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Date: August 29, 2013
Time: 5 hours
Reading Time: 20 minutes

PRELIMINARY EXAMINATION FOR THE Ph.D. DEGREE
MICROECONOMIC THEORY

Answer four questions (out of five)

Question 1. Variations on a theme by Herbert Scarf

Consider an economy with three consumers, labeled 1, 2 and 3, and three goods, labeled 1, 2 and 3. For *i*, *j* = 1, 2, 3, we denote by x_j^i Consumer *i*'s consumption of good *j*, i. e., a superscript indicates the consumer, and a subscript the good. The utility functions of the three consumers, defined on $\Re^3_+$, are as follows.

$$u^1(x_1^1, x_2^1, x_3^1) = \min\{x_1^1, x_2^1\},$$

$$u^2(x_1^2, x_2^2, x_3^2) = \min\{x_1^2, x_3^2\},$$

$$u^3(x_1^3, x_2^3, x_3^3) = \min\{x_2^3, x_3^3\}.$$

Notation. As usual, we denote by (p_1, p_2, p_3) a price vector and by w^i a level of wealth for Consumer *i*, *i* = 1, 2, 3.

1(a). For *i*, *j* = 1, 2, 3, compute the Walrasian demands $\tilde{x}_j^i(p_1, p_2, p_3, w^i)$ for $(p_1, p_2, p_3) >> 0$ and $w^i \geq 0$.

1(b). Is there a positive representative consumer for the unrestricted domain of price-wealth vectors $(p_1, p_2, p_3, w^1, w^2, w^3)$ where $(p_1, p_2, p_3) \in \Re^3_{++}$ and $(w^1, w^2, w^3) \in \Re^3_+$?

If YES, display its indirect utility function and check that it is indeed a positive representative consumer (checking for one of the three goods suffices). If NO, argue why not.

1(c). Is there a positive representative consumer for the restricted domain of price-wealth vectors $(p_1, p_2, p_3, w^1, w^2, w^3)$ where $(p_1, p_2, p_3) \in \Re^3_{++}$ and where, for given nonnegative parameters θ^1, θ^2 and θ^3 such that $\theta^1 + \theta^2 + \theta^3 = 1$, $(w^1, w^2, w^3) \in \Re^3_+$ is restricted to satisfy $w^i = \theta^i[w^1 + w^2 + w^3], i = 1, 2, 3$?

If YES, display its indirect utility function and check that it is indeed a positive representative consumer (checking for one of the three goods suffices). If NO, argue why not.

1(d). We now move to a Jevonsian world where, for i = 1, 2, 3, instead of the money wealth amount w^i, Consumer i is endowed with an initial endowment vector $(\omega_1^i, \omega_2^i, \omega_3^i) \in \Re_+^3$ (but with a zero price-independent component of wealth, i. e., $m^i = 0$). Compute $\hat{x}_j^i(p_1, p_2, p_3)$, Consumer i's Jevonsian demand for good j, for i, j = 1, 2, 3.

1(e). Consider an exchange economy with the above consumers, goods and utility functions for the following initial endowment vectors:

$$(\omega_1^1, \omega_2^1, \omega_3^1) = \left(\frac{6}{8}, \frac{6}{8}, \frac{6}{8}\right),$$

$$(\omega_1^2, \omega_2^2, \omega_3^2) = \left(\frac{1}{8}, \frac{1}{8}, \frac{1}{8}\right),$$

$$(\omega_1^3, \omega_2^3, \omega_3^3) = \left(\frac{1}{8}, \frac{1}{8}, \frac{1}{8}\right).$$

As usual, the social endowment of the three goods is the sum of the individual endowments.

Compute the general competitive equilibrium of this economy, specifying the equilibrium (relative) prices. We admit the possibility that one (but no more than one) price be zero at equilibrium. (Note. If you are unable to answer fully, take $p_3 = 0$, $p_1 = p_2 = 1/2$, and move on.)

1(f). Consider the equilibrium allocation obtained in 1(e). Is it Pareto efficient (or Pareto optimal)? Does it involve any waste? Argue and discuss your answer.

Question 2. Variations on a theme by Joan Robinson

We consider a profit-maximizing firm that produces one output (named *output*) by using one input (named *labor*), according to the continuous, differentiable and concave production function $f : \Re_+ \to \Re : L \mapsto f(L)$, where L denotes the amount of labor used by the firm, and where we assume that $f(0) = 0$. The firm has no market power in the output market, where it faces a price equal to one ($p = 1$), but in the input market it faces a differentiable (inverse) supply-of-labor function $\tilde{w} : \Re_+ \to \Re_+ : L \mapsto \tilde{w}(L)$, where $\tilde{w}(L)$ is the wage per unit of labor that the firm has to pay in order to hire L units of labor, and where we assume that $\tilde{w}'(L) \geq 0$.

2(a). What do we mean when we say that the firm has market power in the input market?

2(b). Write the profit maximizing program of the firm. Obtain its first-order condition and verbally interpret it. Represent the profit maximizing solution in a graph, labeled Figure 2.1, which depicts the relevant marginal magnitudes.

2(c). Rewrite the first-order condition of 2(b) à la Lerner equation, i. e., in such a way that on one side of the equation only the labor elasticity of supply appears. Interpret, separately considering the cases $\tilde{w}'(L) > 0$ and $\tilde{w}'(L) = 0$.

2(d). Assume now and in what follows that the supply-of-labor function $\tilde{w}(L)$ is generated by the UMAX problem of a (representative) consumer who consumes leisure (with amounts denoted x_1) and output (with amounts denoted x_2), with preferences represented by the utility function $u : \Re_+ \times \Re : u(x_1, x_2) = \varphi(x_1) + x_2$, where φ is a differentiable function satisfying φ(0) = 0, and with nonnegative initial endowments ω_1 and ω_2 of leisure and output, respectively. We denote by L^S the consumer's supply of labor, i. e., $x_1 = \omega_1 - L^S$, and define the *disutility-of-labor* function

$$\gamma(L^S) := \varphi(\omega_1) - \varphi(\omega_1 - L^S).$$

The consumer is a price and wage taker, facing the wage rate w in the labor market and the price $p = 1$ in the output market.

Obtain the supply-of-labor function of the consumer under the assumption that the solution to her maximization problem is interior.

2(e). Given an amount of labor L, define the *social surplus* associated with L as $S(L) := f(L) - \gamma(L)$. What amount of labor maximizes social surplus? Graphically represent the surplus-maximizing solution, denoted L^*, and the maximal social surplus, denoted S^*, in a copy of Figure 2.1, to be labeled Figure 2.2.

2(f). Define the *deadweight loss* associated with an amount L of labor as $S^* - S(L)$, i. e., the amount by which the level of social surplus achieved at L falls short of the maximal social surplus. Graphically represent in Figure 2.2 the level of social surplus and the deadweight loss associated with the profit maximizing solution.

2(g). In addition to the assumptions introduced in 2(a)-2(f), postulate now and in what follows that the production function is linear, i. e., $f(L) = \frac{L}{c}$, where c is a positive constant. Express the firm's *profit percentage*, defined as the ratio PROFITS/COSTS, in terms of the elasticity of labor supply, and interpret. (Note that "COSTS" are defined by the outlays incurred by the firm in order to purchase inputs, and "PROFITS" as the difference between revenues and costs.)

2(h). In addition to the assumptions introduced in 2(a)-2(g), postulate now that the leisure valuation function φ is quadratic, i. e., $\varphi(x_1) = qx_1 - \frac{1}{2}k[x_1]^2$, with $q > k\omega_1 > 0$.

Provide a graphical illustration in a figure, labeled Figure 2.3, which specializes Figure 2.2 to the assumptions adopted here.

What is the relation between the surplus-maximizing amount of labor and the profit-maximizing amount of labor?

At the profit maximizing solution, what is the relation between the level of profits and the deadweight loss?

2(i). Under all the assumptions introduced in 2(a)-2(h), express the ratio

$$\frac{DEADWEIGHT\ LOSS}{COSTS}$$

in terms of the elasticity of labor supply.

Question 3: Interest Rate in an Economy With and Without Growth

Consider an economy with I agents living for two periods, $t = 1, 2$. The utility functions of the agents have the separable form

$$u_i(x^i) = v_i(x_1^i) + \delta v_i(x_2^i), \quad i = 1, \ldots, I$$

where the discount factor $\delta > 0$, with $0 < \delta < 1$, is the same for all agents, and $x_t^i \in \mathbb{R}^+$ is the (value of) consumption of agent i in period t, $t = 1, 2$. The functions v_i are differentiable, increasing, concave and such that $\lim v_i'(c) \to \infty$ when $c \to 0$. Agents have initial endowments $\omega^i = (\omega_t^i)_{t=1,2} \in \mathbb{R}_+^2$, $i = 1, \ldots, I$. The only market of the economy is borrowing and lending among agents, with interest rate r. If agent i saves s^i at date 1 (or borrows s^i if $s^i < 0$) he will get (or he will have to pay) $(1 + r)s^i$ at date 2. If $\sum_{i=1}^I \omega_1^i = \sum_{i=1}^I \omega_2^i$, we say that the economy does not grow. If $\sum_{i=1}^I \omega_2^i > \sum_{i=1}^I \omega_1^i$, we say that the economy is growing.

3(a) Show that a competitive two-period equilibrium $(\bar{x}, \bar{s}, \bar{r})$ of the above economy, in which agents take the interest rate as given, is equivalent to a competitive equilibrium $(\bar{x}, \bar{p})$ of the static two-good economy $\mathcal{E}((u^i, \omega^i)_{i=1}^I)$, where $\bar{p}_1 = 1$, $\bar{p}_2 = \frac{1}{1+\bar{r}}$. Deduce that if $(\bar{x}, \bar{s}, \bar{r})$ is a competitive equilibrium, $\bar{x}$ is a Pareto optimal allocation.

3(b) Show that if $x_1^i < x_2^i$ (respectively $x_1^i > x_2^i$), $MRS_{x_1^i, x_2^i}(x^i) > 1/\delta$ (resp. $< 1/\delta$), where $MRS_{x_1^i, x_2^i}(x^i)$ denotes agent i's marginal rate of substitution of date 2 consumption for date 1 consumption at the consumption stream x^i. Deduce that if $\sum_{i=1}^I \omega_1^i = \sum_{i=1}^I \omega_2^i$, a Pareto optimal allocation x is such that $x_1^i = x_2^i$ for all $i = 1, \ldots, I$.

3(c) Deduce the equilibrium interest rate $\bar{r}$ for an economy without growth in terms of the characteristics of the economy.

3(d) Now suppose that $\sum_{i=1}^I \omega_2^i > \sum_{i=1}^I \omega_1^i$. How does the equilibrium interest rate compares with the result of question (c)? Give a proof to justify your answer, and interpret the result.

Question 4: Comparing Lindahl and Subscription Equilibrium.

Consider an economy with $I \geq 2$ agents in which there is one private and one public good. Let x^i denote agent i's consumption of the private good and let y denote the amount of public good, then $u^i(x^i, y)$ is the agent's utility. We assume u^i is strictly monotone, strictly concave, differentiable. Let $\omega^i > 0$ denote agent i's endowment of the private good. The public good, of which there is no initial endowment, can be produced from the private good with constant marginal cost $k > 0$, that is one unit of public good is produced using k units of the private good.

4(a) Assuming that the consumption of each agent in private and public goods is strictly positive, derive the first order conditions for an allocation $(x^1, \ldots, x^I, y)$ to be a Pareto optimum and interpret these conditions.

4(b) Using the private good as numeraire and letting p^i denote the personalized price for agent i, define a Lindahl equilibrium for this economy.

4(c) Show that a Lindahl equilibrium is Pareto optimal (assuming an interior equilibrium).

4(d) Unfortunately information constraints make it difficult to implement a Lindahl equilibrium. An alternative scheme for the provision of the public good is by voluntary subscription. Let z^i be the amount of the private good that agent i is prepared to subscribe towards the production of the public good. In a subscription equilibrium each agent chooses z^i taking the choice of z^j $(j \neq i)$ by all other agents as given. Derive the first order condition of agent i maximization problem in a subscription equilibrium (assuming an interior maximum).

4(e) Suppose all agents have the same utility function and endowment ($u^i = u,\ \omega^i = \omega,\ i = 1, \ldots, I$). Let $(\bar{x}, \bar{y})$ and $(\tilde{x}, \tilde{y})$ denote the symmetric Lindahl and subscription equilibrium respectively, where $\bar{x}$ and $\tilde{x}$ denote the consumption in private good of the representative agent at equilibrium. Show that $\tilde{y} < \bar{y}$. (*Hint*: Give a geometric proof, noting that $(\bar{x}, \bar{y})$ maximizes $u(x, y)$ on a budget line which contains $(\tilde{x}, \tilde{y})$.) Explain the intuition for the result.

Question 5

Consider the following situation of incomplete information. There are three players: an intermediary (I), a seller (S) and a buyer (B). S wants to sell her car, whose quality is known only to her. Neither I nor B know the quality of S's car, but know that it belongs to the set $\{Q_1, Q_2, ..., Q_n\}$ ($n \geq 2$). Both I and B have the same beliefs: they attach probability q_i to quality Q_i (i = 1,…,n). The players are in the same room and play the following game: I suggests a price $p \in P \subseteq [0,\infty)$ and S either says Yes or No. If S says No then the game ends and S gets a payoff of $\varepsilon > 0$, while the other players get a payoff of 0. If S says Yes, then it is B's turn to say Accept or Not Accept. If B says Not Accept, then the game ends and B gets a payoff of $\varepsilon > 0$, while the other players get a payoff of 0. If B says Accept, then S sells the car to B at the price p suggested by I. If the car is sold then I's payoff is 1, B's payoff is $V(Q) - p$ and S's payoff is $p - U(Q)$, where Q denotes the quality of the car. For every i = 1,…,n, $V(Q_i) > U(Q_i) > 0$ and, for every i = 1,…,$n-1$, $U(Q_i) < U(Q_{i+1})$ and $V(Q_i) < V(Q_{i+1})$. The payoffs are von Neumann-Morgenstern payoffs. All of the above is common knowledge among the three players (including the values of $U(Q_i)$ and $V(Q_i)$, for all i = 1,2,…,n).

5(a) Consider first the case n = 2, $\varepsilon = 0.1$, $q_1 = q_2 = 0.5$, $U(Q_1) = 1, U(Q_2) = 4$, $V(Q_1) = 2, V(Q_2) = 5$ and $P = \{3,4\}$. Draw the extensive game that is obtained by applying the Harsanyi transformation to the situation of incomplete information described above.

5(b) Find all the pure-strategy weak sequential equilibria (WSE) of the game of part (a).

From now on consider the case n = 3, $\varepsilon = 0.1$, $U(Q_1) = \frac{1}{2}, U(Q_2) = 3, U(Q_3) = 4$, $V(Q_1) = 2, V(Q_2) = 6, V(Q_3) = 8$ and the game that is obtained by applying the Harsanyi transformation to the situation of incomplete information described above.

5(c) Let $P = \{1,3,4,5\}$. Find necessary and sufficient conditions on q_1 and q_2 for the existence of a pure-strategy WSE where the expected payoff of I is equal to 1. Explain how you derived those conditions. Give one pair of values of q_1 and q_2 that satisfy those conditions.

5(d) Let $P = \{1,3,4,5\}$. Find necessary and sufficient conditions on q_1 and q_2 for the existence of a pure-strategy WSE where the expected payoff of I is equal to q_1. Give one pair of values of q_1 and q_2 that satisfy these conditions. Prove that what you claim to be a WSE is indeed a WSE.

5(e) Suppose that $P = [0,\infty)$ and $q_1 = \frac{1}{2}, q_2 = \frac{1}{6}, q_3 = \frac{1}{3}$. Find the range of prices that are compatible with a Nash equilibrium which is Pareto efficient with probability 1.