University of California, Davis
Department of Economics

PRELIMINARY EXAMINATION FOR THE Ph.D. DEGREE

MACROECONOMICS

July 2, 2018

Directions: The exam consists of six questions. Questions 1,2 concern ECN 200D (Geromichalos), questions 3,4 concern ECN 200E (Cloyne), and questions 5,6 concern ECN 200F (Caramp). You only need to answer five out of the six questions. If you prefer (and have time), you can answer all six questions, and your grade will be based upon the best five scores. Feel free to impose additional structure on the problems below, but please state your assumptions clearly. You have 5 hours to complete the exam and an additional 20 minutes of reading time.

Question 1 (*20 points*)

Consider the Mortensen-Pissarides model in **discrete time**. The labor force is normalized at 1. Let u denote the unemployment rate. There is a large number of firms who can enter the market and search for a worker. Firms who engage in search have to pay a fixed cost k per period until they find a worker. If in any given period a measure v of vacant firms search for workers, then the total number of matches created in the economy is given by

$$m(u, v) = \frac{uv}{u + v}.$$

Each vacant firm has one job opening. Within each match, the firm and the worker bargain (a la Nash) for the wage, w, with η denoting the bargaining power of the worker. If they agree, they move on to production, which will deliver output equal to p per period. All agents discount future at rate $\beta \in (0, 1)$. At the end of every period (after production has taken place), existing matches get destroyed with probability δ.[1]

So far this is just the standard model (in discrete time). We now make two assumptions that depart from the baseline model.

First, the unemployment benefit, z, does not represent utility of leisure or value of home production, as we conveniently assumed in class. Here, z is a payment made by the government and, naturally, this payment needs to be funded somehow. We assume that the government raises these funds by imposing a lump-sum (flat) tax τ (per period) on every matched firm. Thus, the government chooses both z and τ, and must do so in a way so that the budget constraint is satisfied at any t.

The second assumption concerns the duration of unemployment benefits. In particular, we will assume that workers are eligible for unemployment benefits only for one period.[2] (This assumption would be quite realistic for the US, if we were to assume that a period of the model corresponds to 6 months.)

a) Describe the Beveridge curve (BC) of this economy in steady state, i.e., express u as a function of the market tightness $\theta \equiv v/u$.

b) This model predicts that a certain level of unemployment will persist even in the steady state. What is perhaps a little more subtle is that workers who are currently

[1] A worker who was part of a match that got hit by the destruction shock at the end of period t, will be unemployed for sure in $t+1$ and will try to find a job again in (the beginning of) period $t+2$.

[2] Consider again the worker described in footnote 1, i.e., a worker who was part of a match that got hit by the destruction shock at the end of period t. This worker will be unemployed for sure in $t+1$ and will receive z. Starting in period $t+2$, she will try to find a new job. If she is successful, she will move (immediately) into production. If she is unsuccessful, she will remain unemployed for another period, and, importantly, during that period she will *not* be eligible for unemployment benefits. This process will continue until the worker finds employment.

in the pool of unemployment have been unemployed for different periods of time. This is especially relevant in our question, where unemployment benefit eligibility depends on the duration of unemployment. Describe the measure of workers who have been unemployed for i periods, $i = \{1, 2, 3, ...\}$.[3] Verify that your result is correct by adding up the various unemployment durations. (They should add up to the steady state u!)

c) Describe the value function for a vacant firm (V) and a firm that has filled its vacancy (J).

d) Describe the value function of a typical worker in the various states.

e) Exploiting the usual free entry argument, derive the job creation (JC) condition.

f) Describe the wage curve (WC) in this economy.

g) What is the relationship between τ and z, u so that the government's budget constraint is satisfied in every period? Use this condition in order to get rid of τ in the WC and JC expressions you derived earlier.

h) Plot the JC curve in the (w, θ) space. Does it have the standard shape?

i) Plot the WC in the (w, θ) space. Does it have the standard shape?

j) Shortly discuss the existence and uniqueness of a steady state equilibrium.

[3] More precisely, workers who have been unemployed for 1 period are the ones who lost their jobs at the end of the previous period.

Question 2 (*20 points*)

This question studies the co-existence of different currencies. Time is discrete with an infinite horizon. Each period consists of two subperiods. In the day, trade is bilateral and anonymous as in Kiyotaki and Wright (1989) (call this the KW market). At night trade takes place in a Walrasian or centralized market (call this the CM). There are two types of agents, buyers and sellers, and the measure of both is normalized to 1. The per period utility for buyers is $u(q) + U(X) - H$, and for sellers it is $-q + U(X) - H$, where q is the quantity of the day good produced by the seller and consumed by the buyer, X is consumption of the night good (the numeraire), and H is hours worked in the CM. In the CM, all agents have access to a technology that turns one unit of work into a unit of good. The functions u, U satisfy the usual assumptions; I will only spell out the most crucial ones: There exists $X^* \in (0, \infty)$ such that $U'(X^*) = 1$, and we define the first-best quantity traded in the KW market as $q^* \equiv \{q : u'(q^*) = 1\}$.

We will assume that there are two types of money, m_1 and m_2.[4] There are also two types of sellers. For reasons that we will leave out of the model, Type-1 sellers, with measure $\sigma \in [0, 1]$, do not recognize m_2, thus, they accept only the local currency m_1. Type-2 sellers, with measure $1 - \sigma$, recognize and, hence, accept m_2, as well as m_1. Hence, local currency has a liquidity advantage over the foreign one, since it is recognized by all sellers. All buyers meet a seller in the KW market, so that σ is the probability with which a buyer meets a type-1 seller, and $1 - \sigma$ is the probability with which she meets a type-2 seller. In any type of meeting, buyers have **all the bargaining power**.

The rest is standard. Goods are non storable. The supply of each money is controlled by an individual authority, and evolves according to $M_{i,t+1} = (1 + \mu_i)M_{i,t}$. New money (of both types) is introduced, or withdrawn if $\mu_i < 0$, via lump-sum transfers to buyers in the CM. Throughout this question focus on steady states.

a) Describe the value function of a buyer and a seller who enter the Walrasian market with arbitrary money holdings (m_1, m_2).

b) Desribe the terms of trade in each type of KW meeting.

c) Describe the objective function of the typical buyer, $J(m_1', m_2')$.

d) For any given (μ_1, μ_2), $\mu_i \geq \beta - 1$, for all i, describe the steady-state equilibrium, summarized by the variables $\{q_1, q_2, z_1, z_2\}$, where q_i is the amount of special good traded in a KW meeting with a seller of type $i = \{1, 2\}$, and z_i denotes the equilibrium

[4] One possible interpretation is that this is a model of a Latin American economy, and m_1 is the local currency (e.g., the peso) while m_2 is the US dollar. Of course, this is a very simple model, so one should not take this suggestion too literally.

real balances of money $i = 1, 2$. (**Hint:** For now, all you need to do is provide 4 equations, the solution to which yields the equilibrium values for our 4 variables.)

e) I now ask you to characterize the equilibrium in more detail. To that end, let us assume that the KW utility function is quadratic, i.e., $u(q) = (1+\gamma)q - \frac{q^2}{2}$, which implies $q^* = \gamma$. For the various possible combinations of (μ_1, μ_2), $\mu_i \geq \beta - 1$, provide closed-form solutions for (q_1, q_2). Does currency 2 circulate in this economy (i.e., is $z_2 > 0$) in every equilibrium? If not, can you provide a condition on policy parameters that would guarantee $z_2 > 0$?

Question 3 (*20 points*)

Consider the social planner's problem for a real business cycle model. The household makes consumption (C) and leisure ($1 - N$, where N is hours worked) decisions to maximize lifetime utility:

$$E_0 \sum_{t=0}^{\infty} \beta^t u\left(C_t, 1 - N_t\right) \tag{1}$$

Specific functional forms will be given below.

Output is produced using capital K and labor N

$$Y_t = Z_t K_t^{\alpha} N_t^{1-\alpha} \tag{2}$$

Z_t is a TFP shock and is governed by a discrete state Markov chain. Capital evolves:

$$K_{t+1} = (1 - \delta)K_t + I_t \tag{3}$$

but assume full depreciation so $\delta = 1$. There is no trend growth. Finally,

$$Y_t = C_t + I_t$$

First suppose that the utility function u is as follows:

$$\ln\left(C_t - \frac{N_t^2}{2}\right) \tag{4}$$

a) Write down the recursive formulation of planner's problem and derive the first order conditions.

b) Using guess and verify, find the policy functions for investment, consumption and hours worked (**Hint:** first consider the equilibrium condition for hours worked and guess that investment is a constant share of output).

Now suppose the utility function is given by:

$$\ln C_t - \frac{N_t^2}{2} \tag{5}$$

c) Repeat parts (a) and (b) using these new preferences.

d) Compare the business cycle properties implied by these two models and explain how and why a TFP shock might affect output, consumption, investment and hours worked. Some RBC modelers prefer preferences used in parts (a/b) to those in part (c), why might this be the case?

e) If $0 < \delta < 1$ briefly explain how you would solve this model computationally using value function iteration. Give one advantage of this method.

Question 4 (*20 points*)

This question considers a distortionary labor income tax in the New Keynesian model.

The representative household's utility function is:

$$\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\psi}}{1+\psi} \tag{6}$$

The household's budget contraint is:

$$Q_t B_{t+1} + P_t C_t = B_t + (1-\tau_t) w_t P_t N_t + \Pi_t + P_t T_t \tag{7}$$

C is consumption, N is hours worked, w is the real wage, τ is the distortionary labor income tax rate, Π are firm profits distributed lump sum, T are lump-sum taxes. B are bonds that are in zero net supply. P is the aggregate price level. Q_t is the bond price.

In linearized form, the household's Euler equation and labor supply conditions are:

$$E_t \hat{c}_{t+1} - \hat{c}_t = \frac{1}{\sigma}(\hat{i}_t - E_t \hat{\pi}_{t+1}) \tag{8}$$

$$\hat{w}_t = \sigma \hat{c}_t + \psi \hat{n}_t + \hat{\tau}_t \tag{9}$$

The linearized equilibrium conditions for firms are:

$$\hat{y}_t = \hat{n}_t \tag{10}$$

$$\hat{w}_t = \hat{mc}_t \tag{11}$$

$$\hat{\pi}_t = \beta E_t(\hat{\pi}_{t+1}) + \lambda \hat{mc}_t \tag{12}$$

The resource constraint is:

$$\hat{y}_t = \hat{c}_t \tag{13}$$

Monetary policy follows a simple Taylor Rule:

$$\hat{i}_t = \phi_\pi \hat{\pi}_t \tag{14}$$

The (linearized) labor income tax rate follows an AR(1) process

$$\hat{\tau}_t = \rho \hat{\tau}_{t-1} + e_t \tag{15}$$

e_t is i.i.d. and tax revenues are redistributed lump-sum to households.

In percentage deviations from steady state: $\hat{mc}_t$ is real marginal cost, $\hat{c}_t$ is consumption, $\hat{w}_t$ is the real wage, $\hat{n}_t$ is hours worked, $\hat{y}_t$ is output. In deviations from steady state: $\hat{i}_t$ is the nominal interest rate, $\hat{\pi}_t$ is inflation and $\hat{\tau}_t$ is the income tax rate.[5] λ is a function of model parameters, including the degree of price stickiness.[6] Assume that $\phi_\pi > 1$, $0 < \rho < 1$ and $0 < \beta < 1$

a) Using the relevant equations above, show that the natural rate of output in this model depends on the income tax rate. In particular, show:

$$\hat{y}_t^n = -\frac{1}{\sigma + \psi}\hat{\tau}_t \tag{16}$$

b) Show that the Phillips Curve can be written as:

$$\hat{\pi}_t = \beta E_t(\hat{\pi}_{t+1}) + \lambda(\sigma + \psi)\hat{x}_t + \lambda\hat{\tau}_t \tag{17}$$

where $\hat{x}_t$ is the welfare relevant output gap, defined as $\hat{y}_t - \hat{y}_t^e$. (**Hint:** in this model the distortionary tax is the only stochastic element and $\hat{y}_t^e = 0$, which implies $\hat{x}_t = \hat{y}_t = \hat{c}_t$.)

c) Using the method of undetermined coefficients, find the response of the welfare relevant output gap and inflation to an exogenous cut in income taxes when prices are sticky and monetary policy follows the Taylor Rule above. To do this, guess that the solution for each variable is a linear function of the shock $\lambda\hat{\tau}_t$. (**Hint:** you will need to rewrite the consumption Euler equation in terms of the welfare relevant output gap noting that $\hat{x}_t = \hat{y}_t = \hat{c}_t$).

d) Discuss how, and why, labor income tax cuts affect the natural rate of output, the output gap and inflation. Are these results surprising?

e) Now suppose that, instead of following the simple Taylor Rule above, we choose $\hat{x}_t$ and $\hat{\pi}_t$ to maximize welfare and derive the optimal policy under **discretion**. Assume the steady state is efficient. Is it possible to fully stabilize inflation and the output gap ($\hat{x}_t$) with optimal policy? *Briefly* explain. You do not need to derive anything, briefly answer using your economic intuition.

[5] With a zero steady state income tax rate, $\hat{\tau}_t = \tau_t$

[6] $\lambda = \frac{(1-\theta)(1-\beta\theta)}{\theta}$ where θ is the probability that a firm cannot adjust its price.

Question 5 *(20 points)*

This problem analyzes the effect of future financial constraints on current investment decisions.

Consider an economy with two dates, denoted by $t = 1, 2$. There are two goods: consumption and capital. There is a continuum of entrepreneurs and a continuum of consumers. All individuals have linear utility and consume only in period 2, so $U = E[c_2]$. There is a fixed supply of capital $\overline{k}$, initially owned by the consumers. Consumers are endowed with a large amount of consumption good in each period and they can store this good between dates, such that if they store one unit of the consumption good in $t = 1$ they get one unit of the good in $t = 2$. The entrepreneurs have access to a linear technology that produces A units of consumption good in period 2 per unit of capital they own. The consumers have access to a concave technology $G(\tilde{k})$, where $\tilde{k}$ denotes the capital owned by consumers in period 2. Assume $lim_{k\to 0} G'(k) = \infty$ and $G'(\overline{k}) = 0$. The entrepreneurs enter period 1 with a given net worth N_1 in terms of consumption goods. Assume agents can trade a risk-free bond b_2 that pays an interest rate r.

a) Argue that the gross rate of return on the risk-free bond is equal to 1 (i.e., the net return, r, is zero).

b) Suppose that entrepreneurs face no borrowing constraints. State the optimization problems of an entrepreneur and a consumer. Show that the equilibrium capital price is $q_1 = A$ and the entrepreneurs buy k^*, where $G'(\overline{k} - k^*) = A$.

c) Suppose that the entrepreneurs cannot borrow at all, so $q_1 k_2 \leq N_1$. Find the equilibrium price and allocation, show that $q_1 \leq A$ in equilibrium and that the expected utility of the entrepreneur is

$$\frac{A}{q_1} N_1 \tag{18}$$

irrespective of whether the constraint $q_1 k_2 \leq N_1$ binds or not. Show that q_1 is increasing in N_1 for $N_1 < Ak^*$.

Let's add one period prior to period 1, period 0. Now the economy has three dates $t = 0, 1, 2$. In $t = 0$ the entrepreneurs have initial net worth N_0 (in consumption goods). Then they borrow b_1 from the consumers and buy capital k_1 at the price q_0 (capital never depreciates). The technology in period 1 is the same as in period 2: consumers produce using the concave technology $G(k)$ and entrepreneurs have a linear technology. However the productivity of the entrepreneurs at $t = 1$ is a random variable a distributed on $[\underline{a}, \overline{a}]$ with $E[a] = A$. The productivity in $t = 2$ is fixed at A.

The entrepreneurs can only issue non-state contingent, risk-free bonds, so b_1 has to satisfy

$$\underbrace{(q_1(a) + a)k_1 - b_1}_{=N_1(a)} \geq 0 \textit{ for all } a. \tag{19}$$

Note that now the asset price q_1 is in general a function of the shock a and we denote it by $q_1(a)$.

d) Assume that the entrepreneurs' initial net worth N_0 is sufficiently large so that (19) is not binding, and that they are not constrained in period 1. Show that asset prices are $q_0 = 2A$ and $q_1(a) = A$ for all a, and that the capital owned by the entrepreneurs is constant at k^*.

e) Now suppose that the entrepreneurs are fully constrained at date 1, as in part (c), but are completely unconstrained at date 0. Argue informally that now the equilibrium asset price q_1 is weakly increasing in a.

f) Write the entrepreneur's problem at date 0. Assuming the constraint (19) is not binding, derive the first order conditions for the entrepreneur's problem. Derive an expression for the equilibrium price q_0 as a function of $q_1(a)$. **Hint**: Remember that now $N_1(a) = (q_1(a) + a)k_1 - b_1$.

g) Prove that

$$q_0 \leq E[a + q_1(a)] \tag{20}$$

with strict inequality if $q_1(a)$ is not constant. **Hint:** Remember that if x and y are two random variables, then $cov(x, y) = E[xy] - E[x]E[y]$. Why is the date 0 price of capital below its present discounted value, despite the fact that entrepreneurs have linear utility and are unconstrained? Why do entrepreneurs not borrow and leverage their investment more to profit from the positive expected return?

Question 6 (*20 points*)

Consider the following economy. Time is discrete and runs forever, $t = 0, 1, 2, \ldots$ The economy is populated by two types of agents (a measure one of each): farmers and workers. Farmers own a piece of land that pays a stochastic income y_t every period. We assume that y_t is i.i.d. across farmers and time, and that $y_t \sim N(\overline{y}, \sigma^2)$. Farmers use all their time working their land. Their preferences are given by

$$u(c) = -\frac{\exp(-\gamma c)}{\gamma} \tag{21}$$

for some $\gamma > 0$. For simplicity, we assume that consumption of farmers can be negative, that is $c \in \mathbb{R}$. Moreover, farmers can save (or borrow if negative) in a non-state contingent and non-defaultable asset a, which has a rate of return r. Farmers face the following "No-Ponzi" condition on assets

$$\lim_{t\to\infty} \frac{a_t}{(1+r)^t} \geq 0. \tag{22}$$

Moreover, farmers can produce and hold capital, k, which is rented to the representative firm in competitive markets (1 unit of final consumption good produces 1 unit of capital). Let r^K be the rental rate of capital and δ the depreciation rate.

Unlike farmers, workers don't own land, and they use their available time to work in the representative firm. Assume each worker is endowed with one unit of time. They **cannot** trade the asset a, but **they can produce and hold capital**, k (with the same technology as farmers). Their per-period utility is given by $u(c)$, with $u'(c) > 0$, $u''(c) < 0$, $\lim_{c\to 0} u'(c) = \infty$, $\lim_{c\to\infty} u'(c) = 0$.

Finally, there is a representative firm that combines capital and labor to produce final good according to $f(K, L)$, where K is the capital they operate, and L the hours/workers they hire. Assume $f(\cdot)$ satisfies the standard Inada conditions.

Thus, the only financial market available in this economy is the market for assets a, where only farmers can trade. Moreover, there is a market for k, where all agents can trade.

Both type of agents have the same discount factor $\beta \in (0, 1)$.

a) Given a constant path for r and r^K, state the problem of a farmer. Argue that the equilibrium price of capital is equal to 1. Characterize the farmer's problem with the necessary FOCs. Show that in any equilibrium with positive capital it must hold that $r = r^K - \delta$.

b) Let $c^F(a, k, y)$ denote the consumption policy function of the farmers, which depends on asset holdings a, capital holdings k, and land income y. Prove that:

$$c(a, k, y) = \underline{c} + r(a + k) + \frac{r}{1+r} y, \tag{23}$$

where $\underline{c}$ is a constant. Find an expression for $\underline{c}$ in terms of the parameters of the model.

c) Let C^F and K^F denote the aggregate consumption and capital holdings of farmers. Show that in an equilibrium with constant C^F and K^F, it must hold that $\beta(1+r) < 1$. **Hint:** Remember to check the budget constraint.

d) Let's turn to the workers. State their problem. **Hint:** Remember that workers cannot save in asset a; they can only hold capital.

e) Let c^W and k^W denote the consumption and capital holdings of an individual worker. Show that in an equilibrium with constant c^W, it must be that $k^W = 0$. Conclude that an equilibrium with constant C^F, K^F, c^W, and $k^W = 0$ exists **Hint:** Use the result that in equilibrium $\beta(1+r) < 1$.

f) Show that in such an equilibrium, an increase in σ^2 increases the welfare of workers. Explain.