University of California, Davis   Date: September 4, 2014
Department of Economics   Time: 5 hours
Macroeconomics   Reading Time: 20 minutes

PRELIMINARY EXAMINATION FOR THE Ph.D. DEGREE

Directions: Answer all questions. Feel free to impose additional structure on the problems below, but please state your assumptions clearly. Point totals for each question are given in parentheses.

1. (10) Consider a standard Solow growth model with constant labor growth which, in equilibrium, exhibits all of Kaldor's stylized facts of growth (the economy is on a balanced growth path). If a general, constant returns to scale production function is used (that is, we are not restricting technology to be a Cobb-Douglas production function), provide an intuitive explanation for why technological progress must be labor augmenting only. Or, more specifically, what features of a balanced growth path and the model combine to rule out capital augmenting technological progress?

2. (20) Consider the following variant of Stockman's cash-in-advance model. The primary distinction is that asset and goods markets are assumed to meet sequentially with the asset market preceding the goods market. The assets in the economy are: capital (k_t), one-period nominal bonds (B_t), and money (M_t). Nominal bonds costs 1\$ in period t and return \$$N_t$ in next period's asset market. In the asset market, agents also receive the returns from capital (i.e. the revenue from the sale of output in last period's goods market and the sale of undepreciated capital) and the lump-sum monetary transfer. They combine this with any money left over from the previous goods market to buy new capital, bonds and money. Next agents visit the goods market where money is used to finance consumption (i.e. consumption is subject to the cash-in-advance constraint). The aggregate money stock grows at the constant rate $\mu > 0$ and capital depreciates at the rate $\delta \in (0,1)$. (Note that, as in Stockman, agents' use capital to produce output via the production function $y_t = f(k_t)$. This output is sold in the goods market.) Households have standard time separable preferences as in Stockman.

   (a) Set up the agent's maximization problem as a dynamic programming problem. (Hint: Note that the budget constraint relevant to the asset market does not include consumption.)

   (b) Prove that, in this economy, the Lagrange multiplier associated with the budget constraint is equal to agent's marginal utility of consumption. Compare this result to that in Stockman's model.

   (c) Define a steady-state equilibrium. Prove that money is not superneutral. Again, compare this result to that in Stockman's paper.

   (d) Prove that $N_t = 1$ (i.e. net nominal interest rates are zero) if and only if the cash-in-advance constraint is not binding. Explain.

3. (20) Consider a representative agent economy in which, every period, agents allocate income between consumption, capital, bonds, and equity. Capital acquired in period t is used to produce output in period $t + 1$; the production function is given by

$$y_t = z_t k_t$$

where z_t denotes a random shock to technology. (The depreciation rate of capital is 100%.) It is assumed that z_t can take on two values $z_1 < z_2$. The evolution of z_t is governed by a symmetric transition probability matrix with diagonal elements $\pi > 1/2$. One and two period bonds are traded. Both bonds cost one unit of consumption in period t and return $R1_t$ and $R2_t^2$, respectively, upon maturity. The remaining asset, equity, sells at the price q_t and pays a dividend each period given by $d_t = \alpha y_t$ where $\alpha \in (0, 1)$. Households choose consumption and assets in order to maximize expected lifetime utility given by:

$$E \left\{ \sum_{t=0}^{\infty} \beta^t \ln c_t; \quad \beta \in (0, 1) \right\}$$

Given this environment, answer the following questions:

(a) Express the household's maximization problem as a dynamic programming problem. (In setting up the budget constraint, assume that two-period bonds are sold after holding them for one period.)

(b) Define a recursive competitive equilibrium. Be precise in identifying the elements in your definition.

(c) Solve the economy as a social planner problem and derive the equilibrium policy functions for capital and consumption.

(d) Characterize the behavior of one-period interest rates in equilibrium. Explain your result.

(e) Compare the average return from selling a two-period bond after holding it for one period to that from holding a one-period bond to maturity. Explain.

(f) Prove that, if $d_t = c_t$, then $q_t = k_{t+1}$. Explain.

4. (20) Consider the standard growth model in discrete time. There is a large number of identical households normalized to 1. Each household wants to maximize life-time discounted utility

$$U(\{c_t\}_{t=0}^{\infty}) = \sum_{t=0}^{\infty} \beta^t u(c_t), \ \beta \in (0,1).$$

Each household has an initial capital k_0 at time 0, and one unit of productive time in each period that can be devoted to work. Final output is produced using capital and labor, according to a CRS production function F. This technology is owned by firms (whose measure does not really matter because of the CRS assumption). Output can be consumed (c_t) or invested (i_t). Households own the capital (so they make the investment decision), and they rent it out to firms. Let $\delta \in (0,1)$ denote the depreciation rate of capital. Households own the firms, i.e. they are claimants to the firms' profits, but these profits will be zero in equilibrium.

The function u is twice continuously differentiable and bounded, with $u'(c) > 0$, $u''(c) < 0$, $u'(0) = \infty$, and $u'(\infty) = 0$. We will not be specific about the properties of F, but instead we will introduce (just like we did in class) the more useful function $f(x) \equiv F(x,1) + (1-\delta)x$, $\forall x \in \mathbb{R}_+$. The function f is twice continuously differentiable with $f'(x) > 0$, $f''(x) < 0$, $f(0) = 0$, $f'(0) = \infty$, and $f'(\infty) = 1 - \delta$.

There is also a government who taxes households' income, from both labor *and* capital, at the common and constant rate $\tau \in [0,1]$. The government returns all the tax revenues, T, to the households in the form of lump-sum transfers. Throughout this question focus on recursive competitive equilibrium (RCE).

(a) Write down the problem of the household recursively.[1] Carefully define the state variables and distinguish between aggregate and individual states. Then, carefully define a RCE.

(b) Write down the dynamic equation that the aggregate capital stock follows in this economy. **Hint:** Obtain the Euler equation for the typical household and impose the RCE conditions in order to end up with an expression that involves only the aggregate capital K and functions that are part of the definition of RCE, (and, of course, the various functions and parameters that are given in the model).

(c) Now focus on steady-states. Describe the steady-state equilibrium value of the aggregate capital stock in this economy, and denote it by $K^*(\tau)$. **Hint:** If the formula you arrived at involves the function F, it will be useful to replace it with the function f in order to answer the next parts.

(d) What is the value of K^* when $\tau = 0$? When $\tau = 1$? What is the intuition of the latter result?

(e) Provide a formula for the derivative $dK^*(\tau)/d\tau$. What is the sign of this expression? Does it make economic sense?

(f) Assume that $F(K,N) = K^a N^{1-a}$, $a \in (0,1)$, and let $\delta = 1$. Calculate the government's total tax revenue, T, and plot it as a function of the tax rate τ (this is the so-called Laffer curve). If $a = 1/2$ which value of τ maximizes tax revenues?

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[1] Here firms face a static problem. I am not asking you to explicitly spell it out, but it will be critical for correctly defining a RCE.

5. (20) Consider the discrete time, infinite horizon monetary-search model we saw in class. Each period is divided into two subperiods. In the day, trade is bilateral and anonymous. At night trade takes place in a standard perfectly competitive market. There are two types of agents, buyers and sellers, and the measure of both is normalized to the unit. The per period utility for buyers is $u(q) + U(X) - H$, and for sellers it is $-q + U(X) - H$, where q is the quantity of the day good (consumed by the buyer and produced by the seller), X is consumption of the night good (i.e. the numeraire), and H is hours worked in the night market. We make some standard assumptions: u, U are twice continuously differentiable with $u(0) = 0$, $u' > 0$, $u'(0) = \infty$, $u'(\infty) = 0$, $U' > 0$, $u'' < 0$, and $U'' \leq 0$. Let $q^* \equiv \{q : u'(q^*) = 1\}$. Also, there exists $X^* \in (0, \infty)$ such that $U'(X^*) = 1$.

In the day market, the probability with which a buyer meets with a seller is σ. In any such meeting the buyer makes a take-it-or-leave-it (TIOLI) offer to the seller, i.e. she gets to keep any surplus generated in the match. Goods are non storable, but there exits a storable and recognizable object, called fiat money, that can serve as a means of payment. The supply of money is controlled by the monetary authority according to the rule $M_{t+1} = (1 + \mu)M_t$, with $\mu > \beta - 1$. New money is introduced (or withdrawn if $\mu < 0$) via lump-sum transfers (taxes) to buyers in the night market.

What is interesting is that buyers can use not only cash, but also credit. Let $D > 0$ denote the *exogenous* credit limit that a buyer can use in order to purchase q from a seller. This credit takes the form of a (credible) promise to repay the seller with numeraire good in the forthcoming night market (sometimes referred to as an IOU).[2] As an example, consider a meeting between a buyer and a seller, and suppose that the two parties have agreed that a quantity q of the day good will be produced, in return for a payment equal to π, measured in terms of the numeraire. Then, the buyer can pay the seller with any combination of credit, d, and (real) dollars, z, such that $d + z = \pi$, subject to the constraint that the credit limit is not violated, i.e. $d \leq D$.

Intuition: Summing up, here we study a model where money and credit compete as means of payment. It is important to understand that in this model credit has an advantage over money: if an agent wants to use money, she must bring it with her from the previous period and pay the holding cost (nominal interest rate), while she is not even certain that she will need it. In contrast, the buyer can use the credit only after she finds out that she has matched with a seller. Hence, credit sounds like a much better deal, except that, for reasons that we will not discuss here, it is limited.

(a) Describe the value function of the typical buyer and seller in the night market (recall that these agents do not only carry money, as in the baseline model, but they might also have some debt/credit from the preceding round of trade, and this should show up as a state variable in their value function).

(b) Describe the terms of trade in the day market (recall that the buyer makes a TIOLI offer).

(c) Describe the objective function of the typical buyer. **Hint:** There is no need to derive the objective function formally. Full credit will be given if you "guess" it correctly.

(d) Suppose that $D \geq q^*$. What do equilibrium real balances (of money), z, look like?

(e) Suppose that $D < q^*$. Describe the steady-state equilibrium values of q and z as functions of monetary policy.

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[2] If you like, you can think that in the back round there is a financial institution which guarantees that the buyer will repay her debt, otherwise the institution will. What is important is that when a seller accepts credit $d \leq D$ from a buyer, she is 100% certain that she will receive d units of the numeraire in the forthcoming night market, i.e. the possibility of default is excluded.

6. (10) Consider the Mortensen-Pissarides model in continuous time. Labor force is normalized to 1. Unemployed workers, with measure $u \leq 1$, search for jobs, and firms with vacancies, with measure v, search for unemployed workers. A CRS matching function, $m(u,v) = u^a v^{1-a}$, brings together unemployed workers and vacant firms. It is convenient to define the market tightness $\theta \equiv v/u$.

A large measure of firms decide whether to enter the labor market with exactly one vacancy. When a firm meets an unemployed worker a job is formed. The output of a job is $p > 0$ per unit of time. However, while the vacancy is unfilled, firms have to pay a search cost, given by $pc > 0$, per unit of time. Moreover, existing jobs are destroyed at the exogenous (Poisson) rate $\lambda > 0$. Once a shock arrives, the worker goes back to the pool of unemployment, and the firm exits the labor market. Unemployed workers enjoy a utility of $z < p$ per unit of time. The discount rate of agents is $r > 0$.

(a) In this environment (and focusing on steady-states), what problem would a benevolent Social Planner solve in order to maximize social welfare? Assume that the Planner is also subject to the matching frictions that characterize this environment.

(b) Describe the (unique) value of θ that the Social Planner chooses in order to maximize the objective function defined earlier.[3]

(c) Consider an environment where matched firms and workers bargain (a la Nash) over the wage. In this environment, would you expect the market equilibrium to coincide with the Planner's allocation? Why?

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[3] Consider the continuous time maximization problem

$$\int_0^\infty e^{-rt} u(c(t), x(t)) dt,$$

subject to $\dot{x}(t) = \gamma(x(t), c(t))$, where c denotes the control and x the state variable (respectively). Then, defining the Hamiltonian function $H = e^{-rt} u(c(t), x(t)) + \mu \gamma(x(t), c(t))$, and referring to μ as the "co-state" variable, the solution to the dynamic problem in question is fully characterized by the solution to the following two equations:

$$\frac{\partial H}{\partial c} = 0, \quad \frac{\partial H}{\partial x} = -\dot{\mu}.$$