with subscript indicating the period, m denotes the gift that a rich agent makes to charities and ℓ is leisure. The term $\delta \log(m)$ is called in the literature the 'warm-glow' effect: rich people receive utility from the act of giving—a positive emotional feeling people get when helping others. This term is present when we consider the economy with voluntary giving by the rich but is not present when the rich are taxed by the government which does the redistribution. In this case we assume that $\delta = 0$. We could consider a case where rich agents give to charity in addition to paying taxes but to simplify, we assume an either/or situation: either no tax and voluntary giving or redistribution through taxation and no voluntary giving.

Part 1: Voluntary Giving. In this part we assume that rich agents give to the poor because of the 'warm-glow' effect and there is no taxation.

(3a) Study the profit maximization problem of the firm and show how equilibrium prices (r, w) relate to the marginal products of capital and labor. Do not try to solve for the input demands as functions of prices and level of production. You will not need it.

(3b) Study the maximum problem of the representative rich agent and deduce the supply of capital and gifts as a function of the prices.

(3c) Study the maximum problem of the representative worker and deduce the supply of labor as function of prices.

(3d) Using the previous questions find the competitive equilibrium of this economy. Argue that the consumption of the agents of each type depends in an intuitive way of the parameters $(e, \alpha, \beta, \gamma, \delta, n_r, n_w, n_p)$. (You can take as given that $\dfrac{\delta}{(\gamma+\delta)^{1-\alpha}(1+\gamma+\delta)^{\alpha}}$ is increasing in δ).

(3e) Calculate $\bar{\tau} = \dfrac{\bar{m}}{\bar{r}\,\bar{k}}$, where $\bar{r}, \bar{k}, \bar{m}$ denote respectively the equilibrium price of capital, choice of capital, and of gift of the representative rich. $\bar{\tau}$ is the rate at which the rich agents voluntarily tax themselves to give to the poor.

Part 2: Capital Taxation. In this part we assume that the redistribution is done through taxation and there is no gift. To be able to compare the two situations, the tax rate is fixed at $\bar{t}$. Redoing step (3b) of the previous part (be careful in re-writing the FOC) calculate the amount of capital in the equilibrium with tax. With a minimum of new calculations, compare qualitatively the equilibrium with voluntary redistribution and the equilibrium with tax and comment of the differences.

Question 4

Food&Quiet (from now on FQ) is a restaurant that has been operating for a number of years. It has developed a regular clientele and, until last year, its yearly profits have been \$*R*. Things have changed this year: the MegaSound discotheque (from now on MS) opened just opposite FQ and changed the character of the neighborhood: the noise and traffic have made the experience of eating at FQ much less pleasant. As a consequence, FQ has suffered a loss in profits equal to \$*L* (with $0 < L \leq R$). MS, on the other hand, has been quite successful, with a yearly profit of \$*P*. FQ threatened to sue MS, but in the end - in order to avoid expensive litigation – they agreed to submit to an arbitrator, the highly regarded retired judge Solomon. Since both *L* and *P* are private and non-verifiable information to the respective parties, the judge decides to have the two play the following game. FQ will report a non-negative number ℓ which will be interpreted by the judge as the **profit loss** suffered by FQ. MS will report a non-negative number π that the judge will interpret as the yearly **profit** of MS. The reports are made simultaneously and independently. Note that nothing prevents either party from making a false report (since the true values are private information). The judge then will make the following decision: if $\ell > \pi$ then MS will have to shut down; if $\pi \geq \ell$ then MS will be allowed to remain open, but it will have to pay a yearly compensation to FQ in the amount of ℓ. Thus the payoff functions are as follows:

$$\text{for FQ: } \begin{cases} R - L + \ell & \text{if } \pi \geq \ell \\ R & \text{if } \ell > \pi \end{cases}, \qquad \text{for MS: } \begin{cases} P - \ell & \text{if } \pi \geq \ell \\ 0 & \text{if } \ell > \pi \end{cases}.$$

(4.a) Is it a dominant strategy for FQ to make a truthful report, that is, to choose $\ell = L$? [Prove your claim and explain if you are referring to strict or weak dominance.]

(4.b) Is it a dominant strategy for MS to make a truthful report, that is, to choose $\pi = P$? [Prove your claim and explain if you are referring to strict or weak dominance.]

(4.c) For the case where $P > L$ (recall that *P* and L denote the *true* amounts), find **all** the pure-strategy Nash equilibria. [Prove that what you claim to be Nash equilibria are indeed Nash equilibria and that there are no other Nash equilibria.]

(4.d) For the case where $P < L$, find **all** the pure-strategy Nash equilibria. [Prove that they are Nash equilibria and that there are no other Nash equilibria.]

(4.e) Assuming that FQ and MS end up playing a pure-strategy Nash equilibrium, is the outcome Pareto efficient?

(4.f) Now restrict attention to the case where it is common knowledge among the judge, FQ and MS that there are only two possible values for each of *L* and *P*: $L \in \{L_1, L_2\}$ and $P \in \{P_1, P_2\}$ with $L_1 < P_1 < L_2 < P_2$.

(4.f.1) Use states and information partitions to represent the state of knowledge of the judge, FQ and MS.

(4.f.2) Use the Harsanyi transformation to represent the game described above (where the players are MS and FQ), assuming that each party assigns equal probability to each state he is uncertain about.

Question 5

In an attempt to reduce the deficit, the government of Italy has decided to sell a 14th century palace near Rome. The palace is in disrepair and is not generating any revenue for the government. From now on we'll call the government Player G. A Chinese millionaire has offered to purchase the palace for $\$p$. Alternatively, Player G can organize an auction among n interested parties ($n \geq 2$). The participants to the auction (we'll call them players) have been randomly assigned labels 1,2,…,n. Player i is willing to pay up to $\$p_i$ for the palace, where p_i is a positive integer. For the auction assume the following: (1) it is a simultaneous, sealed-bid second-price auction, (2) bids must be non-negative integers, (3) each player only cares about his own wealth, (4) the tie-breaking rule for the auction is that the palace is given to that player who has the lowest index (e.g. if the highest bid was submitted by Players 3, 5 and 12 then the palace is given to Player 3). All of the above is commonly known among everybody involved, as is the fact that for every $i, j \in \{1,...,n\}$ with $i \neq j$, $p_i \neq p_j$. We shall consider various scenarios. In all scenarios you can assume that the p_i's are common knowledge.

Scenario 1. Player *G* first decides whether to sell the palace to the Chinese millionaire or make a public and irrevocable decision to auction it.

(5.a) Draw the extensive form of this game for the case where $n = 2$ and the only possible bids are \$1 and \$2. [List payoffs in the following order: first *G* then 1 then 2.]

(5.b) For the general case where $n \geq 2$ and every positive integer is a possible bid, find a pure-strategy subgame-perfect equilibrium of this game. What are the players' payoffs at the equilibrium?

Scenario 2. Here we assume that $n = 2$ and $p_1 > p_2 + 1 > 2$. First Player *G* decides whether to sell the palace to the Chinese or make a public and irrevocable decision to auction it. In the latter case he first asks Player 2 to publicly announce whether or not he is going to participate in the auction. If Player 2 says Yes then he has to pay \$1 to Player *G* as a participation fee, which is non-refundable. If he says No then she is out of the game. After player 2 has made his announcement (and paid his fee if he decided to participate) Player 1 is asked to make the same decision (participate and pay a non-refundable fee of \$1 to Player *G* or stay out); Player 1 knows player 2's decision when he makes his own decision. After both players have made their decisions, player *G* proceeds as follows: (1) if both 1 and 2 said Yes then he makes them play a simultaneous second-price auction, (2) if only one player said Yes then he is asked to put an amount $\$x$ of his choice in an envelope (where x is a positive integer) and give it to Player *G* in exchange for the palace, (3) if both 1 and 2 said No then *G* is no longer bound by his commitment to auction the palace and he sells it to the Chinese.

(5.c) Draw the extensive form of this game for the case where the only possible bids are \$1 and \$2 and also $x \in \{1,2\}$ [List payoffs in the following order: first *G* then 1 then 2.]

(5.d) For the general case where all possible bids are allowed (subject to being positive integers) and x can be any positive integer, find a pure-strategy subgame-perfect equilibrium of this game. What are the players' payoff at the equilibrium?

Scenario 3. Same as Scenario 2; the only difference is that if both Players 1 and 2 decide to participate in the auction then Player *G* gives to the loser the fraction a (with $0 < a < 1$) of the amount paid by the winner *in the auction* (note that player *G* still keeps 100% of the participation fees). This is publicly announced at the beginning and is an irrevocable commitment.

(5.e) For the general case where all possible bids are allowed (subject to being positive integers) find a subgame-perfect equilibrium of this game. What are the players' payoff at the equilibrium?