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Department of Economics
Macroeconomics

Date: June 29, 2015
Time: 5 hours
Reading Time: 20 minutes

PRELIMINARY EXAMINATION FOR THE Ph.D. DEGREE

Directions: Answer all questions. Feel free to impose additional structure on the problems below, but please state your assumptions clearly. Point totals for each question are given in parentheses.

1. (10) Briefly discuss the following statements (keep your answers short and concise):

   (a) The consumption-based capital asset pricing model is inconsistent with high volatility of stock prices.
   Answer: I disagree with this. Consider the Lucas tree model - that is, a representative agent exchange economy so that output = consumption = dividend = y_t. Given this scenario, the price of equity, q_t, would be a function of the endowment and given by

   $$q(y_t) = \frac{\beta E_t [U'(y_{t+1})(q(y_{t+1}) + y_{t+1})]}{U'(y_t)}$$

   To simplify things, assume that the endowment is i.i.d. so that the numerator is a constant. Then the volatility of stock prices will be determined by the change in agents' marginal utility of consumption. Furthermore, the elasticity of stock prices with respect to the endowment is

   $$\frac{dq}{dy}\frac{y_t}{q_t} = -\frac{U''(y_t)}{U'(y_t)}y_t = relative\ risk\ aversioin$$

   So high volatility of stock prices is possible if agents have high relative risk aversion. Things are more complicated if the endowment is serially correlated but an argument based on the Euler equation for asset prices would be the way to go..

   (b) In standard real business cycle models, the MPK is highly procyclical. This implies that interest rates (i.e. real) will be as well.

   Answer: This is not necessarily the case. The interest rate in RBC models is determined by the yield on consumption loans with the real interest rate being the price of current consumption relative to future consumption. Suppose the technology shock is high - this implies that the current MPK is high. But it also implies that consumption is relatively high (especially if the shocks have little persistence). Hence consumption will be relatively more abundant today implying a low real interest rate - real interest rates are predicted to be countercyclical. Again, things get messy if shocks exhibit serial correlation, but a correct answer will follow this kind of logic.

2. (20) An economy is populated by identical, infinitely-lived agents (there is no population growth) that maximize the present discounted value of lifetime utility given by

$$\sum_{t=0}^{\infty} \beta^t \ln c_t; \ \beta \in (0,1)$$

where c_t denotes consumption. Output is produced via a standard Cobb-Douglas production function:

$$y_t = k_t^{\alpha}$$

where k_t denotes the beginning of period capital stock. (Implicitly it is assumed that labor supply is inelastically supplied by households to firms and that the labor input has been normalized to unity. Hence the labor market is ignored in this analysis.) In addition to consumption, households choose investment. This produces next period's capital stock using the following production function:

$$k_{t+1} = k_t^{\delta_t} i_t^{1-\delta_t}$$

where δ_t denotes a stochastic depreciation factor. It is assumed that this shock, the only source of uncertainty in the economy, is an *i.i.d.* random variable. Given this environment, do the following:

(a) Solve for the recursive competitive equilibrium by solving the associated social planner's problem. In setting up the dynamic programming problem, use two constraints: the typical resource constraint and the capital production constraint. Denote the Lagrange multiplier on the budget constraint as λ_t while the Lagrange multiplier on the capital production constraint is given by the product of $\lambda_t q_t$ where q_t is the shadow price of capital (in terms of consumption).

(b) Derive the associated Euler equations for the social planner problem. Give an intuitive explanation for the determination of q_t.

(c) Define and solve for the recursive competitive equilibrium in this economy. (Note: This is simplified by first using output (y_t) as a state variable and then employing the guess and verify solution method.)

(d) Give an intuitive explanation for the behavior of consumption and investment in this economy.

Answer: The associated social planner problem is:

$$V(k_t, \delta_t) = \max_{(c_t, i_t, k_{t+1})} \left\{ \begin{array}{c} \ln c_t + \beta E\left[V(k_{t+1}, \delta_{t+1})\right] \\ +\lambda_t \left[k_t^{\alpha} - c_t - i_t\right] \\ +\lambda_t q_t \left[k_t^{\delta_t} i_t^{1-\delta_t} - k_{t+1}\right] \end{array} \right\}$$

The associated necessary conditions (after using the envelope theorem) are:

$$\frac{1}{q_t} = (1-\delta_t) k_t^{\delta_t} i_t^{-\delta_t} \tag{1}$$

$$\frac{q_t}{c_t} = \beta E\left[\frac{1}{c_{t+1}}\left(\alpha \frac{y_{t+1}}{k_{t+1}} + q_{t+1}\delta_{t+1}\frac{k_{t+2}}{k_{t+1}}\right)\right] \tag{2}$$

Note that eq.(1) implies that the price of consumption in terms of capital (the inverse of q_t) is simply the MP of investment with regard to new capital. This makes sense. And note that in eq.(2) I have made use of both production functions. Multiplying both sides of eq.(1) by i_t implies that $q_t = \left(\frac{1}{1-\delta_t}\right)\frac{i_t}{k_{t+1}}$. Using this in the Euler equation for capital yields:

$$\left(\frac{1}{1-\delta_t}\right)\frac{i_t}{c_t} = \beta E\left[\alpha \frac{y_{t+1}}{c_{t+1}} + \left(\frac{\delta_{t+1}}{1-\delta_{t+1}}\right)\frac{i_{t+1}}{c_{t+1}}\right] \tag{3}$$

The state vector in the dynamic programming problem is identified as (k_t, δ_t) but the role of beginning of period capital is entirely captured through its effect on output. So use this to define the state vector as (y_t, δ_t). Then a Recursive Competitive Equilibrium is defined, in general, by three functions: $c(y_t, \delta_t), i(y_t, \delta_t)$, and $q(y_t, \delta_t)$. (This is a cursory definition of a RCE. A complete answer would discuss the policy functions for the agent and how the relevant state variables are the individual and aggregate capital stocks, in addition to the δ shock. Also a law of motion for the aggregate capital stock would be known by agents and, in equilibrium, there would be consistency between the agents and aggregate law of motion for the capital stock. Something along those lines would be more appropriate.) However, given the resource constraint, once the function for consumption is determined, then the investment function is also determined. And, due to eq.(1), the consumption function and production function also imply the equilibrium function for $q(y_t, \delta_t)$. So, more precisely, solving for the consumption function, $c(y_t, \delta_t)$, does solve for the RCE. This task is made easier by the setup: it is reasonable, as we saw many times in class), to conjecture that this function is homogeneous of degree 1 in y_t. That is, $c(y_t, \delta_t) = y_t \omega(\delta_t)$. If this conjecture is correct, then the necessary conditions will define $\omega(\delta_t)$. Note that $i(y_t, \delta_t) = (1 - \omega(\delta_t)) y_t$ and using this in eq.(3) yields:

$$\left(\frac{1}{1-\delta_t}\right)\left(\frac{1-\omega(\delta_t)}{\omega(\delta_t)}\right) = \beta E\left[\alpha\frac{1}{\omega(\delta_{t+1})} + \left(\frac{\delta_{t+1}}{1-\delta_{t+1}}\right)\left(\frac{1-\omega(\delta_{t+1})}{\omega(\delta_{t+1})}\right)\right] = \Omega \tag{4}$$

Where Ω is a constant due to the assumption of *i.i.d.* depreciation shocks. So, we have:

$$\omega(\delta_t) = [1 + \Omega(1-\delta_t)]^{-1}$$

This verifies the conjecture. Note that this implies that a depreciation shock results in a greater fraction of output being consumed. This makes sense given that a higher value of δ_t (ceteris paribus) reduces the MP of investment (or, as seen in eq.(1), increases the price of capital in terms of consumption (q_t)).

3. (20) Consider a standard Solow growth model that is augmented with labor migration. As is typical, the aggregate production function is given by $Y = (AL)^{\alpha} K^{1-\alpha}$ where Y is output, A is effectiveness of labor, L is labor, and K is capital. Also, as is typical, the law of motion for aggregate physical capital is given by $\dot{K} = sY - \delta K$ where $\dot{K} = dK/dt$, s is the savings rate and δ is the depreciation rate. The effectiveness of labor grows at the constant rate of $g : \dot{A} = gA$. In addition to population growth (given by the rate n), the country experiences migration M so that $\dot{L} = (nL + M)$. Migrants bring no physical capital and assume that the migration rate is positively related to the capital per worker. In particular, assume that the migration rate $m = M/L$ is given by $m = b\log(1+k)$ where $b > 0$ and $k = K/(AL)$ is the capital per effective units of labor. Given this, do the following:

   (a) Derive the expression for $\dot{k}$ in this economy. Compare this to the expression in the standard Solow model.

   (b) As in the standard Solow model, analyze graphically the behavior of the economy in a graph with k on the horizontal axis. Let k^*denote the balanced growth path level of k and compare k^*in the economy with migration to that in the standard Solow model.

   (c) Analyze using a phase diagram the stability properties of the balanced growth path for the case with migration.

   (d) Linearize the $\dot{k}$ function around the balanced growth path, define the speed of convergence of the economy to its steady-state and calculate it. Compare the speed of convergence in this economy to that in the standard Solow model.

ANSWER: Given the definition of k, we have that:

$$\dot{k} = \frac{dk}{dt} = sk^{1-\alpha} - k\left(\delta + g + \frac{\dot{L}}{L}\right)$$

where I have used the definition of $\dot{K}$ and the intensive form of the production function. Hence we have:

$$\begin{aligned} Solow &: \quad \dot{k} = sk^{1-\alpha} - k(\delta + g + n) \\ Migration &: \quad \dot{k} = sk^{1-\alpha} - k(\delta + g + n + b\log(1+k)) \end{aligned}$$

For the Solow model, the balanced growth path level of capital per labor efficiency units is given by:

$$sk_S^{*1-\alpha} = k_S^*(\delta + g + n)$$

This is the standard graph in which the saving function (a concave function) intersects with the linear function defined by the right hand side expression. For the model with migration we have:

$$sk_M^{*1-\alpha} = k_M^*(\delta + g + n + b\log(1 + k_M^*))$$

The right hand side expression defines a convex function in k which lies above the function defined in the Solow model. Hence we have:

$$k_M^* < k_S^*$$

The phase diagram analysis with $\dot{k}$ on the vertical axis and k on the horizontal follows directly from these functions and is straightforward. Linearizing the two expressions for $\dot{k}$ and using the steady-state level of k^* for the two models yields:

$$\begin{aligned} Solow &: \quad \frac{\dot{k}}{k} = -(\alpha)(\delta + g + n)\left(\frac{k - k_S^*}{k}\right) \\ Migration &: \quad \frac{\dot{k}}{k} = -\left((\alpha)\Gamma + b\frac{k_M^*}{1 + k_M^*}\right)\left(\frac{k - k_M^*}{k}\right) \end{aligned}$$

where $\Gamma = (\delta + g + n + b\log(1 + k_M^*))$ for the model with migration. Because of the additional positive terms in the model with migration, the implication is that the speed of convergence is greater relative to the basic Solow model.

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ANSWER KEY FOR QUESTION 4,5, 6

Question 4

a) We have the following value functions:

$$\begin{aligned} rV &= -pc + q(\theta)(M - V), & (1)\\ rM &= a(J - M), & (2)\\ rJ &= p - w - \lambda J. & (3) \end{aligned}$$

The intuition is straightforward. For instance, V is as in the lectures, except when the worker arrives, the firm goes to state M not J. At state M the firm is not paying any wage but also not producing. It is basically waiting for the training to be over, in which case it moves to the state J. J is as in the lectures.

b) For the worker we have

$$\begin{aligned} rU &= z + \theta q(\theta)(T - U), & (4)\\ rT &= a(W - T), & (5)\\ rW &= w + \lambda(U - W), & (6) \end{aligned}$$

where the interpretation of these functions is the same as in part (a).

c) The free-entry condition $V = 0$ implies that

$$M = \frac{pc}{q(\theta)}. \qquad (7)$$

Combining this with (2) implies that

$$J = \frac{a+r}{a}\frac{pc}{q(\theta)}. \qquad (8)$$

Now combine (8) and (3) to get the job creation curve:

$$w = p - \frac{(r+\lambda)pc}{q(\theta)}\frac{a+r}{a}.$$

As I hinted, this gives you the standard JC curve of the textbook as $a \to \infty$. Regardless of the term a showing up, this is still a negatively-sloped curve in the (θ, w) space.

d) The firm and worker are solving[1]

$$\max_{w}(T - U)^{\beta} M^{1-\beta}. \qquad (9)$$

[1] This is exactly what I was describing in the footnote. Although the two parties are negotiating over something that will be paid in the future, the problem is to split the surplus generated at the moment when the two are bargaining. That surplus is $T - U$ for the worker and M for the firm. Notice that this makes prefect sense, since the term T includes the term W (the value of a productive worker) and it is only adjusted to reflect the waiting time till production starts (a similar comment applies for the terms M and J).

Like in the lecture notes, this problem leads to

$$(1-\beta)(T-U) = \beta M. \tag{10}$$

After a little algebra (the steps of which are identical to the ones seen many times in class and homeworks), we get:

$$w = \beta p + (1-\beta) rU \left(1 + \frac{r+\lambda}{a}\right). \tag{11}$$

As in the lecture notes, the last task is to get rid of rU. Use (10) to write (4) as

$$rU = z + \theta q(\theta) \frac{\beta}{1-\beta} M,$$

and use (7) to write the last expression as

$$rU = z + \frac{\beta \theta p c}{1-\beta}.$$

Plugging the last expression for rU back into (11) yields the wage curve:

$$w = \beta p + \left(1 + \frac{r+\lambda}{a}\right) \left[\beta p c \theta + (1-\beta) z\right],$$

which also coincides with the textbook formula as $a \to \infty$. Regardless of the term a showing up, this is still a positively-sloped curve in the (θ, w) space.

e) We have seen that the WC is positively sloped and the JC negatively sloped. So for existence, we basically need to make sure that the intercept of the JC curve when $\theta = 0$ lies above the intercept of the WC curve when $\theta = 0$. If this is true the equilibrium will exist and it will be unique (I asked you to not get into too much detail, so basically there is no need to see what happens as $\theta \to \infty$). When $\theta = 0$ the WC gives:

$$w_{WC}(0) = \beta p + \left(1 + \frac{r+\lambda}{a}\right)(1-\beta) z.$$

When $\theta = 0$ the JC gives:

$$w_{JC}(0) = p.$$

So for existence we need

$$w_{WC}(0) > w_{JC}(0) \;\Leftrightarrow\; \beta p + \left(1 + \frac{r+\lambda}{a}\right)(1-\beta) z > p,$$

which after a little algebra yields the much simpler and intuitive condition

$$p > \left(1 + \frac{r+\lambda}{a}\right) z.$$

With $a < \infty$ having $p > z$ is not good enough for an equilibrium to exist. The p has to be even bigger in order to compensate for the fact that jobs do not start immediately, but have to go through the training period. The lower the a, the higher the p must be (for given z). Of course, as $a \to \infty$, we get the usual sufficient condition $p > z$.

f) If a goes down it is easy (and intuitive) to see that both the WC and the JC will shift to the left. Hence, without any doubt we will have a reduction in θ, which is not surprising because a decrease in a is essentially a decrease in productivity (so fewer firms wish to enter in this market).

Since both curves shift to the left, the effect on equilibrium wage is unclear, which is an interesting result. What is going on here is that there are two opposing forces. On the one hand, the lower a, which effectively means a lower productivity, lowers θ and tends to lower the wages too (through the JC effect). On the other hand, the lower a means that workers must wait longer before they start getting paid, and so they will demand higher compensations in order to agree to work for the firm. Which force "wins" is unclear. Identifying these two forces is more than enough to get full credit here.

g) Let u, t, e be the measure of workers in the states of unemployment, training, and employment, respectively ("employment" here means workers who are producing and getting paid). We have $\theta q(\theta)u$ workers moving out of U and into the T state (no worker goes directly from U to W). We also have at workers moving from T into the state of employment. Finally, we have λe workers moving from employment to unemployment. To equate the inflow and outflow of workers at states U and T, respectively, we need

$$\theta q(\theta)u = \lambda e. \tag{12}$$
$$\theta q(\theta)u = at. \tag{13}$$

These two equations together with $u+t+e=1$ form a very simple system of 3 equations and three unknowns. Solving it with respect to u yields the Beveridge curve:

$$u = \frac{a\lambda}{a\lambda + \theta q(\theta)(a+\lambda)}.$$

As always, taking the limit as a goes to infinity will give you the well-known textbook Beveridge curve. Also, this is a standard downward sloping curve in the (v, u) space.

Regaridng the effect of a on unemployment, it is easy to see that a lower a will shift the BC to the right. Moreover, we know from part (f) that the equilibrium θ went down. Hence, with no doubt equilibrium unemployment will go up.

Question 5

a) Since getting the budget constraint right here is very important let's write that separately first. The household has an income which it can allocate to either c or i. However, as I clearly implied in the hint, for any extra unit that goes into i the household's income (or resources) will go down by τ. Hence, we have

$$c + i = w + rk - \tau i + T,$$

which can be re-written as

$$c + i(1+\tau) = w + rk + T,$$

and since we know that $k' = (1-\delta)k + i$ (or, if you prefer $i = k' - (1-\delta)k$), the budget constraint becomes

$$c = w + [r + (1-\delta)(1+\tau)]k - (1+\tau)k' + T.$$

Hence, the typical household's problem can be written recursively as

$$\begin{aligned} V(k,K) &= \max_{c,k'} \left\{ u(c) + \beta V(k',K') \right\} \\ s.t.\ \ c &= w + [r + (1-\delta)(1+\tau)]k - (1+\tau)k' + T, && (14) \\ K' &= H(K), && (15) \\ w &= w(K) = F_2(K,1), && (16) \\ r &= r(K) = F_1(K,1), && (17) \\ T &= T(K) = \tau[H(K) - (1-\delta)K], && (18) \end{aligned}$$

where k is the individual capital, and K is the aggregate capital. Moreover, (14) is the household's budget constraint, (15) is the aggregate law of motion of capital, (16) and (17) follow directly from market clearing, and (18) is the government revenue as a function of the aggregate state.

The definition of a RCE is standard, and can be found in the lecture notes (or in problem 4 of PS 6). In the definition of RCE, the most important part is to clarify that consistency requires $g(K,K) = H(K)$, where H was defined above, and g is the typical household's policy function.

b) The Euler equation for the typical household is given by

$$u'(c)(1+\tau) = \beta[F_1(K',1) + (1-\delta)(1+\tau)]\, u'(c').$$

But the objective here was to express everything as a function of the aggregate capital stock only. To that end, notice that we can use the budget constraint to write consumption in a more useful form. First impose consistency (i.e., $k = K$) to obtain

$$c = w + [r + (1-\delta)(1+\tau)]K - (1+\tau)K' + T.$$

Next, replace the prices and the total revenue T with the terms that include the aggregate capital stock (i.e., $r = F_1(K,1)$, $w = F_2(K,1)$, and $T = \tau[K' - (1-\delta)K]$). We obtain:

$$\begin{aligned} c &= F_2(K,1) + [F_1(K,1) + (1-\delta)(1+\tau)]K - (1+\tau)K' + \tau[K' - (1-\delta)K] = \\ &= F_2(K,1) + F_1(K,1)K + (1-\delta)K - K'. \end{aligned}$$

Finally, exploiting Euler's Theorem and the definition of $f(K)$, we find that

$$c = f(K) - K'.$$

Using this expression back into the Euler equation, and noticing that $f'(K) = F_1(K,1) + 1 - \delta$, we obtain the second-order difference equation, which describes the law of motion of aggregate capital:

$$u'(f(K) - K')(1+\tau) = \beta[f'(K) + \tau(1-\delta)]\, u'(f(K') - K'').$$

c) In the steady-state equilibrium $c = c'$ and $K = K' = K''$. Imposing this condition on the Euler equation, it is easy to show that the steady state capital stock is given by:

$$K^*(\tau) \equiv \left\{ K : f'(K) = \frac{1 + \tau[1 - \beta(1-\delta)]}{\beta} \right\}. \tag{19}$$

d) When $\tau = 0$, it is easy to check that $K^*(0)$ solves $f'(K^*(0)) = 1/\beta$, which, of course, is exactly the same as the steady-state level of capital in the baseline model with no government and taxes (or, if you prefer, the same as in the social planner's problem). If $\tau = 1$, we have $f'(K^*) = [2 - \beta(1-\delta)]/\beta$.

e) No. As we just saw, for $\tau = 1$ we have $f'(K^*) = [2 - \beta(1-\delta)]/\beta$, which means that $K^*(1) > 0$. The reason why a proportional tax (equal to 100%) on capital income and on investment do not lead to the same result is simple. The tax on capital income is a tax on ALL the household's capital. The tax on investment is just a tax on new capital that comes to replace the depreciated one, so clearly it is not as severe.

f) With $F(K,N) = K^a N^{1-a}$, we have $f(K) = K^a + (1-\delta)K$ and $f'(K) = aK^{a-1} + 1 - \delta$. Using these functional forms in (19), we get

$$K^*(\tau) = \left[\frac{a\beta}{(1+\tau)[1 - \beta(1-\delta)]} \right]^{\frac{1}{1-a}}. \tag{20}$$

g) In the steady state, $T = T(K^*) = \tau\delta K^*$, which makes a lot of sense, since in the steady state investment is just enough to replace the depreciated capital. Replacing for K^* from (20) (with $a = 1/2$), we can write the tax revenue as

$$T = \frac{\delta\beta^2}{4[1 - \beta(1-\delta)]^2} \frac{\tau}{(1+\tau)^2}. \tag{21}$$

The first fraction on the LHS just a constant multiplier, hence, taking the FOC on this expression is very easy. If you do it, you will find that the tax rate that maximizes T is $\tau = 1$, i.e., if the governments objective is to maximize tax revenue they should set a 100% tax. The Laffer curve is just an increasing (and concave) function throughout the whole domain $\tau \in [0,1]$.

Question 6

a) The state variables for a buyer who enters the CM market are her money holdings, m, and (potentially) some debt, d, (or credit, if $d < 0$) that she is carrying over from the LM which occurred earlier in the day. Hence,

$$W(m,d) = \max_{X,H,\hat{m}} \left\{ U(X) - H + \beta \left[\pi \Omega^C(\hat{m}) + (1-\pi)\Omega^N(\hat{m}) \right] \right\},$$
$$s.t. \quad X + \phi \hat{m} = H + \phi m + T - d.$$

This is the only part in which I asked you to work out the VF a bit more (by asking you to show the linearity property). If you substitute for H in the objective function, and observe that (as always) we have $X = X^*$ at the optimum, we can re-write

$$W(m,d) = \Lambda + \phi m - d,$$

which establishes the linearity property, and where we have defined

$$\Lambda \equiv U(X^*) - X^* + T + \max_{\hat{m}} \left\{ -\phi \hat{m} + \beta \left[\pi \Omega^C(\hat{m}) + (1-\pi)\Omega^N(\hat{m}) \right] \right\}.$$

b) As I revealed in the hint, q is a function of the money holdings of the (C-type) buyer, and, of course, the same is true of p, since (q,p) are the terms of trade in a typical DM meeting and they are jointly determined.

Regaring l, d, these terms typically depend on the money holdings of both types. Clearly, the N-type's holdings matter because they affect the size of the loan that she can give to the C-type (the loan cannot physically exceed the amount that the N-type is carrying). However, the amount that the C-type carries also matters (in a more indirect sense) because the C-type's initial money holdings determine how much money she will need to borrow.[2]

c) As in the case of W, the state variables for a buyer who enters the DM are the units of money held, m (because this determines how much good she can purchase), and whatever debt she is carrying over from the preceding LM (because this affects her continuation value). We have

$$V(m,d) = u(q) + W(m-p, d).$$

Of course, here p, q are functions of m (as explained in part (b)), but full credit was given even if you were not explicit about this.

d) As I already explained in the question, the only state variable here is the buyer's money holdings m. If the buyer turns out to be a C-type, we have

$$\Omega^C(m) = \frac{f(\pi, 1-\pi)}{\pi} V(m+l, d) + \left[1 - \frac{f(\pi, 1-\pi)}{\pi} \right] V(m, 0),$$

[2] To see this point more clearly, assume that the C-type has brought with her 10 dollars, but in order to purchase the first best, q^*, she needs 15 dollars (in the exam I did not define q^*, but, again, in the exam you did not need to provide all these details). Then, a C-type who meets an N-type in the LM would like to borrow 5 dollars from her. However, if the C-type had 15 dollars with her, she would not need to borrow anything. Hence, the C-type's money also matters.

where the term f/π is just the probability with which the typical C-type will match with someone in the LM.

Finally, let's go to the VF of the typical N-type in the LM. We have

$$\Omega^N(m) = \frac{f(\pi, 1-\pi)}{1-\pi} W(m-l, -d) + \left[1 - \frac{f(\pi, 1-\pi)}{1-\pi}\right] W(m, 0).$$

Notice that unlike the C-type, whose continuation value is represented by the function V (because she is going in the DM to consume), the N-type's continuation value is represented by W because this agent proceeds directly to the CM (i.e., does not wish to consume in the DM of the current period). Also, as in part (c), here it is OK if you were not too precise about the fact that the terms l, d may be functions of m (but if you did, good for you!)