PRELIMINARY EXAMINATION FOR THE Ph.D. DEGREE
MACROECONOMICS

August 24, 2018

Directions: The exam consists of six questions. Questions 1,2 concern ECN 200D (Geromichalos), questions 3,4 concern ECN 200E (Cloyne), and questions 5,6 concern ECN 200F (Caramp). You only need to answer five out of the six questions. If you prefer (and have time), you can answer all six questions, and your grade will be based upon the best five scores. Feel free to impose additional structure on the problems below, but please state your assumptions clearly. You have 5 hours to complete the exam and an additional 20 minutes of reading time.
Question 1 (20 points)

Consider the Mortensen-Pissarides model in continuous time. Labor force is normalized to 1. Unemployed workers, with measure $u \leq 1$, and firms with one vacancy each and total measure $v$ search for each other, and $v$ is determined endogenously by free entry. A CRS matching function, $m(u,v)$, brings together unemployed workers and vacant firms; $m$ is increasing in both arguments. As is standard, let $\theta \equiv v/u$ denote the market tightness and $q(\theta) = m/v$ the arrival rate of unemployed workers to the typical firm.

What is different here compared to the baseline model is that a “match” and a “productive job” are not equivalent by default. When a worker and a vacant firm meet, the firm must train the worker before she can start producing. A formed match turns into a productive job at a stochastic rate, $a \in (0, +\infty)$, so that $1/a$ can be thought of as the average time necessary for the training to be completed. Assume that the firm and the worker determine the wage level when they first meet (i.e., even before training starts), through Nash bargaining, with $\beta \in (0, 1)$ representing the worker’s power. However, the wage upon which they have agreed will only be paid to the worker when she starts producing.$^1$

To close the model, we will make a few more standard assumptions. The output of a productive job is $p > 0$ per unit of time, and while a firm is searching for a worker it has to pay a search (or recruiting) cost, $pc > 0$, per unit of time. Firms that are training their workers do not pay this cost (they are done recruiting). Productive jobs are exogenously destroyed at rate $\lambda > 0$ (only productive jobs are subject to this shock; matches at the training stage cannot be terminated). All agents discount future at the rate $r > 0$, and unemployed workers enjoy a benefit $z > 0$ per unit of time. While at the training stage the worker does not receive an unemployment benefit (a trainee is not unemployed).

a) Define the value function of the typical firm for all the possible states of the world it may find itself in.

b) Do the same for the typical worker.

c) Combine the free entry condition with the expressions you provided in part (a) in order to derive the job creation (JC) curve of this economy.

d) Using the same methodology as in the lectures (adjusted to accommodate the differences in the new environment), derive the wage curve (WC) for this economy.

$^1$ Hence, two parties who met at time, say, $t$ are negotiating over an object that will be paid in the future (at time $t + 1/a$, in expected terms). But, as is always the case, the Nash Bargaining problem is to split the generated surplus as of time $t$. 
e) Provide a restriction on parameter values such that a steady state equilibrium pair \((w, \theta)\) exists. Is it unique? (No need for a lengthy discussion.)

f) What is the effect of a decrease in \(a\) on the equilibrium \(w\) and \(\theta\)? Explain analytically and intuitively (but shortly).

g) Describe the Beveridge curve (BC) of this economy by looking at the flows of workers in and out of the various states. What effect will the decrease in \(a\) (discussed in the previous part) have on equilibrium unemployment?
Question 2 \((20 \text{ points})\)

This question studies the co-existence of money and credit. Time is discrete with an infinite horizon. Each period consists of two subperiods. In the day, trade is partially bilateral and anonymous as in Kiyotaki and Wright (1991) (call this the KW market). At night trade takes place in a Walrasian or centralized market (call this the CM). There are two types of agents, buyers and sellers, and the measure of both is normalized to 1. The per period utility for buyers is \(u(q) + U(X) - H\), and for sellers it is \(-q + U(X) - H\), where \(q\) is the quantity of the day good produced by the seller and consumed by the buyer, \(X\) is consumption of the night good (the numeraire), and \(H\) is hours worked in the CM. In the CM, all agents have access to a technology that turns one unit of work into a unit of good. The functions \(u, U\) satisfy the usual assumptions; I will only spell out the most crucial ones: There exists \(X^* \in (0, \infty)\) such that \(U'(X^*) = 1\), and we define the first-best quantity traded in the KW market as \(q^* \equiv \{q : u'(q^*) = 1\}\).

The difference compared to the baseline model is that there are two types of sellers. Type-0 sellers, with measure \(\sigma \in [0, 1]\), accept credit. More precisely, in meetings with a type-0 seller (type-0 meetings), no medium of exchange (MOE) is necessary, and the buyer can purchase day good by promising to repay the seller in the forthcoming CM with numeraire good (this arrangement is called an IOU). The buyer can promise to repay any amount (no credit limit), and her promise is credible (buyers never default). Type-1 sellers, with measure \(1 - \sigma\), never accept credit, hence, any purchase of the day good must be paid for on the spot \((\text{quid pro quo})\) with money. All buyers meet a seller in the KW market, so that \(\sigma\) is the probability with which a buyer meets a type-0 seller, and \(1 - \sigma\) is the probability with which she meets a type-1 seller.

The rest is standard. Goods are non storable. There exits a storable and recognizable object, fiat money, that can serve as a MOE in type-1 meetings. Money supply is controlled by a monetary authority, and we consider simple policies of the form \(M_{t+1} = (1 + \mu)M_t, \mu > \beta - 1\). New money is introduced, or withdrawn if \(\mu < 0\), via lump-sum transfers to buyers in the CM. Let \(\phi\) denote the unit price of money (in terms of the numeraire). In KW meetings buyers have all the bargaining power.

a) Describe the CM value function of the typical buyer. (Be sure to correctly identify the state variables.) Show that this value function is linear in its argument(s).

b) Describe the CM value functions of the typical seller of type 0 and 1. As in part (a), show that these value functions are linear.

c) Let \(q_j\) denote the quantity of good traded in a type-\(j = 0, 1\) meeting. Let \(d\) denote the amount of numeraire the buyer promises to repay in the forthcoming CM
in exchange for \( q_0 \), and let \( x \) denote the amount of money the buyer pays in exchange for \( q_1 \). Describe the bargaining solution in a typical type-\( j = 0, 1 \) meeting.\(^2\)

d) Describe the objective function of the typical buyer, \( J(m') \), where the prime denotes next period’s choices.\(^3\)

e) Describe the equilibrium variables \( q_0, q_1 \) as functions of the model’s parameters, including the nominal interest rate, \( i \). How is \( q_1 \) related to the real balances \( z \equiv \phi m \)?

f) Does a monetary equilibrium (i.e., an equilibrium with \( z > 0 \)) always exist? If not, describe the set of parameter values (including the policy parameter \( i \)) for which such an equilibrium exists.

Finally, define the welfare function of this economy as the measure of the various KW market meetings times the net surplus generated in each meeting, i.e.,

\[
W = \sigma[u(q_0) - q_0] + (1 - \sigma)[u(q_1) - q_1].
\]

g) Can you describe the sign of the term \( \partial W / \partial \sigma \) for the various values of \( \sigma \)?\(^4\)

\(^2\)Hint: Feel free to analyze the bargaining problems in detail, but I think that with a little intuitive thinking you can answer this question in a couple of minutes.

\(^3\)Hint: Again, feel free to do all the work that leads to the objective function, but here I think it is easy to guess the form of \( J \), and I will give full credit for a correct guess.

\(^4\)Hint: If an analytical solution is hard, I will give partial credit for an intuitive explanation.
Consider a decentralized real business cycle model. The representative household chooses consumption \((c)\) and leisure \((L = 1 - N, \text{ where } N \text{ is hours worked})\) to maximize (detrended) lifetime utility:

\[
E_0 \sum_{t=0}^{\infty} \beta^t \left( \log c_t + \frac{\theta}{1-\eta} (L_t^{1-\eta} - 1) \right)
\]

subject to the detrended household budget constraint:

\[
c_t + \gamma k_{t+1} = w_t N_t + (1 - \delta(u_t))k_t + r_k u_t k_t + \pi_t
\]

where \(u_t\) is the degree of capital utilization and is also chosen by the household. \(w\) is the real wage, \(N\) is hours worked, \(k\) is capital, \(r_k\) is the rental price of capital and \(\pi\) are profits from firms. Detrended capital evolves as follows:

\[
\gamma k_{t+1} = (1 - \delta(u_t))k_t + i_t
\]

where \(X_t/X_{t-1} = \gamma\) is the deterministic growth rate of labor augmenting technological change. Assume \(\gamma = 1\).

Competitive firms produce output using capital services \(u_t k_t\) and labor \(N_t\). The detrended production function is

\[
y_t = A_t (u_t k_t)^{\alpha} (N_t)^{1-\alpha}
\]

where TFP, denoted by \(A_t\), is stochastic and follows an AR(1) process in logs. In steady state \(A = 1\).

Households own the capital stock, invest and decide on the degree of utilization. Utilization is costly and causes capital to depreciate more quickly, as indicated by the depreciation cost function \(\delta(u_t)\), where \(\delta(1) = \delta\) in steady state. Firms choose capital to rent from households, understanding that they will be able to utilize it to a certain degree. You can therefore think of the firm as choosing their demand for labor \(N_t\) and their demand for capital services \(u_t k_t\).

a) Write down the household’s problem in recursive form and write down the firm’s maximization problem. Derive the household’s first order conditions and the firm’s optimal hiring rules.

b) Carefully define a recursive competitive equilibrium. Take care to distinguish between the aggregate and individual state variables and explain any market clearing conditions.
c) By linearizing the equilibrium condition governing the optimal degree of utilization, show that

\[ \hat{y}_t = \hat{a}_t + \alpha \hat{k}_t + (1 - \alpha)\hat{N}_t + \frac{\alpha}{(1 - \alpha + \xi)}(\hat{a}_t - (1 - \alpha)\hat{k}_t + (1 - \alpha)\hat{N}_t) \]  

(3)

where \( \xi = \frac{\delta''}{\delta'} \) and \( \delta' \) and \( \delta'' \) refer to the first and second derivatives of the \( \delta \) function with respect to \( u_t \) in steady state. As usual, variables with a hat denote percentage deviations from steady state. (Hint: once you have linearized the equilibrium condition governing the optimal degree of utilization, you can combine this with the linearized production function \( \hat{y}_t = \hat{a}_t + \alpha \hat{u}_t + \alpha \hat{k}_t + (1 - \alpha)\hat{N}_t \).

d) Discuss how the addition of variable capital utilization helps the RBC model explain the business cycle facts in the data? If you can, explain why in the limit \( \xi = \infty \) and \( \xi = 0 \) correspond to the cases of no variable capital utilization and full variable capital utilization.

e) Briefly explain how you would solve this model using a linearization-based method and how you would produce impulse response functions for the effects of a temporary one percent shock to \( \hat{a}_t \).
**Question 4 (20 points)**

This question considers a TFP shock in the New Keynesian model.

The representative household’s utility function is:

\[
\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\psi}}{1+\psi}
\]  

In linearized form, the equilibrium conditions for this model are as follows. The household’s Euler equation and labor supply conditions are:

\[
E_t \hat{c}_{t+1} - \hat{c}_t = \frac{1}{\sigma}(\hat{i}_t - E_t \hat{\pi}_{t+1})
\]  

\[
\hat{w}_t = \sigma \hat{c}_t + \psi \hat{n}_t
\]

The linearized equilibrium conditions for firms are:

\[
\hat{y}_t = \hat{a}_t + \hat{n}_t
\]

\[
\hat{m}c_t = \hat{w}_t - \hat{a}_t
\]

\[
\hat{\pi}_t = \beta E_t (\hat{\pi}_{t+1}) + \lambda \hat{m}c_t
\]

The resource constraint is:

\[
\hat{y}_t = \hat{c}_t
\]

Monetary policy follows a simple Taylor Rule:

\[
\hat{i}_t = \phi \hat{\pi}_t
\]

(Linearized) TFP follows an AR(1) process

\[
\hat{a}_t = \rho \hat{a}_{t-1} + \epsilon_t
\]

\(\epsilon_t\) is i.i.d.

In percentage deviations from steady state: \(\hat{m}c_t\) is real marginal cost, \(\hat{c}_t\) is consumption, \(\hat{w}_t\) is the real wage, \(\hat{n}_t\) is hours worked, \(\hat{y}_t\) is output and \(\hat{a}_t\) is Total Factor Productivity. In deviations from steady state: \(\hat{i}_t\) is the nominal interest rate, \(\hat{\pi}_t\) is inflation. \(\lambda\) is a function of model parameters, including the degree of price stickiness.\(^5\)

Assume that \(\phi > 1\), \(0 < \rho < 1\) and \(0 < \beta < 1\)

\(^5\lambda = \frac{(1-\theta)(1-\beta\theta)}{\theta}\) where \(\theta\) is the probability that a firm cannot adjust its price.
a) Using the equilibrium conditions above, show that this model can be represented by the standard 3 equations

\[ E_t \tilde{y}_{t+1} - \tilde{y}_t = \frac{1}{\sigma} (\tilde{i}_t - E_t \tilde{\pi}_{t+1} - \hat{r}_n) \]  \hspace{1cm} (13)

\[ \tilde{\pi}_t = \beta E_t (\tilde{\pi}_{t+1}) + \kappa \tilde{y}_t \]  \hspace{1cm} (14)

\[ \hat{i}_t = \phi \pi \tilde{\pi}_t \]  \hspace{1cm} (15)

plus expressions for the natural rate of output:

\[ \hat{y}_n = \frac{1 + \psi}{\sigma + \psi} \hat{a}_t \]  \hspace{1cm} (16)

the natural real rate of interest:

\[ \hat{r}_n = -\sigma (1 - \rho) \frac{1 + \psi}{\sigma + \psi} \hat{a}_t \]  \hspace{1cm} (17)

and the output gap

\[ \tilde{y}_t = \hat{y}_t - \hat{y}_n \]  \hspace{1cm} (18)

where \( \hat{a}_t \) follows the process in equation 12. (\textbf{Hint:} You may want to start by finding the natural rate of output (equation 16) and then writing the household Euler equation in terms of (linearized) output, the natural rate of output, the nominal interest rate, TFP and expected inflation.)

b) Using the method of undetermined coefficients, find the response of the output gap and inflation to an exogenous increase in \( \hat{a}_t \) when prices are sticky and monetary policy follows the Taylor Rule above. To do this, guess that the solution for each variable is a linear function of the shock \( \hat{a}_t \):

\[ \tilde{y}_t = \Lambda_y \hat{a}_t \]

\[ \tilde{\pi}_t = \Lambda_\pi \hat{a}_t \]

c) Interpret your results in (b). In particular, carefully explain how, and why, TFP shocks affect the output gap and inflation in this model.

d) Instead of following the Taylor Rule above, policy is now set optimally. One result is that, under optimal policy, the real interest rate tracks the natural real interest rate. From your knowledge of this model and optimal policy, what is the optimal path for the output gap and inflation in response to a TFP shock and why (you do not need to derive anything)?

e) Suppose the monetary policymaker wants to implement optimal policy using an interest rate rule for \( \hat{i}_t \). Explain why the policymaker cannot simply use a rule which attempts to set the nominal interest rate (\( \hat{i}_t \)) equal to the natural real interest rate (\( \hat{r}_n \)). What other component(s) should the rule contain and why (you do not need to derive anything)?
Question 5 (20 points)

Consider the fixed-investment model with two alternative projects: the two projects have the same investment cost \( I \) and the same payoffs, \( R \) in the case of success and 0 in the case of failure. The entrepreneur has initial wealth \( N < I \) and must collect \( I - N \) from risk-neutral financiers who demand a rate of return equal to 0. Project 1 has probability of success \( p_H \) if the entrepreneur works and \( p_L = p_H - \Delta p \) if she shirks. Similarly, the probabilities of success are \( q_H \) and \( q_L = q_H - \Delta q \) for project 2, where

\[ \Delta q = \Delta p. \]

Project 1 (respectively, 2) delivers private benefit \( B \) (respectively, \( b \)) when the entrepreneur shirks; no private benefit accrues in either project if the entrepreneur works. We make the following assumptions:

1. project 1 has a higher probability of success

\[ p_H > q_H, \]

2. both projects have positive NPV if the entrepreneur works, but negative if she shirks

\[ p_H R > q_H R > I > p_L R + B > q_L R + b. \]

3. pledgeable income is higher for project 2 but lower than the cost of investment

\[ p_H \left( R - \frac{B}{\Delta p} \right) < q_H \left( R - \frac{b}{\Delta q} \right) < I. \]

Assume that at most one project can be implemented (because, say, the entrepreneur has limited attention), and (except in question d)) that the financiers can verify which project, if any, is implemented.

a) Suppose effort is observable. Show that only project 1 is financed, independently of \( N \).

b) From now on, effort is not observable. Characterize the financing problem between the entrepreneur and the financiers. In particular, clearly state: i) the resource constraint; ii) the financiers’ participation constraint; iii) the entrepreneur’s incentive compatibility constraints.

c) Divide the set of possible net worths \( N, [0, I) \), into three regions, \( [0, \bar{N}_q) \), \( [\bar{N}_q, \bar{N}_p) \), and \( [\bar{N}_p, I) \) and show that the equilibrium investment policies in these regions are ”not invest,” ”invest in project 2,” and ”invest in project 1.” Find the expressions for \( \bar{N}_q \) and \( \bar{N}_p \). Why is project 2 sometimes financed?
d) In this question only, suppose that the financiers cannot verify which project the entrepreneur is choosing (they only observe success/failure). Argue that nothing is altered if \( N \geq N_p \). Formally show that if \( N \in [N_q, N_p) \), the entrepreneur doesn’t get financing.

e) Suppose now that the private benefit of shirking on project 1 can be reduced from \( B \) to \( b \) by using a monitoring technology. This technology has an implementation cost of \( c \). Assume that

\[
p_H R - c > q_H R > I
\]

and

\[
p_H \left( R - \frac{b}{\Delta p} \right) < I + c
\]

Show that monitoring is useful if and only if

\[
c < p_H \frac{B - b}{\Delta p}
\]

Is there any level of \( N \) and \( c \) such that project 2 is implemented?
Question 6 (20 points)

Consider the following OLG economy. Time is discrete and goes from \( t = 0 \) to infinity. Agents live for two periods, and there is a constant measure 1 of individuals in each generation. Young agents born in \( t \) have preferences over consumption streams of a single good that are ordered by \( u(c^t_i) + u(c^t_{i+1}) \) where \( u(c) = \frac{c^{1-\sigma}}{1-\sigma} \), and where \( c^t_i \) is the consumption of an agent born at \( i \) in time \( t \). Assume that \( \sigma > 0 \) and that when \( \sigma = 1 \), \( u(c) = \ln c \). Each young agent born at \( t \geq 0 \) has identical preferences and endowment pattern \((w_1, w_2)\), where \( w_1 \) is the endowment when young and \( w_2 \) is the endowment when old. Assume \( 0 < w_2 < w_1 \). In addition, there are some initial old agents at \( t = 0 \) who are endowed with \( w_2 \) of consumption good, and who order consumption streams by \( u(c_{0}^{-1}) \). The initial old at \( t = 0 \) are also endowed with \( M \) units of an irreproducible and useless asset (that pays zero dividend and does not depreciate). We will call this asset ”money”. The stock of money is constant over time. There is no other store of value in the economy. Let \( p_t \) denote the price of money.

a) Consider a perfect foresight economy. State the problem of an agent born in period \( t \). Characterize the solution to this problem for a given sequence of prices of money.

b) Define and characterize an equilibrium for this economy.

c) Guess and verify a steady-state equilibrium in which money has a constant positive price.

d) Show that there exists an equilibrium in which agents believe that with probability \( \pi \) the price of money in the following period will be \( p > 0 \) and with probability \( 1 - \pi \) the price will be zero. What happens after the price is zero? How does the price \( p \) change with \( \pi \)?

e) Assume \( \sigma = 1 \) and \( w_2 = 0 \). Show that there are equilibria in which in some periods agents are optimistic (so that \( \pi_t \) is high) and the price of money is high, and some periods in which they are pessimistic (so that \( \pi_t \) is low) and the price of money is low. Argue that this model produces non-fundamental fluctuation in asset prices.