University of California, Davis | Date: June 22, 2009
Department of Agricultural and Resource Economics
Department of Economics | Time: 5 hours
Microeconomics | Reading Time: 20 minutes

PRELIMINARY EXAMINATION FOR THE Ph.D. DEGREE

Please answer **four** of the following five parts

Part I

(a) There are N goods. Good 1 is leisure, measured in hours; its consumption is denoted x_1, and its dollar price (per hour) $p_1 > 0$. Goods 2 to N are consumption goods, with dollar prices $p_2, \ldots, p_N$, all positive. The consumer is endowed with $\omega_1 > 0$ units of leisure and zero units of any other good ($\omega_2 = \ldots = \omega_N = 0$). The consumer is also endowed with m dollars of price-independent wealth. We define the variable $L \equiv \omega_1 - x_1$ and call it the supply of labor. The consumer's preferences are represented by a differentiable utility function $u : X \to \Re$, increasing in all its arguments in the interior of X, where $X \subset \Re_+^N$ denotes her consumption set. Denote the Jevonsian supply-of-labor function by $\hat{L}(p_1, p_2, ..., p_N)$ (we take the parameters m and ω_1 as given, so that they do not appear as arguments).

(a) 1. Write the Slutsky equation for the decomposition of the "total effect" $\frac{\partial \hat{L}}{\partial p_1}$ of a change in p_1 on the supply of labor, expressing the substitution and the wealth effects in terms of the partial derivatives of the Hicksian and Walrasian demand-for-leisure functions.

(a) 2. Assume that leisure is a normal good. What can you say about the sign of the substitution and wealth effects of (a)1? What can you say about the sign of the total effect $\frac{\partial \hat{L}}{\partial p_1}$? Interpret in words.

(b) We now consider a new model. We still have prices $(p_1, p_2, \ldots, p_N) >> 0$, an endowment vector $(\omega_1, 0, \ldots 0)$, and the consumer's price-independent wealth m. But we introduce two differences.

First, <u>leisure no longer enters the utility function,</u> so that preferences are defined on an open subset $Z \subset \Re_+^{N-1}$, with typical element $(x_2, \ldots, x_N)$, and represented by a differentiable, strictly quasiconcave and strictly increasing utility function $\beta : Z \to \Re$.

Second, <u>the consumption of any commodity</u> $j \in \{2, \ldots, N\}$ <u>takes time</u>, so that the available amount of leisure ω_1 must be allocated between the supply of labor in the market, denoted L, which is paid at rate p_1 per hour, and unpaid time devoted to consumption activities. More specifically, the data of the economy include, for j = 2,…, N, a nonnegative coefficient t_j expressing the per-unit amount of time required by the consumption of good of j, so that the consumption of x_j units of good j requires spending $t_j \; x_j$ units of time, in addition to spending $p_j \; x_j$ dollars.

(b) 1. Write the consumer optimization problem that yields her demand for goods $\breve{x}_j(p_1, p_2, ..., p_N), j = 2, ..., N$ and her supply of labor $\breve{L}(p_1, p_2, ..., p_N)$ (again, we take the parameters m and ω_1 as given). Assume that the solution exists.

(b) 2. Argue that the time endowment ω_1 is totally spent.

(b) 3. Combine the budget constraint and the time constraint into a single equality constraint involving (x_2,…, x_N), and interpret.

(b) 4. Denote a (N-1) dimensional price vector by $(\pi_2, ..., \pi_N)$, and a wealth magnitude by w, and define the Walrasian demand function $(\tilde{x}_2(\pi_2, \cdots, \pi_N, w), \ldots, \tilde{x}_N(\pi_2, \cdots, \pi_N, w))$, as usual, by the solution to the problem $\max \beta(x_2, ..., x_N)$ subject to $\sum_{j=2}^{N} \pi_j x_j = w$. Similarly, define the Hicksian demand function $(h_2(\pi_2, ..., \pi_N, \overline{u}), ..., h_N(\pi_2, ..., \pi_N, \overline{u}))$ by the solution to the problem $\min \sum_{j=2}^{N} \pi_j x_j$ subject to $\beta(x_2, ..., x_N) = \overline{u}$. Using (b)3, express the functions $\breve{x}_j(p_1, p_2, ..., p_N), j = 2, ..., N$ and $\breve{L}(p_1, p_2, ..., p_N)$ in terms of the Walrasian demand functions $\tilde{x}_j, j = 2, \cdots, N$.

(b) 5. Express $\frac{\partial \breve{L}}{\partial p_1}$ in terms of the derivatives of the Walrasian demand functions $(\tilde{x}_2, \cdots, \tilde{x}_N)$.

(b) 6. By using the Slutsky decomposition of the total effects $\frac{\partial \tilde{x}_j}{\partial \pi_k}(j, k = 2, \ldots, N)$, write $\frac{\partial \breve{L}}{\partial p_1}$ as the sum of a "substitution term" involving the derivatives $\frac{\partial h_j}{\partial \pi_k}$ of Hicksian demand, and a "wealth term" involving the derivatives $\frac{\partial \tilde{x}_j}{\partial w}$ of Walrasian demand.

(b) 7. Assume that goods 2, …, N are normal. What can you say about the sign of the substitution and wealth effects obtained in (b).6? What can you say about the sign of the total effect $\frac{\partial \breve{L}}{\partial p_1}$? Compare with (a) above.

Part II

There are two goods, good 1 and good 2, and good 2 is the numeraire good. There are I consumers. For i = 1,…, I, consumer i's utility function is $u^i : \Re_+ \times \Re \to \Re : u^i(x_1^i, x_2^i) = b^i(x_1^i) + x_2^i$, where b^i is differentiable, increasing and strictly concave. All consumers are price takers. We assume that the price of the numeraire good is equal to 1, and we denote by p the price of good 1. Denote by $\tilde{x}_1^i(p)$ consumer i's Walrasian demand for good 1.

There is a single firm which produces good 1 by using the numeraire as an input, with cost function $C(y)$, where y is the amount of good 1 produced. We assume in what follows that first-order equalities characterize the solution to every optimization problem.

Given a price p, we define the markup as $\frac{p - C'}{p}$, where the marginal cost C' is evaluated at the amount of output equal to aggregate demand.

(a) Write the first order condition of the consumer's optimization problem that yields her Walrasian demand for good 1.

(b) Suppose that prices are regulated in order to maximize the sum of consumer surplus and profits. What can you say about the resulting markup?

(c) Let consumer i own a share $\theta^i \geq 0$ in the profits of the firm (i = 1,…, I , $\sum_{i=1}^{I} \theta^i = 1$). As a consumer, she buys the good in the market, where she is a price taker, but she can vote at the shareholders' meeting on the price that the firm will charge. Derive the equation that characterizes the best price for consumer-shareholder i.

(d) Suppose that, at the shareholders' meeting, all shareholders unanimously agree on a price. What can you say about the resulting markup? What can you say about the share θ^i of consumer-shareholder i? Interpret. Hint: Add up the FOC.

(e) We now specialize the model to a very simple case, where $b^i(x_1^i) = ax_1^i - \frac{1}{2}(x_1^i)^2$, i = 1,…, I, and $C(y) = cy$, where $a > c$. But we assume that only a fraction σ of the population of I consumers are shareholders in the firm, each owning a share $\theta = \frac{1}{\sigma I}$ in the firm's profits.

(e) 1. Compute the monopoly profit-maximizing price p^M.

(e) 2. Show that all shareholders agree on a price $p(\sigma)$, and compute it. How does $p(\sigma)$ vary with σ? What are the limits of $p(\sigma)$ as $\sigma \to 0$? As $\sigma \to 1$? Comment.

Part 3

When discussing the market for electricity it is often asserted that the demand should be rationed at peak times because of the limited capacity of production. We show that competitive prices can do the job provided they are sufficiently refined. To see this, consider an economy with $T+1$ goods: good 0 is a numeraire good and goods $1,\ldots,T$ represent consumption of electricity at time $t=1,\ldots,T$ (for example consumption in the day time and consumption at night). The production of electricity requires building a plant of capacity K, where K represents the maximum amount of electricity that can be produced at any time. Building a plant of capacity K requires ρK units of numeraire good and then the cost of producing one unit of electricity at any time is γ units of numeraire. Since it is not optimal to build a larger capacity than the maximum amount of electricity produced at any time, the production set for electricity is

$$Y=\left\{(-z_0,y_1,y_2,\ldots,y_T)\in\mathbb{R}_-\times\mathbb{R}_+^T \;\middle|\; z_0\geq\rho\Big(\max_{1\leq t\leq T}y_t\Big)+\gamma\sum_{t=1}^{T}y_t\right\}$$

(a) Show that the production set for electricity is convex and exhibits constant returns.

(b) Since it is equivalent to assume that there is one price-taking firm or a lot of small firms producing electricity, to simplify we assume that there is one firm which maximizes its profit taking prices as given. To simplify the study of the firm's profit maximization, from now on we take $T=2$ and $p=(1,p_1,p_2)$.

 (i) In the plane (y_1,y_2), draw the iso-cost curves of the firm. Make sure that you get the shape right.

 (ii) Add an iso-revenue line and find the condition on the ratio of the prices p_1/p_2 which guarantees that profit maximization occurs along the diagonal $y_1=y_2$.

 (iii) In the case considered in (ii) find the additional condition on the prices which ensures that there is a non zero solution to profit maximization.

(c) There are I agents in the economy. Agent i ($1\leq i\leq I$) has an endowment ω^i of the numeraire good and a utility function

$$u^i(x^i)=(x_0^i)^{\alpha_0^i}(x_1^i)^{\alpha_1^i}(x_2^i)^{\alpha_2^i},\qquad \alpha_0^i+\alpha_1^i+\alpha_2^i=1.$$

We assume that

$$1<\frac{\sum_{i=1}^{I}\alpha_1^i\omega^i}{\sum_{i=1}^{I}\alpha_2^i\omega^i}<\frac{\gamma+\rho}{\gamma}$$

i.e. time 1 is the period when electricity is most needed (day time) but the difference in needs is not extreme. Show that there is a competitive equilibrium such that the consumption is the same at period 1 and at period 2, so that the capacity is optimally used in equilibrium. (Actually this is the only equilibrium but we do not have time to prove it.)

Part 4

We will show that the use of lotteries by institutions which provide privately financed public goods (e.g. charities) results in a higher level of public good financing than relying on voluntary contributions. Consider a simple model with two goods, one private, one public, one unit of private good producing one unit of public good. There are I agents with identical endowment ω of private good and identical, quasi-linear preferences

$$u(x_i, y) = x_i + \gamma \log(y), \qquad \gamma > 0$$

where x_i is the consumption of private good, y the quantity of public good, and $\gamma < \omega$. Consider only symmetric outcomes where all agents consume the same amount of the private good.

(a) Find the Pareto optimal level of public good y^*.

(b) Calculate the amount $\bar{y}$ of public good which will be provided by a charity if it relies on voluntary contributions by the I agents. Explain why $\bar{y}$ is less than y^*.

(c) Suppose that instead of relying on contributions, the charity organizes a lottery with a prize R. It can print as many lottery tickets, sold for one unit of private good each, as the agents want to buy. If agent i buys z_i tickets and the total number sold is Z, then the probability that agent i will win the prize in a random draw of the lottery is

$$\pi(z_i, Z) = \frac{z_i}{Z}$$

Participating in the lottery makes agent i's consumption x_i random. Agents are assumed to maximize expected utility with the von-Neumann-Morgerstern utility function u and, since u is quasi-linear, agents are risk neutral. An equilibrium with lottery is an outcome where the charity uses the receipts from the lottery tickets to cover both the cost R of the prize and the cost of producing the public good y (agents know this), and each agent chooses the optimal number of lottery tickets to buy, given the number of tickets bought by the other agents and the prize R.

(i) Write the first-order condition for the optimal number z_i of lottery tickets for the typical agent, and using symmetry express it as a function of $Z = Iz_i$.

(ii) Expressing this relation as a second degree polynomial in the variable $Z - R$, show that the total amount of money $\bar{Z}_L$ collected by the lottery satisfies $\gamma < \bar{Z}_L - R < I\gamma$. (You do not need to compute $\bar{Z}_L$).

(iii) Interpret the result in (ii). Explain this result in terms of marginal costs and marginal benefits.

Part 5

The purpose of this exercise is to study Betrand-Nash equilibria when some consumers do not pay attention to small price differences and to relate the "Bertrand paradox" to consumers' sensitivity to price differences.

There are two firms that produce a homogeneous product. Let p_i be the price of firm i $(i = 1, 2)$. Assume for simplicity that the firms have **zero costs**. There are N consumers with unit demands, each with the same reservation price for the product, denoted by r. That is, each consumer buys one unit at most from one of the two firms if and only if at least one of the prices is less than or equal to r. When $p_1 = p_2 \leq r$, 50% of the consumers go to firm 1 and 50% to firm 2. What happens when the two prices are different? Some consumers are very sensitive to price differences, while others are not. For example, if $p_1 = p_2 + 0.01$ then some consumers might prefer firm 2 because they save 1 penny, but probably most consumers would be indifferent between the two firms. Let $f : [0, r] \to \mathbb{R}^+$ be a density function (thus $f(x) \geq 0$, for all $x \in [0, r]$, and $\int_0^r f(x)dx = 1$) that measures the price-difference sensitivity of consumers. For example, suppose that $p_1 < p_2 < r$. Then $\int_0^{p_2 - p_1} f(x)dx$ gives the fraction of consumers who prefer the cheaper firm (firm 1), while the others are indifferent between the two firms. Assume that 50% of the indifferent consumers go to one firm and 50% go to the other firm.

In questions (b)-(e) we focus on the possibility of a Nash equilibrium where both firms charge the highest possible price, while in questions (f) and (g) we look at a Nash equilibrium where both firms charge a price equal to marginal cost.

(a) Write down the demand function of each firm (cover all the possibilities, that is, all pairs (p_1, p_2) with $p_i \in [0, \infty)$ for every $i = 1, 2$).

(b) Suppose that $r = 120$, $\int_0^{20} f(x)dx = 0.4$, $f(20) = 0.014$. Is $p_1 = p_2 = 120$ a Nash equilibrium?

(c) Suppose that f is continuous and that the function $g(x) = x[1 + F(r - x)]$ $\Big($where F is the c.d.f., that is, $F(x) = \int_0^x f(t)dt\Big)$ is convex in the interval $(0, r)$. Show that $p_1 = p_2 = r$ is a Nash equilibrium.

(d) Suppose that f is continuous and that the function g (defined in the previous question) is concave in the interval $(0, r)$. Give a necessary and sufficient condition for $p_1 = p_2 = r$ to be a Nash equilibrium.

(e) Assume that f is constant (uniform distribution). Is $p_1 = p_2 = r$ a Nash equilibrium?

(f) Assume that $\int_0^x f(t)dt < 1$ for x sufficiently close to 0. Is $p_1 = p_2 = 0$ a Nash equilibrium?

(g) What assumptions on f would yield the "Bertrand paradox"?