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PRELIMINARY EXAMINATION FOR THE Ph.D. DEGREE,
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Directions: Answer all questions. Feel free to impose additional structure on the problems below, but please state your assumptions clearly. Point totals for each question are given in parentheses. You have 5 hours to complete the exam and an additional 20 minutes of reading time.

Question 1 (*10 points*)

Consider the standard deterministic growth model in discrete time. There is a large number of identical households (normalized to 1). Each household wants to maximize life-time discounted utility

$$U(\{c_t\}_{t=0}^{\infty}) = \sum_{t=0}^{\infty} \beta^t (\ln c_t + \gamma \ln c_{t-1}), \quad \gamma > 0,$$

that is, households preferences are characterized by "habit persistence". Each household has an initial capital stock x_0 at time 0, and one unit of productive time in each period, that can be devoted to work. Final output is produced using capital and labor services,

$$y_t = F(k_t, n_t) = k_t^a n_t^{1-a}.$$

This technology is owned by firms whose number will be determined in equilibrium. Output can be consumed (c_t) or invested (i_t). We assume that households own the capital stock (so they make the investment decision) and rent out capital services to the firms. We also assume that the capital stock (x_t) fully depreciates at the end of a given period, i.e. $\delta = 1$. Finally, it is assumed that households own the firms, i.e. they are claimants to the firms' profits.

a) In this economy, why is it a good idea to describe the AD equilibrium capital stock allocation by solving the (easier) Social Planner's Problem?

b) Fully characterize (i.e. find a closed form solution for) the equilibrium allocation of the capital stock.[1]

c) What is the capital stock equal to as $t \to \infty$? What is the ADE value of the rental rate of capital and the rental rate of labor as $t \to \infty$?

[1] **Hint:** Derive the Euler equation, and "guess and verify" a policy rule of the form $k_{t+1} = g k_t^a$, where g is an unknown to be determined.

Question 2 (*20 points*)

Consider the Mortensen-Pissarides model in continuous time. Labor force is normalized to 1, but here there are N types of workers (this will be the only difference in comparison to the baseline model seen in class). A worker of type i enjoys a benefit equal to z_i while unemployed, and $z_1 < z_2 < ... < z_N$. The measure of workers of type i is given by π_i, with $\sum_{i=1}^{N} \pi_i = 1$. Since matching is random, the probability with which a firm matches with a certain type i depends only on the relative measure of type i workers in the aggregate pool of unemployed.

The rest of the model is standrard. Unemployed workers and firms (with one vacancy each) search for each other. Let the measure of unemployed workers be denoted by $u = \sum_{i=1}^{N} u_i$, where u_i is the measure of unemployed workers of type i. Also, let the measure of vacant firms be denoted by v, which will be determined endogenously by free entry. A CRS and increasing (in both arguments) matching function, $m(u, v)$, brings together unemployed workers and vacant firms. It will be useful to define the market tightness as $\theta \equiv v/u$. Once a match has been formed, the wage is determined through Nash bargaining, with $\beta \in (0, 1)$ representing the worker's power.

The output of *all* jobs is $p > 0$ per unit of time, i.e., p does not depend on the worker's type and $p > z_i$, for all i. Also, while a firm is searching for a worker it has to pay a search (or recruiting) cost, $pc > 0$, per unit of time. *All* jobs are exogenously destroyed at rate $\lambda > 0$ (again, independently of the worker's type). All agents discount future at the rate $r > 0$. Throughout this question focus on steady state equilibria.

a) Based only on your knowledge of the environment (i.e., without analyzing the model), explain whether the following statement is true or false: "In this economy, all workers will be paid the same wage, since they are equally productive".

b) Write down the value function of a firm with an unfilled vacancy (V) and the value function of a firm matched with a worker of type i (J_i).

c) Write down the value function of a worker of type i while unemployed (U_i) and while employed (W_i).

d) Combine the free entry condition (i.e., $V = 0$) with the expressions for J_i provided earlier in order to derive the job creation (JC) curve for this economy.[2]

e) Using the same methodoly as in the lectures (adjusted to accommodate the differences in the new environemnt), derive the wage curve (WC) for this economy.[3]

f) Combine the JC curve and the WC curve determined in the previous parts in order to provide an equation that (implicitly) determines the equilibrium θ.

g) Describe the equilibrium wage for a worker of type i (w_i) as a fucntion of the model's parameters and the equilibrium θ (which was implicitly determined in part f).

h) Is w_i increasing or decreasing in z_i? What happens to the distribution of equilibrium wages as the distribution of workers' types becomes more dispersed?

[2] **Hint:** The right-hand side (RHS) of the JC curve should look identical to the one seen in class. On the left-hand side (LHS), instead of just w, you should have an expression that represents the average wage that a firm *expects* to pay.

[3] **Hint:** There will be two differences compared to the WC seen in class: 1) On the LHS, instead of just w, you should have the average wage that a firm expects to pay (the same as in part d); 2) On the RHS, instead of just z, you should have the *average* unemployment benefit, i.e., $\bar{z} \equiv \sum_{i=1}^{N} \pi_i z_i$.

Question 3 (*20 points*)

This question studies the co-existence of money and credit. Time is discrete with an infinite horizon. Each period consists of two subperiods. In the day, trade is *partially* bilateral and anonymous as in Kiyotaki and Wright (1989) (call this the KW market). At night trade takes place in a Walrasian or centralized market (call this the CM). There are two types of agents, buyers and sellers, and the measure of both is normalized to 1. The per period utility for buyers is $u(q)+U(X)-H$, and for sellers it is $-q+U(X)-H$, where q is the quantity of the day good produced by the seller and consumed by the buyer, X is consumption of the night good (the numeraire), and H is hours worked in the CM. In the CM, all agents have access to a technology that turns one unit of work into a unit of good. The functions u, U satisfy the usual assumptions; I will only spell out the most crucial ones: There exists $X^* \in (0,\infty)$ such that $U'(X^*) = 1$, and we define the first-best quantity traded in the KW market as $q^* \equiv \{q : u'(q^*) = 1\}$.

The **only difference** comapared to the baseline model is that there are two types of sellers. Type-0 sellers, with measure $\sigma \in [0,1]$, accept credit. More precisely, in meetings with a type-0 seller (type-0 meetings), no medium of exchange (MOE) is necessary, and the buyer can purchase day good by promising to repay the seller in the **forthcoming CM with numeraire good** (this arrangement is called an IOU). The buyer can promise to repay *any* amount (no credit limit), and her promise is credible (buyers never default). Type-1 sellers, with measure $1-\sigma$, never accept credit, hence, any purchase of the day good must be paid for on the spot (*quid pro quo*) with money. *All* buyers meet a seller in the KW market, so that σ is the probability with which a buyer meets a type-0 seller, and $1-\sigma$ is the probability with which she meets a type-1 seller.

The rest is standard. Goods are non storable, but there exits a storable and recognizable object, fiat money, that can serve as a MOE in type-1 meetings. Money supply is controlled by a monetary authority, and we consider simple policies of the form $M_{t+1} = (1+\mu)M_t$, $\mu > \beta - 1$. New money is introduced, or withdrawn if $\mu < 0$, via lump-sum transfers to buyers in the CM. Let ϕ denote the unit price of money (in terms of the numeraire). In any type of meeting, buyers have **all the bargaining power**.

a) Describe the CM value function, $W(m,d)$, of a typical buyer, where m denotes her money holdings as she enters the CM, and d (for 'debt') denotes the amount of numeraire good she may owe to a (type-0) seller that she met earlier in the KW market. Show that $W(m,d)$ is linear in both arguments.

Now let $W^{S0}(d)$ and $W^{S1}(m)$ denote the CM value functions for sellers of type 0 and 1, respectively (d stands for the amount of numeraire good to be paid to the type-0 seller in the CM, and m stands for the amount of money that the type-1 seller received in the KW market). To answer the next question you can *take as given* that $W^{S0}(d) = \Lambda^{S0} + d$, and $W^{S1}(m) = \Lambda^{S1} + \phi m$ (treat the Λ terms as constants).

b) Let q_j denote the quantity of day good traded in a type-$j = 0,1$ meeting. Let d denote the amount of numeraire the buyer promises to repay in the forthcoming CM in exchange for q_0, and let x denote the amount of money the buyer pays in exchange for q_1. Describe the bargaining solution in a typical type-$j = 0,1$ meeting (recall that, under our assumptions, sellers will not carry any money as they enter the KW market).[4]

c) Describe the objective function of the typical buyer, $J(\hat{m})$, where the hat denotes next period's choice.[5]

d) Describe the equilibrium variables q_0, q_1 as functions of the model's parameters, including the nominal interest rate, i. How is q_1 related to the real balances $z \equiv \phi m$?[6]

To simplify the analysis hereafter, assume that u is quadratic: $u(q) = -\frac{q^2}{2} + (1+\gamma)q$, $\gamma > 0$. Notice that this specification implies $q^* = \gamma$, $u'(q) - 1 = \gamma - q$, and $u(q^*) - q^* = \frac{\gamma^2}{2}$.

e) Given this utility specification find closed-form solutions for q_0, q_1.

f) For any γ, i, with $i < \gamma$, describe the set of values of σ for which a monetary equilibrium exists (i.e., for which $z > 0$).[7]

Finally, define the welfare function of this economy as the measure of the various KW market meetings times the net surplus generated in each meeting, i.e.,

$$\mathcal{W} = \sigma[u(q_0) - q_0] + (1-\sigma)[u(q_1) - q_1].$$

g) Since σ is the fraction of type-0 sellers, who accept credit, and since in type-0 meetings the buyer is never liquidity constrained, intuition suggests that welfare should increase in σ. It turns out that this intuition is wrong! Show that, for $\sigma \in [0, \bar{\sigma}]$, we have $\partial \mathcal{W}/\partial \sigma < 0$, and discuss the intuition behind this result.[8]

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[4] **Hint:** Feel free to analyze the bargaining problems in detail, but I think that with a little intuitive thinking you can answer this question in a couple of minutes.

[5] **Hint:** Again, feel free to do all the work that leads to the objective function, but here I think it is easy to guess the form of J, and I will give full credit for a correct guess.

[6] **Hint:** To bring i into the analysis use the Fisher equation to link i to μ.

[7] **Hint:** If σ is too large, agents may not find it worthwhile to carry money, especially if i is large. So there will be an upper bound, call it $\bar{\sigma}$, above which a monetary equilibrium would cease to exist.

[8] **Hint:** Showing that $\partial \mathcal{W}/\partial \sigma < 0$, for all $\sigma \in [0, \bar{\sigma}]$, is not hard but requires some algebra. If you cannot show the result for all σ, I will give partial credit for showing that $\partial \mathcal{W}/\partial \sigma < 0$ at a specific value of σ, and your safest bet is to focus on $\sigma = \bar{\sigma}$.

Question 4 (*20 points*)

Consider the planner's problem for a real business cycle model with inelastic labor supply (essentially the stochastic growth model) and no trend growth. Preferences are given by:

$$\ln C_t \tag{1}$$

Output is produced using capital K

$$Y_t = A_t K_t^{\alpha} \tag{2}$$

where K_t is the capital stock at the start of period t, and A_t is a TFP shock and is governed by a discrete state Markov chain. There are adjustment costs to capital, which evolves according to the following production function:

$$K_{t+1} = K_t^{\delta} I_t^{1-\delta} \tag{3}$$

When $\delta = 0$, this becomes the simple model we saw in class with full depreciation (i.e. where $K_{t+1} = I_t$). The resource constraint is

$$Y_t = C_t + I_t$$

a) Write down the recursive formulation of planner's problem. Use two constraints: the typical resource constraint and the capital production function. Denote the Lagrange multiplier on the resource constraint as λ_t and the one on the capital production constraint as $\lambda_t q_t$.

b) Derive the first order conditions. Provide an intuitive explanation of q_t.

c) Using guess and verify, solve the model and find the policy functions for K_{t+1}, I_t and C_t (Hint: start by guessing that investment and consumption are a constant share of output).

d) Explain why we can interpret δ as a parameter that affects the degree of capital adjustment costs in this model. Briefly discuss how, and why, the responses of consumption and investment to TFP shocks vary with δ. Why do some macroeconomic modelers prefer to include capital adjustment costs in their models?

Question 5 (*20 points*)

Consider the following set of *linearized* equilibrium conditions for the standard New Keynesian model. The only difference from the model we saw in class is that the government can now purchase a basket of goods G_t, which is completely funded by lump sum taxes. This is like a pure government demand shock: assume that G_t is not productive and does not provide utility.

In percentage deviations from steady state: $\hat{\phi}_t$ is real marginal cost, $\hat{c}_t$ is consumption, $\hat{w}_t$ is the real wage, $\hat{n}_t$ is hours worked, $\hat{y}_t$ is output. In deviations from steady state: $\hat{i}_t$ is the nominal interest rate, $\hat{\pi}_t$ is inflation. σ and ψ come from household preferences $\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\psi}}{1+\psi}$.

Households

$$E_t\hat{c}_{t+1} - \hat{c}_t = \frac{1}{\sigma}(\hat{i}_t - E_t\hat{\pi}_{t+1}) \tag{4}$$

$$\hat{w}_t = \sigma\hat{c}_t + \psi\hat{n}_t \tag{5}$$

Firms

$$\hat{y}_t = \hat{n}_t \tag{6}$$

$$\hat{w}_t = \hat{\phi}_t \tag{7}$$

$$\hat{\pi}_t = \beta E_t(\hat{\pi}_{t+1}) + \kappa\tilde{y}_t \tag{8}$$

κ is inversely related to the degree of degree of price stickiness and $\tilde{y}_t$ is the output gap (relative to the model with flexible prices)

$$\tilde{y}_t = \hat{y}_t - \hat{y}_t^n \tag{9}$$

Resource constraint

$$\hat{y}_t = \gamma_c\hat{c}_t + \hat{g}_t \tag{10}$$

γ_c is the steady state share of consumption in output. To make the math easier, $\hat{g}_t$ is the deviation of government spending from steady state relative to output.

Policy:

$$\hat{i}_t = \phi_\pi\hat{\pi}_t \tag{11}$$

where $\phi_\pi > 1$. Government spending follows an AR(1) process

$$\hat{g}_t = \rho\hat{g}_{t-1} + e_t \tag{12}$$

a) Show that the natural level of output can be written as

$$\hat{y}_t^n = \Gamma\hat{g}_t$$

$$\Gamma \equiv \frac{\sigma}{\sigma + \psi\gamma_c}$$

Explain the mechanism through which an increase in government spending leads to an

increase in output in this model with flexible prices (similar to the RBC model).
(Hint: start by combining equations 5, 6, 7 and 10 and note that, under flexible prices $\hat{\phi}_t = 0$).

For the rest of this question note that this model can be simplified to the familiar 3 equations:

$$E_t\tilde{y}_{t+1} - \tilde{y}_t = \frac{\gamma_c}{\sigma}(\hat{i}_t - E_t\hat{\pi}_{t+1} - \hat{r}_t^n) \tag{13}$$

$$\hat{\pi}_t = \beta E_t(\hat{\pi}_{t+1}) + \kappa\tilde{y}_t \tag{14}$$

$$\hat{i}_t = \phi_\pi\hat{\pi}_t \tag{15}$$

Plus the process for government spending

$$\hat{g}_t = \rho\hat{g}_{t-1} + e_t \tag{16}$$

and a definition of the natural real rate of interest

$$\hat{r}_t^n = \frac{\sigma}{\gamma_c}(1-\Gamma)(1-\rho)\hat{g}_t \tag{17}$$

b) Using the method of undetermined coefficients, find the response of the output gap and inflation to an exogenous increase in $\hat{g}_t$ when prices are sticky and monetary policy follows the Taylor Rule above. To do this, guess that the solution for each variable is a linear function of the shock $\hat{g}_t$:

$$\tilde{y}_t = \Lambda_y\hat{g}_t$$

$$\hat{\pi}_t = \Lambda_\pi\hat{g}_t$$

(Hint: follow the steps used in the problem set. Start by substituting the guesses, the monetary policy rule and the AR(1) process for $\hat{g}_t$ into the dynamic equations (13) and (14).)

c) How, and why, does the response of GDP differ from the model with flexible prices? Do positive government spending shocks increase inflation? Provide economic intuition and (if you can) discuss the solution you found in part (b).

d) In principle, could monetary policy fully stabilize the output gap and inflation after a government spending shock? (Hint: think about how the 3-equation setup above looks like the cases we studied in class)? Would there be any additional benefit from conducting optimal monetary policy under commitment?

Question 6 (*10 points*)

This question is about the standard decentralized real business cycle model. You do not need to derive anything for this question and keep your answers clear and concise.

a) *Briefly* explain the mechanisms through which TFP shocks affect output, consumption, hours worked and investment in the standard RBC model. How well does the model replicate the business cycle facts seen in the data? How would adding habits in consumption affect the dynamics of consumption and investment?

b) Suppose you want to solve the model using computational methods. Explain one approach, the advantages of this method and the steps you would need to take.