University of California, Davis
Department of Economics

PRELIMINARY EXAMINATION FOR THE Ph.D. DEGREE

MACROECONOMICS

June 28, 2021

Directions: The exam consists of three questions. Question 1 concerns ECN 200D (Geromichalos), question 2 concerns ECN 200E (Cloyne), and question 3 concerns ECN 200F (Caramp). You only need to answer two out of the three questions. If you prefer (and have time), you can answer all three questions and your grade will be based upon the best two scores. Feel free to impose additional structure on the problems below, but please state your assumptions clearly. You have 3 hours to complete the exam and an additional 20 minutes of reading time.

Question 1 (*50 points*)

Consider the following extension of the Mortensen-Pissarides model in continuous time. Labor force is normalized to 1 and there are two types of workers: Healthy (H), with measure $\pi \in (0,1)$, and unhealthy (U), with measure $1-\pi$. The only difference between the two types is that, while employed, U-workers may get sick and not be able to work. If a firm is employing a U-worker, that worker will get sick (probabilistically) at a Poisson rate σ, and the sickness period will end (again, probabilistically), at rate ε. While on sick leave the U-worker does not work/produce, but the firm is obligated to pay him/her the full salary. Firms searching for workers cannot discriminate between the two types. This implies that the matching process is "unbiased": when a firm matches with a worker, the probability that this worker is of type H depends only on the relative measure of H-workers in the pool of unemployed.[1]

When at work all workers produce an amount p of the numeraire good per unit of time. Within a matched firm-worker pair the wage is determined through Nash bargaining, and $\beta \in (0,1)$ captures the worker's bargaining power (which is independent of the type). There is a large measure of firms that can enter the market and search for workers. A firm can enter the labor market with exactly one vacancy, and the total measure of vacancies v will be determined endogenously by free entry. A matching function brings together unemployed workers and vacant firms, and the total number of matches is given by $m = m(u,v)$, where m is CRS and increasing in both arguments, and u is the total number of unemployed workers. As is standard, let $\theta \equiv v/u$ denote the market tightness. While a firm is searching for a worker it has to pay a search cost, $pc > 0$, per unit of time. Productive jobs are exogenously destroyed at Poisson rate $\lambda > 0$. Jobs that involve a U-worker on leave are also subject to the λ shock (hence, a U-worker may move to unemployment while on a sick leave).

All agents discount future at the rate $r > 0$, and all unemployed workers enjoy a benefit $z > 0$ per unit of time. For this question you can focus on steady states. You can also take as given that u_H/u (the fraction of unemployed healthy workers in the aggregate pool of unemployed) is simply equal to π, as is standard in a model with worker heterogeneity and unbiased matching.

a) Let V, J_H, J_U, L be the value functions for a firm that is vacant, employing an H-worker, employing a U-worker who currently produces, and employing a U-worker who is currently on leave, respectively. Describe these functions.

b) Let U_H, W_H be the value functions for an H-worker who is unemployed and employed respectively. Let U_U, W_U, P be the value functions for a U-worker who is unemployed, employed and currently able to work, and employed but currently on sick leave, respectively. Describe these functions.

[1] However, the wage the firm will pay the worker **depends on her type**. Thus, when we say "firms cannot discriminate" we are talking strictly about the search and matching process, not the wage negotiation.

c) Exploiting the free entry condition, derive the Job Creation (JC) curve for this economy. This will be an equation that involves the variables w_H, w_U, θ.[2]

d) Derive the wage curve (WC) for this economy. Notice that here we are not asking for two equations that link w_H and w_U with θ, but for a unique equation that links θ with an appropriately weighted average of w_H and w_U.[3]

e) Use your findings in parts (c) and (d) to provide a condition that uniquely determines the equilibrium θ. Test that your condition is correct by evaluating appropriate extreme values of certain parameters and making sure that the limiting results coincide with those in the baseline model.[4]

f) Now assume that the government passes a new law dictating that a worker's wage cannot be based upon her type. Simply put, workers of all types must be paid the same wage. Discuss how this law may affect equilibrium **welfare** compared to the baseline model. (No need for any math or value functions here. Just provide an intuitive description of how you think equilibrium will be affected.)

[2] **Hint:** The free entry condition will yield a formula that involves J_H and J_U. Characterizing the first is standard. To get an expression for the second one, look jointly at the value functions J_U, L that you described in part (a).

[3] **Hint:** Start by solving the bargaining problems for each type of worker. At some point, you will need to replace the term W_U. To get a useful expression for that term, look jointly at the value functions W_U, P that you described in part (b).

[4] **Hint:** In this model, when the parameters π, σ and ε obtain certain (extreme) values, the model collapses to the baseline model. What are these values?

Question 2 (*50 points*)

The U.S. government is considering a large increase in public investment. Using the New Keynesian model, this question asks you to analyze the macroeconomic effects of an increase in the public provision of capital and how monetary policy might respond.

As usual, there are a continuum of identical households. The representative household makes consumption (C) and labor supply (N) decisions to maximize lifetime expected utility:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\mu \ln C_t + (1-\mu)\ln(1-N_t)\right) \tag{1}$$

subject to their budget constraint:

$$C_t + B_t = w_t N_t + (1+i_{t-1})\frac{P_{t-1}}{P_t} B_{t-1} + D_t + T_t \tag{2}$$

where w_t is the real wage, N_t is hours worked, B_t are real bond holdings at the end of period t, i_{t-1} is the nominal interest rate paid between $t-1$ and t, P_t is the price of the final consumption good and D_t are real profits from firms that are distributed lump sum. Assume $0 < \beta < 1$ and $0 < \mu < 1$. T_t are real lump sum taxes that the government levies on households to fund investment in public capital ($K_{G,t}$) which is used by firms. The government runs a balanced budget and issues no debt.

The production side of the model is similar to the standard New Keynesian environment, except for the inclusion of public capital in the intermediate goods production process. Monopolistically competitive intermediate goods firms produce an intermediate good using labor as usual and public capital which is as an "external input" and not chosen by firms. The production function for product variety j is given by:

$$y_t(j) = K_{G,t}^{\chi} n_t(j) \tag{3}$$

Public capital $K_{G,t}$ is common across all firms and evolves as follows:

$$K_{G,t} = (1-\delta)K_{G,t-1} + G_t \tag{4}$$

G_t is government investment in period t. For simplicity, assume full depreciation $\delta = 1$ and $K_{G,t} = G_t$. χ is the elasticity of output with respect to public capital and $0 \leq \chi \leq 1$.

The rest of the model is standard and intermediate goods firms face a probability that they cannot adjust their price each period (the Calvo pricing mechanism). As usual, intermediate goods are aggregated into a final (homogenous) consumption good by final goods firms using the constant elasticity (CES) aggregator.

The production side of the economy, when aggregated and linearized, can be described by the following set of linearized equilibrium conditions (the production function, the optimal hiring condition for labor and the dynamic evolution of prices):

$$\hat{y}_t = \chi \hat{k}_{G,t} + \hat{n}_t \tag{5}$$

$$\hat{w}_t = \hat{mc}_t + \chi \hat{k}_{G,t} \tag{6}$$

$$\hat{\pi}_t = \beta E_t(\hat{\pi}_{t+1}) + \lambda \hat{mc}_t \tag{7}$$

The resource constraint is:

$$\hat{y}_t = \gamma_c \hat{c}_t + \hat{g}_t \tag{8}$$

Monetary policy follows a simple Taylor Rule:

$$\hat{i}_t = \phi_\pi \hat{\pi}_t \tag{9}$$

Government investment (linearized) follows:

$$\hat{g}_t = \rho \hat{g}_{t-1} + e_t \tag{10}$$

e_t is i.i.d. $\hat{g}_t$ is the change in government investment relative to steady state as a proportion of steady state output: $\hat{g}_t = \frac{G_t - G}{Y}$. This implies that, with 100% depreciation each period, $\hat{k}_{G,t} = \frac{1}{\gamma_g}\hat{g}_t$ where γ_g is the steady state share of government investment in GDP and assume $0 < \gamma_g < 1$. Government investment is financed entirely by lump sum taxes on households. In percentage deviations from steady state: $\hat{mc}_t$ is real marginal cost, $\hat{c}_t$ is consumption, $\hat{w}_t$ is the real wage, $\hat{n}_t$ is hours worked, $\hat{y}_t$ is output and $\hat{k}_{G,t}$ is public capital. In deviations from steady state: $\hat{i}_t$ is the nominal interest rate and $\hat{\pi}_t$ is inflation ($\hat{\pi}_t = \hat{p}_t - \hat{p}_{t-1}$; $\hat{p}_t$ is the percentage deviation of the price level from steady state). γ_c is the steady state share of consumption in GDP. λ is a function of model parameters, including the degree of price stickiness.[5] Assume that $\phi_\pi > 1$, $0 < \rho < 1$.

a) First consider the representative household's problem. Write down the household's problem in *recursive form* and derive the household's first order conditions.

b) Show that the linearized first order condition for labor supply from part (a) is:

$$\hat{w}_t = \hat{c}_t + \psi \hat{n}_t$$

where $\psi = \frac{N}{1-N}$. N denotes steady state hours worked, $0 < N < 1$ and $\psi > 0$. Using this and the relevant equations above show that, with flexible prices, the natural level of output can be written as

$$\hat{y}_t^n = \Gamma \hat{g}_t \tag{11}$$

where

$$\Gamma \equiv \frac{1}{1+\psi\gamma_c}\left(1 + \frac{\gamma_c(1+\psi)\chi}{\gamma_g}\right)$$

[5] $\lambda = \frac{(1-\theta)(1-\beta\theta)}{\theta}$ where θ is the probability that a firm cannot adjust its price.

c) The full sticky price model can be simplified to 3 equations:

$$E_t\tilde{y}_{t+1} - \tilde{y}_t = \gamma_c(\hat{i}_t - E_t\hat{\pi}_{t+1} - \frac{1}{\gamma_c}(1-\Gamma)(1-\rho)\hat{g}_t) \tag{12}$$

$$\hat{\pi}_t = \beta E_t(\hat{\pi}_{t+1}) + \kappa\tilde{y}_t \tag{13}$$

$$\hat{i}_t = \phi_\pi\hat{\pi}_t \tag{14}$$

plus the stochastic process for government investment (equation (10)). $\tilde{y}_t = \hat{y}_t - \hat{y}_t^n$ is the output gap. $\kappa = \frac{1+\psi\gamma_c}{\gamma_c}\lambda$.

Using the method of undetermined coefficients, find the response of the output gap and inflation to an exogenous increase in government investment when prices are sticky and monetary policy follows the Taylor Rule above. To do this, guess that the solution for each variable is a linear function of the government investment shock $\hat{g}_t$.

d) Carefully discuss how, and why, an increase in government investment affects the natural level of output, the output gap and inflation in this model. Take care to explain how your answer depends on the value of χ (Hint: you may want to consider two cases: $\chi = 0$ vs. $\chi = 1$ and examine the expression for Γ).

e) According to the monetary policy rule in equation (9), show what happens to the nominal interest rate following an increase in government investment and discuss whether this depends on the value of χ. Explain. Can you think of a policy rule that would implement full stabilization of inflation and the output gap in this model? Explain. (Hint: for this second part you do not need to derive anything. You should be able to answer this from your knowledge of the model.)

Question 3 (*50 points*)

Consider an economy that lasts three periods, $0, 1, 2$. The economy is populated by an entrepreneur who has access to an investment project. The entrepreneur has linear preferences and consumes only at $t = 2$. The project needs an initial fixed investment of $I > 0$ in period 0. In period 1, the project can be either continued or liquidated. If it is continued, it pays 0 in period 1 and R in period 2, with $R > I$. If it is liquidated, it pays 0 in periods 1 and 2. The entrepreneur has zero initial wealth and receives no endowment in any period.

To fund the project, the entrepreneur can borrow from a continuum of risk-neutral financiers with preferences represented by the utility function $E[c_0+c_1+c_2]$. Financiers have a large endowment of the final consumption good in each period, so the gross interest rate on risk-free debt is always 1. The entrepreneur can only borrow short-term. At the beginning of each period t, she chooses the number of short-term bonds to sell. Denote by b_{t+1} the face value of the bonds, and by q_t the price. Then, the entrepreneur receives $q_t b_{t+1}$ in period t. If the revenue from the sales of new bonds is insufficient to cover the repayment of bonds coming to maturity, the entrepreneur defaults (i.e., she does not repay the debt), and the project is liquidated. For now, assume that R is fully pledgeable and focus on perfect foresight equilibria.

a) Consider the entrepreneur's problem in period 1 assuming that she invested in the project in period 0 and sold bonds for b_1. What is the value of b_1? What is the minimum price q_1 that allows the entrepreneur to avoid default? Express your answers as a function of q_0.

b) Show that it is optimal for the entrepreneur to invest in period 0 if and only if

$$q_0 \geq \frac{1}{q_1}\frac{I}{R}$$

c) Show that there exists an equilibrium in which $q_0 = q_1 = 1$. **Hint:** Do not forget to check the financiers' problem.

d) Consider again the entrepreneur's problem in period 1 assuming that she invested in the project in period 0. Show that there exists an equilibrium in the continuation economy starting in period 1 such that $q_1 = 0$ and the entrepreneur has to liquidate the project. We will call this a "debt run" equilibrium (in contrast to the "bank run" equilibrium seen in class). Can there be an equilibrium in which the entrepreneur undertakes the project in period 0 and there is a debt run in period 1 with certainty? Why?

e) Suppose that the government announces in period 0 that it will guarantee the debt of the entrepreneur in period 1. That is, the government promises to pay the entrepreneur's debt if the entrepreneur goes bankrupt. Show that the equilibrium of the economy is unique. What are the equilibrium prices? What is the fiscal cost of this policy? **Hint:** Assume that any revenue that the government needs is collected by taxing the financiers lump-sum.

Now let's change the environment a bit and assume that, in period 2, the project succeeds with probability $p < 1$ and pays R, but with probability $1 - p$, it fails and pays 0. Suppose that the probability p depends on whether the entrepreneur chooses the good or bad project in period 0. In particular, assume that if the agent chooses the good project, the project succeeds with probability $p^H \in (0, 1)$. If the entrepreneur chooses the bad project, the projects fail with certainty ($p^L = 0$), but the entrepreneur gets a private benefit of $B > 0$, with $B < I$. In case of liquidation, the project yields 0 as in the baseline case. Assume $p_H R > I$.

f) Characterize the choice of the entrepreneur if the government does not guarantee the debt.

i. Under what conditions does the entrepreneur get funding?

ii. Does the entrepreneur ever misbehave in equilibrium?

iii. Are debt runs possible? You don't need to do the math, just 1-2 sentences with the logic is enough.

g) Suppose the government guarantees the entrepreneur's debt.

i. Are the conditions for the entrepreneur to get funding different than in f)?

ii. Does the entrepreneur ever misbehave in equilibrium?

iii. Explain in words how government intervention can affect incentives.

g) Go back to the baseline case in which the entrepreneur cannot misbehave and $p = 1$. Take any $\pi \in \left(0, \frac{R-I}{R}\right]$. Show that there is an equilibrium in which $q_0 < 1$ and $q_1 = 1$ with probability $1 - \pi$ and $q_1 = 0$ with probability π. What happens if $\pi > \frac{R-I}{R}$?

MACROECONOMICS PRELIM, JUNE 2021

ANSWER KEY

QUESTION 1

Athanasios Geromichalos (ageromich@ucdavis.edu)

a) Let V, J_H, J_U and L be the value functions for a firm that is vacant, employing a healthy worker, employing an unhealthy worker who is currently working, and employing an unhealthy worker who is currently on sick leave, respectively. As explained in the question, wages for healthy and unhealthy workers will be differently determined through bargaining. Let each be denoted by w_H and w_U. Each value function is defined as follows:

$$\begin{aligned} rV &= -pc + q(\theta)\Big(\frac{u_H}{u}J_H + \frac{u_U}{u}J_U - V\Big) = -pc + q(\theta)(\pi J_H + (1-\pi)J_U - V) \\ rJ_H &= p - w_H - \lambda J_H \\ rJ_U &= p - w_U - \lambda J_U + \sigma(L - J_U) \\ rL &= -w_U - \lambda L + \varepsilon(J_U - L). \end{aligned}$$

b) Let U_H and W_H be the value functions for a healthy worker who is unemployed and employed, respectively. Each is defined as follows:

$$\begin{aligned} rU_H &= z + \theta q(\theta)(W_H - U_H) \\ rW_H &= w_H + \lambda(U_H - W_H). \end{aligned}$$

Let U_U, W_U and P be the value functions for an unhealthy worker who is unemployed, employed and currently working, and employed and currently on sick leave, respectively. Each is defined as follows:

$$\begin{aligned} rU_U &= z + \theta q(\theta)(W_U - U_U) \\ rW_U &= w_U + \lambda(U_U - W_U) + \sigma(P - W_U) \\ rP &= w_U + \lambda(U_U - P) + \varepsilon(W_U - P). \end{aligned}$$

c) Free entry implies $V = 0$ in equilibrium. The value function V together with $V = 0$ yields

$$\frac{pc}{q(\theta)} = \pi J_H + (1-\pi)J_U.$$

This equation is intuitive, saying that, because of free entry, the expected value of getting a worker should boil down to the expected amount of search costs that have to be incurred in order to get a worker. Since the expected vacancy duration is $1/q(\theta)$, the expected search costs are $pc/q(\theta)$, and the expected value of having a worker employed is $\pi J_H + (1-\pi)J_U$.

Now we need expressions for J_H and J_U. J_H can be obtained from the value function J_H:

$$J_H = \frac{p - w_H}{r + \lambda},$$

which says that the present value of hiring a healthy worker is instantaneous profits $p - w_H$ summed over a continuous time with the effective discount rate $r + \lambda$.

J_U can be obtained by combining the value functions J_U and L. First, subtracting the latter from the former generates

$$J_U - L = \frac{p}{r + \lambda + \sigma + \varepsilon}.$$

This is the value that the firm would lose if the worker goes on sick leave when a firm is hiring an unhealthy worker. Then, plugging this into the value function J_U gives us an expression for J_U:

$$J_U = \frac{p - w_U - \sigma \dfrac{p}{r + \lambda + \sigma + \varepsilon}}{r + \lambda},$$

which also has a straightforward interpretation. Instantaneously, a firm hiring an unhealthy worker who is currently working earns $p - w_U$. Besides this, however, a firm could lose some values $p/(r + \lambda + \sigma + \varepsilon)(= J_U - L)$ when the worker goes on sick leave with a rate σ. Considering these together and summing over a continuous time with the effective discount rate $r + \lambda$ gives us the above present value of having an unhealthy worker who is employed and currently working.

Putting what we have got all together produces the job creation (JC) curve:

$$\pi w_H + (1 - \pi) w_U = p - (1 - \pi)\sigma \frac{p}{r + \lambda + \sigma + \varepsilon} - (r + \lambda)\frac{pc}{q(\theta)}. \tag{JC}$$

The intuition behind the JC curve is simple. The expected wage that firms are willing to pay should reflect the instantaneous value of operating firms, which are expected production minus the expected search costs $pc/q(\theta)$ spread equally over a time period by being multiplied by $r + \lambda$. One remark is that here, unlike the baseline model, the expected production is not p but $p - (1 - \pi)\sigma p/(r + \lambda + \sigma + \varepsilon)$. It is because healthy workers can produce p, whereas unhealthy workers are expected to produce less than p by the amount that they are not going to be able to produce while on sick leave, $\sigma p/(r + \lambda + \sigma + \varepsilon)$, and the probability of being matched with an unhealthy worker is $1 - \pi$.

d) First let us derive the wage curve (WC) for healthy workers. The Nash bargaining problem between a healthy worker and a firm and its corresponding FOC are

$$\max_{w_H} (W_H - U_H)^{\beta} J_H^{1-\beta} \quad \Longrightarrow \quad (1 - \beta)(W_H - U_H) = \beta J_H.$$

This is the exactly same problem as the baseline model, and one can easily check

$$w_H = \beta p + (1 - \beta) z + \beta \theta q(\theta) J_H.$$

Next let us turn to the WC for unhealthy workers. The Natsh bargaining problem between an unhealthy worker and a firm and its corresponding FOC are

$$\max_{w_U}(W_U - U_U)^\beta J_U^{1-\beta} \quad \implies \quad (1-\beta)(W_U - U_U) = \beta J_U.$$

Now we need an expression to replace W_U. To this end, look jointly at the value functions W_U and P. Subtracting one from the other gives $W_U - P = 0$, i.e.,

$$W_U = P.$$

This is intuitive in that, from the workers' perspective, the value of being employed is same whether they are working or on sick leave since jobs could be destructed in both situations and they will be paid anyway in both situations. After some algebra, one can reach the following expression for w_U:

$$w_U = \beta p - \beta\sigma\frac{p}{r+\lambda+\sigma+\varepsilon} + (1-\beta)z + \beta\theta q(\theta)J_U.$$

Now, we finally obtain the WC as a weighted average of w_H and w_U:

$$\begin{aligned}
\pi w_H + (1-\pi)w_U &= \beta p - \beta(1-\pi)\sigma\frac{p}{r+\lambda+\sigma+\varepsilon} + (1-\beta)z + \beta\theta q(\theta)[\pi J_H + (1-\pi)J_U] \\
&= \beta\left(p - (1-\pi)\sigma\frac{p}{r+\lambda+\sigma+\varepsilon}\right) + (1-\beta)z + \beta pc\theta \qquad \text{(WC)} \\
&\left(= z + \beta\left(p - (1-\pi)\sigma\frac{p}{r+\lambda+\sigma+\varepsilon} - z + pc\theta\right)\right),
\end{aligned}$$

where in the second equality we have used $pc/q(\theta) = \pi J_H + (1-\pi)J_U$. The intuition is again straightforward. The expected wage that will be paid to workers should be equal to the outside option of workers, z, plus a share equal to the workers' bargaining power β of the total surplus created by forming a match. The surplus is equal to expected production net of the unemployment benefit z plus the savings of recruiting costs $pc\theta$. Recall that in this model the expected production is not p but $p-(1-\pi)\sigma p/(r+\lambda+\sigma+\varepsilon)$.

e) Equating the JC curve and the WC provides the condition that uniquely determines the equilibrium θ:

$$p - (1-\pi)\sigma\frac{p}{r+\lambda+\sigma+\varepsilon} - (r+\lambda)\frac{pc}{q(\theta)} = \beta\left(p - (1-\pi)\sigma\frac{p}{r+\lambda+\sigma+\varepsilon}\right) + (1-\beta)z + \beta pc\theta.$$

The only difference from the baseline model is that, instead of p, here we have

$$p - (1-\pi)\sigma\frac{p}{r+\lambda+\sigma+\varepsilon},$$

which is the expected production that reflects the expected production loss from unhealthy workers' sick leave, considering that firms could be matched with unhealthy workers. This term converges to p as $\pi \to 1$, $\sigma \to 0$, or $\varepsilon \to \infty$. The rationale behind each extreme case is as follows.

The result would be the same as the baseline model if there were no unhealthy workers from the beginning ($\pi \to 1$), or if unhealthy workers are never on sick leave. This could be achieved in two ways: (1) unhealthy workers never go on sick leave ($\sigma \to 0$), or (2) unhealthy workers come back to work right away after sick leave ($\varepsilon \to \infty$).

f) If the wage cannot be contingent upon the worker's type, now firms must pay U-workers the same wage as H-workers. In the baseline model, firms knew they may match with the (ultimately) less productive U-workers, but they also knew they would have to pay them a lower salary. This is not the case anymore, and it is reasonable to expect that, under the new specification, firm entry will be discouraged (as firms know they will have to pay U-workers higher wages). With lower firm entry we know for sure that output will go down and unemployment will go up, but the question is not about GDP or unemployment, it is about welfare! We know that in the MP model entry of firms could be too low or too high depending on parameters, most importantly, the firms' bargaining power $1 - \beta$.

Thus, if we are in a world where $\beta > \eta$ (i.e., β is higher than the value described by the Hosios condition), we know that firm entry is insufficient, and the new law will make the inefficiency even more severe. But if we are in a world where $\beta < \eta$, we know that firm entry is higher than optimal. In this case, it may very well be the case that the new law can improve welfare by discouraging firm entry.

This result is sometimes known as the Theorem of the second best, roughly meaning that if you live in a world with a well-documented inefficiency (here the search externality of the MP model), adding a new source of inefficiency or friction (here the law that forbids firms to make wages contingent on the worker's type) may mitigate the first inefficiency and ultimately improve welfare. There are many applications of this theorem in macroeconomics, and this is just one of them.

Question 2

James Cloyne (jcloyne@ucdavis.edu)

The U.S. government is considering a large increase in public investment. Using the New Keynesian model, this question asks you to analyze the macroeconomic effects of an increase in the public provision of capital and how monetary policy might respond.

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subject to their budget constraint:

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where w_t is the real wage, N_t is hours worked, B_t are real bond holdings at the end of period t, i_{t-1} is the nominal interest rate paid between $t-1$ and t, P_t is the price of the final consumption good and D_t are real profits from firms that are distributed lump sum. Assume $0 < \beta < 1$ and $0 < \mu < 1$. T_t are real lump sum taxes that the government levies on households to fund investment in public capital ($K_{G,t}$) which is used by firms. The government runs a balanced budget and issues no debt.

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Monetary policy follows a simple Taylor Rule: $$\hat{i}_t = \phi_\pi \hat{\pi}_t \tag{9}$$

Government investment (linearized) follows: $$\hat{g}_t = \rho \hat{g}_{t-1} + e_t \tag{10}$$

e_t is i.i.d. $\hat{g}_t$ is the change in government investment relative to steady state as a proportion of steady state output: $\hat{g}_t = \frac{G_t - G}{Y}$. This implies that, with 100% depreciation each period, $\hat{k}_{G,t} = \frac{1}{\gamma_g}\hat{g}_t$ where γ_g is the steady state share of government investment in GDP and assume $0 < \gamma_g < 1$. Government investment is financed entirely by lump sum taxes on households. In percentage deviations from steady state: $\hat{mc}_t$ is real marginal cost, $\hat{c}_t$ is consumption, $\hat{w}_t$ is the real wage, $\hat{n}_t$ is hours worked, $\hat{y}_t$ is output and $\hat{k}_{G,t}$ is public capital. In deviations from steady state: $\hat{i}_t$ is the nominal interest rate and $\hat{\pi}_t$ is inflation ($\hat{\pi}_t = \hat{p}_t - \hat{p}_{t-1}$; $\hat{p}_t$ is the percentage deviation of the price level from steady state). γ_c is the steady state share of consumption in GDP. λ is a function of model parameters, including the degree of price stickiness.[1] Assume that $\phi_\pi > 1$, $0 < \rho < 1$.

a) First consider the representative household's problem. Write down the household's problem in *recursive form* and derive the household's first order conditions.

Answer

$$V(B_{t-1}, K^G_{t-1}, G_t, \mathcal{B}_{t-1}) = \max_{C_t, B_t, N_t} \left\{ \mu \ln C_t + (1-\mu)\ln(1-N_t) + \beta E_t V(B_t, K^G_t, G_{t+1}, \mathcal{B}_t) \right\} \tag{11}$$

subject to

$$C_t + B_t = w_t N_t + (1 + i_{t-1})\frac{P_{t-1}}{P_t}B_{t-1} + D_t + T_t \tag{12}$$

where $\mathcal{B}$ are aggregate bond holdings. Setting this up as a Lagrangian (using λ_t as the multiplier), we can derive the first order conditions:

[1] $\lambda = \frac{(1-\theta)(1-\beta\theta)}{\theta}$ where θ is the probability that a firm cannot adjust its price.

$$\frac{\mu}{C_t} = \lambda_t \tag{13}$$

$$\frac{(1-\mu)}{1-N_t} = \lambda_t w_t \tag{14}$$

$$\lambda_t = \beta E_t \frac{\partial V}{\partial B_t} \tag{15}$$

Using the envelope condition (take partial derivative of the value function today wrt B_{t-1} and shift forward one period):

$$\lambda_t = \beta E_t \lambda_{t+1}(1+i_t)\frac{P_t}{P_{t+1}} \tag{16}$$

b) Show that the linearized first order condition for labor supply from part (a) is:

$$\hat{w}_t = \hat{c}_t + \psi \hat{n}_t$$

where $\psi = \frac{N}{1-N}$. N denotes steady state hours worked, $0 < N < 1$ and $\psi > 0$. Using this and the relevant equation above show that, with flexible prices, the natural level of output can be written as

$$\hat{y}_t^n = \Gamma \hat{g}_t \tag{17}$$

where

$$\Gamma \equiv \frac{1}{1+\psi\gamma_c}\left(1 + \frac{\gamma_c(1+\psi)\chi}{\gamma_g}\right)$$

Answer

After combining equations 13 and 14 taking the total derivative yields:

$$\frac{w\mu}{c}(\hat{w}_t - \hat{c}_t) = N\hat{n}_t \frac{1-\mu}{(1-N)^2} \tag{18}$$

The steady state terms cancel out (noting that $\frac{w\mu}{c} = \frac{1-\mu}{1-N}$) leaving:

$$\hat{w}_t = \hat{c}_t + \psi \hat{n}_t \tag{19}$$

where $\psi = \frac{N}{1-N}$

Next, note that

$$\hat{w}_t = \hat{mc}_t + \chi \hat{k}_{G,t} \tag{20}$$

Furthermore, the natural rate of output occurs under flexible prices, so $\hat{mc}_t = 0$. Because all firms are free to set the same price, there is no markup dispersion and

$\hat{mc}_t = 0$. Also making use of the production function and the resource constraint yields:

$$\chi \hat{k}_{G,t} = \frac{1}{\gamma_c}(\hat{y}_t - \hat{g}_t) + \psi(\hat{y}_t - \chi \hat{k}_{G,t}) \tag{21}$$

Solving for $\hat{y}_t$ and putting a superscript n to denote the level of output under the assumption of flexible prices yields the result in the question:

$$\hat{y}_t^n = \Gamma \hat{g}_t \tag{22}$$

where

$$\Gamma \equiv \frac{1}{1+\psi\gamma_c}\left(1 + \frac{\gamma_c(1+\psi)\chi}{\gamma_g}\right)$$

c) The full sticky price model can be simplified to 3 equations:

$$E_t\tilde{y}_{t+1} - \tilde{y}_t = \gamma_c(\hat{i}_t - E_t\hat{\pi}_{t+1} - \frac{1}{\gamma_c}(1-\Gamma)(1-\rho)\hat{g}_t) \tag{23}$$

$$\hat{\pi}_t = \beta E_t(\hat{\pi}_{t+1}) + \kappa\tilde{y}_t \tag{24}$$

$$\hat{i}_t = \phi_\pi \hat{\pi}_t \tag{25}$$

plus the stochastic process for government investment (equation (10)). $\tilde{y}_t = \hat{y}_t - \hat{y}_t^n$ is the output gap. $\kappa = \frac{1+\psi\gamma_c}{\gamma_c}\lambda$.

Using the method of undetermined coefficients, find the response of the output gap and inflation to an exogenous increase in government investment when prices are sticky and monetary policy follows the Taylor Rule above. To do this, guess that the solution for each variable is a linear function of the government investment shock $\hat{g}_t$.

Answer

Using the guesses into the Phillips Curve:

$$\Lambda_\pi \hat{g}_t = (\beta\Lambda_\pi\rho + \kappa\Lambda_y)\hat{g}_t$$

The solution must satisfy:

$$\Lambda_\pi = \frac{\kappa}{1-\beta\rho}\Lambda_y$$

Next, using the guesses, the AR(1) process and the policy rule in the dynamics IS curve:

$$\Lambda_y\hat{g}_t = \Lambda_y\rho\hat{g}_t - \gamma_c(\phi_\pi\Lambda_\pi\hat{g}_t - \Lambda_\pi\rho\hat{g}_t) - (\Gamma-1)(1-\rho)\hat{g}_t$$

Simplifying and substituting the result we found for Λ_π, the solution must satisfy:

$$\Lambda_y = \frac{(1-\rho)(1-\Gamma)}{1-\rho+\Psi}$$

where

$$\Psi = \frac{\kappa\gamma_c(\phi-\rho)}{(1-\beta\rho)}$$

and, from above,

$$\Lambda_\pi = \frac{\kappa}{1-\beta\rho}\Lambda_y$$

At this stage, the effect on the output gap and inflation of an increase in public investment is ambiguous. Note that the key question is whether Γ is less than one or not. If $\Gamma < 1$ these two expressions are positive. But Γ may also be greater than 1 depending on the value of χ.

d) Carefully discuss how, and why, an increase in public investment affects the natural level of output, the output gap and inflation in this model. Take care to explain how your answer depends on the value of χ (Hint: you may want to consider two cases: $\chi = 0$ vs. $\chi = 1$ and examine the expression for Γ).

Answer

As noted in part (c), without some further assumptions on χ it is not possible to know whether the increase in $\hat{g}_t$ raises or lowers the output gap and inflation. This shock has two different types of effects. First, this shock influences the economy in the same way as a pure government consumption shock. This can be seen when $\chi = 0$. In this case, the government spending shock is a pure demand shock (assuming the government spending is not useful, as is the case when $\chi = 0$). When prices are flexible all firms adjust prices and the natural rate of output rises because households want to supply more labor. But when prices are sticky, firms who don't adjust prices raise production following the increase in demand. Labor demand therefore increases. Inflation also increases with the demand shock (some firms adjust prices), although sticker prices will tend to limit the effect on inflation. In addition to the neoclassical effects on the natural rate, there is also a Keynesian-type demand effect. Because the output gap is positive, output increases by more in the sticky price model. Mathematically we can see this as follows. From the answers to parts (b) and (c), when $\chi = 0$, $\Gamma < 1$. The output gap and inflation always rise (under this monetary policy rule).

Now consider $\chi > 0$. This means the additional government spending influences productivity. In fact, because $K_{G,t} = G_t$, G_t (partly) shows up like a TFP shock. We know that technology shocks raise the natural rate of output but also generate falling prices. With price stickiness, output does not increase as much as it would if prices could fall freely. As a result, inflation and the output gap are negative. As χ gets larger, this effect starts to dominate the demand effect discussed above. For a large enough χ, $\Gamma > 1$ and inflation and the output gap become negative. Of course, the natural level of output still unambiguously increases (as can be seen from the equation in part (b)). The effect on the natural level is increasing in χ because a higher χ implies a larger effect of public capital on firm productivity.

e) According to the monetary policy rule in equation (9), show what happens to the nominal interest rate following an increase in public investment and discuss whether this depends on the value of χ. Explain. Can you think of a policy rule that would implement full stabilization of inflation and the output gap in this model? Explain. (Hint: for this second part you do not need to derive anything. You should be able to answer this from your knowledge of the model.)

Answer

According to the policy rule in equation (9), the path of interest rates mirrors the path for inflation found in part (c). When χ is sufficiently large, the inflation responsive is negative and interest rates decline. This mirrors the result for a TFP shock in the standard model: an improvement in TFP leads to falling interest rates, prices and the output gap. When χ is small, the demand effects start to dominate and prices and the output gap increase. In this case, interest rates also increase.

From the 3-equation setup in part (c) you can see that this is still like a shock to the natural real rate of interest, although it has elements more like a demand shock and other elements more like a TFP shock. It is potentially feasible to fully stabilize the output gap and inflation by setting the real interest rate equal to the natural real interest rate. Simply following this rule would not deliver a unique equilibrium, so the rule needs to be augmented by an inflation feedback term. A rule such as:

$$\hat{i}_t = \hat{r}_t^n + \phi\hat{\pi}_t$$

with $\phi > 1$ would implement full stabilization of inflation and the output gap as the unique equilibrium.

Question 3

Nicolas Caramp (ncaramp@ucdavis.edu)

a) *Consider the entrepreneur's problem in period* 1 *assuming that she invested in the project in period* 0 *and sold bonds for* b_1*. What is the value of* b_1*? What is the minimum price* q_1 *that allows the entrepreneur to avoid default? Express your answers as a function of* q_0*.*

The entrepreneur's budget constraint in period 0 is

$$xI \leq q_0 b_1$$

where $x = 1$ if the entrepreneur invests in the project and $x = 0$ if she doesn't. Assuming that $x = 1$, we get

$$b_1 \geq \frac{I}{q_0}.$$

Note that setting $b_1 > \frac{I}{q_0}$ is suboptimal, as the entrepreneur values consumption only in period 2. Then,

$$b_1 = \frac{I}{q_0}.$$

b) *Show that it is optimal for the entrepreneur to invest in period* 0 *if and only if*

$$q_0 \geq \frac{1}{q_1}\frac{I}{R}$$

If the entrepreneur doesn't invest in period 0, she gets $c_2 = 0$. If she invests, she sells bonds for $b_1 = \frac{I}{q_0}$ in period 0, which she needs to roll over in period 2:

$$b_1 \leq q_1 b_2 \implies b_2 = \frac{I}{q_0 q_1}.$$

Hence, consumption in period 2 would be

$$c_2 = R - b_2 \geq 0 \iff R - \frac{I}{q_0 q_1} \geq 0$$

or

$$q_0 \geq \frac{1}{q_1}\frac{I}{R}.$$

c) *Show that there exists an equilibrium in which* $q_0 = q_1 = 1$. ***Hint:*** *Do not forget to check the financiers' problem.*

If $q_0 = q_1 = 1$, we have that $c_2 = R - I > 0$, hence the entrepreneur wants to undertake the project.

The financiers solve

$$\max_{c_1, c_2, c_3, b_1, b_2} c_0 + c_1 + c_2$$

subject to

$$\begin{aligned} c_0 + q_0 b_1 &\leq e \\ c_1 + q_1 b_2 &\leq e + b_1 \\ c_2 &\leq e + b_2 \end{aligned}$$

The FOCs are

$$\begin{aligned} (c_0) &: 1 \leq \lambda_0 \\ (c_1) &: 1 \leq \lambda_1 \\ (c_2) &: 1 \leq \lambda_2 \\ (b_1) &: -q_0 \lambda_0 + \lambda_1 = 0 \\ (b_2) &: -q_1 \lambda_1 + \lambda_2 = 0 \end{aligned}$$

The large endowment assumption implies that $c_0, c_1, c_2 > 0$, hence,

$$\lambda_0 = 1, \quad \lambda_1 = 1, \quad \lambda_2 = 1$$

hence, the FOCs are satisfied only if

$$q_0 = 1, \quad q_1 = 1,$$

consistent with the guess.

d) *Consider again the entrepreneur's problem in period* 1 *assuming that she invested in the project in period* 0*. Show that there exists an equilibrium in the continuation economy starting in period* 1 *such that* $q_1 = 0$ *and the entrepreneur has to liquidate the project. We will call this a "debt run" equilibrium (in contrast to the "bank run" equilibrium seen in class). Can there be an equilibrium in which the entrepreneur undertakes the project in period* 0 *and there is a debt run in period* 1 *with certainty? Why?*

Suppose we are in $t = 1$ and the entrepreneur invested in the project in period 0. Suppose the financiers believe that the entrepreneur will not repay the debt in period 2. In that case, the cost of lending is q_1 in period 1 per unit of bond and the benefit is zero. Thus, optimality requires $q_1 = 0$. In this case, the entrepreneur gets no funding and she liquidates the project and defaults on b_1. If the financiers anticipate this outcome, they would not lend to the entrepreneur in period 0. Thus, an equilibrium cannot feature a run with certainty.

e) *Suppose that the government announces in period* 0 *that it will guarantee the debt of the entrepreneur in period* 1*. That is, the government promises to pay the entrepreneur's debt if the entrepreneur goes bankrupt. Show that the equilibrium of the economy is unique. What are the equilibrium prices? What is the fiscal cost of this policy?* ***Hint:*** *Assume that any revenue that the government needs is collected by taxing the financiers lump-sum.*

If the government guarantees all debts, then they become risk-free, so $q_0 = q_1 = 1$. We know from question b) that when facing this prices, the entrepreneur invests and is able to roll over her debt. Since the entrepreneur never defaults in this case, the fiscal cost of this policy is zero.

Now let's change the environment a bit and assume that, in period 2*, the project succeeds with probability* $p < 1$ *and pays* R*, but with probability* $1 - p$*, it fails and pays* 0*. Suppose that the probability* p *depends on whether the entrepreneur chooses the good or bad project in period* 0*. In particular, assume that if the agent chooses the good project, the project succeeds with probability* $p^H \in (0, 1)$*. If the entrepreneur chooses the bad project, the projects fail with certainty* ($p^L = 0$)*, but the entrepreneur gets a private benefit of* $B > 0$*, with* $B < I$*. In case of liquidation, the project yields* 0 *as in the baseline case. Assume* $p_H R > I$*.*

f) *Characterize the choice of the entrepreneur if the government does not guarantee the debt.*
i. Under what conditions does the entrepreneur get funding?
ii. Does the entrepreneur ever misbehave in equilibrium?
iii. Are debt runs possible? You don't need to do the math, just 1-2 sentences with the logic is enough.

i. The entrepreneur behaves if and only if

$$p_H R^E \geq B \text{ or } R^E \geq \frac{B}{p_H}$$

Since $R^E + R^F = R$, this implies

$$R^F \leq R - \frac{B}{p_H}$$

Since the entrepreneur has to repay b_2 from the proceeds of the project

$$\frac{I}{p_H q_0 q_1} \leq R - \frac{B}{p_H}$$

In the best case scenario for the entrepreneur in which $q_0 = q_1 = 1$,

$$I \leq p_H R - B$$

which is satsifed for some parameter values.

ii. No, it never misbehaves (the financiers never lend if they anticipate misbehavior).

iii. Yes, it is still possible that financiers coordinate and don't lend, so that the entrepreneur is not able to roll-over the debt. The logic is similar to the one in question d).

g) *Suppose the government guarantees the entrepreneur's debt.*
i. Are the conditions for the entrepreneur to get funding different than in f)?
ii. Does the entrepreneur ever misbehave in equilibrium?
iii. Explain in words how government intervention can affect incentives.

i. Even if $I > p_H R - B$, the financiers will lend to the entrepreneur because their debt is guaranteed.

ii. In this case, the entrepreneur will misbehave in period 1 and the project will fail with certainty in period 2.

iii. By guaranteeing the entrepreneur's debt, the government killed the monitoring incentives of the financiers.

h) *Go back to the baseline case in which the entrepreneur cannot misbehave and* $p = 1$. *Take any* $\pi \in \left(0, \frac{R-I}{R}\right]$. *Show that there is an equilibrium in which* $q_0 < 1$ *and* $q_1 = 1$ *with probability* $1 - \pi$ *and* $q_1 = 0$ *with probability* π. *What happens if* $\pi > \frac{R-I}{R}$*?*

The financiers' problem implies that they will lend in period 0 if and only if

$$q_0 \leq 1 - \pi,$$

since the entrepreneur repays only if $q_1 = 1$. Competition implies that $q_0 = 1 - \pi$. On the other hand, it is optimal for the entrepreneur to invest only if when $q_1 = 1$, consumption in period 2 is non negative:

$$c_2 \geq 0 \iff q_0 \geq \frac{I}{R}$$

Putting these two expressions together, we get

$$1 - \pi \geq \frac{I}{R}$$

or

$$\pi \leq \frac{R - I}{R}.$$

If $\pi > \frac{R-I}{R}$, q_0 would need to be so low to compensate the financiers, that it's not worth for the entrepreneur to invest in period 0 (even if they undertook the project, they would at least partially default in period 2, which forces $q_0 = 0$). Interestingly, this conditions hold even though the project is **always** efficient.