University of California, Davis
Department of Economics
Macroeconomics

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Reading Time: 20 minutes

PRELIMINARY EXAMINATION FOR THE Ph.D. DEGREE

Directions: Answer all questions. Feel free to impose additional structure on the problems below, but please state your assumptions clearly. Point totals for each question are given in parentheses.

1. (10) In Mehra and Prescott's analysis of the equity premium, they employed a discrete time asset pricing model in which the exogenous process for consumption growth was assumed to be a two-state Markov process. Denote the two growth rates as $\lambda_1 < \lambda_2$ and define the transition probabilities as $\pi_{ij} = \Pr(\lambda_{t+1} = \lambda_j | \lambda_t = \lambda_i)$ where $(i,j) = (1,2)$. Let β denote the representative agent's subjective discount factor and γ denote the agent's relative risk aversion parameter (assumed to be constant). In order for the equilibrium price function to exist, Mehra and Prescott assumed that $\beta\pi_{ij}\lambda_j^{1-\gamma} < 1$ for all (i,j). (Actually, they made a slightly weaker assumption but let's use the stronger form presented here.) Why was this assumption useful in establishing equilibrium? Be precise in your answer.

2. (20) Consider a continuous-time optimal growth model in which the aggregate production function is of the form: $Y(t) = K(t)^{\alpha}(A(t)L(t))^{1-\alpha}$ where Y denotes output, K is the capital stock, L is labor and A is labor-augmenting technology. Assume that L and A grow exogenously at the rates n and a respectively. Capital depreciates at the rate δ. The representative household in the economy has lifetime preferences given by:

$$\int_{t=0}^{\infty} e^{-\rho t}\left[\frac{\tilde{C}(t)^{1-\gamma}}{1-\gamma}\right]L(t)$$

where $\rho \in (0,1)$, $\gamma > 0$ and $\gamma \neq 1$, and $\tilde{C}(t)$ is per-capita consumption. In addition to households, a government exists which purchases $G(t)$ units of output. This amount is growing at the rate $n+a$ (i.e. the growth rate of government purchases is equal to the sum of the population growth rate and the growth rate of technology). Government purchases are financed via lump-sum taxes on households. Given this environment, do the following

   (a) Solve the model as a social planner problem. Write down the associated present-value Hamiltonian and derive the necessary conditions.

   (b) Define a steady-state equilibrium and derive the phase diagram associated with this economy.

   (c) Suppose that, in period t_k the level of government purchases jumps unexpectedly to $G'(t_k) > G(t_k)$. This has no effect on the growth rate of government purchases. Describe the effect that this has on equilibrium (steady-state and any transition to a new steady-state) and use the phase diagram developed in (b) to support your analysis.

   (d) Suppose now that, in period t_m, the capital depreciation rate falls to $\delta' < \delta$. Again analyze the effects that this has on equilibrium (both transition and steady-state) using the phase diagram for this economy. Discuss the differences between your answers for (c) and (d).

   (e) Show that, in steady-state equilibrium, this model replicates all of Kaldor's stylized facts of growth.

3. (20) Consider a Lucas-tree type economy in which the level of the endowment is independently and identically distributed with support $x_t \in (x', x'')$. Agents trade one- and two-period bonds that have prices p_{1t} and p_{2t}, respectively, and return 1 unit of consumption at maturity. In addition, agents can purchase a one-period futures contract for the price f_{1t}. A futures contract purchased at time t states that the owner agrees to give up f_{1t} units of consumption in period $t+1$ for the return of 1 unit of consumption in period $t+2$. Given this setup, do the following:

(a) Assuming standard preferences (i.e. infinitely lived, risk-averse agents), set up the household's maximization problem as a dynamic programming problem. Derive and interpret the associated necessary conditions.

(b) Define a recursive competitive equilibrium in this economy.

(c) Characterize the equilibrium behavior of bond prices and the price of the future contract. Derive an exact relationship between these prices; interpret this relationship.

(d) Is it the case that $f_{1t} = E_t\left[p_{1t+1}\right]$? Or, is the price of the futures contract in period t an unbiased estimator of the price of a one-period bond in period $t+1$? Explain.

4. (20) Consider the standard growth model in discrete time. There is a large number of identical households (normalized to 1). Each household wants to maximize life-time discounted utility

$$U(\{c_t\}_{t=0}^{\infty}) = \sum_{t=0}^{\infty} \beta^t u(c_t), \;\; \beta \in (0,1).$$

Each household has an initial capital stock x_0 at time 0, and one unit of productive time in each period, that can be devoted to work. Final output is produced using capital and labor services,

$$y_t = F(k_t, n_t),$$

where F is a CRS production function. This technology is owned by firms whose number will be determined in equilibrium. Output can be consumed (c_t) or invested (i_t). We assume that households own the capital stock (so they make the investment decision) and rent out capital services to the firms. The depreciation rate of the capital stock (x_t) is denoted by $\delta \in [0,1]$.[1] Finally, we assume that households own the firms, i.e. they are claimants to the firms' profits. The functions u and F have the usual nice properties.[2]
a) First consider an Arrow-Debreu world. Describe the households' and firms' problems and carefully define an AD equilibrium. How many firms operate in this equilibrium?
b) Write down the problem of the household recursively.[3] Be sure to carefully define the state variables and distinguish between aggregate and individual states. Define a recursive competitive equilibrium (RCE).

For the rest of this question focus again on an Arrow-Debreu setting.
c) In this economy, why is it a good idea to describe the AD equilibrium capital stock allocation by solving the (easier) Social Planner's Problem?

From now on assume that $F(k_t, n_t) = k_t^a n_t^{1-a}$, $a \in (0,1)$, and $\delta = 1$. Also, assume that the households' preferences are characterized by "habit persistence". In particular, households wish to maximize

$$\sum_{t=0}^{\infty} \beta^t (\ln c_t + \gamma \ln c_{t-1}), \;\; \gamma > 0.$$

d) Fully characterize (i.e. find a closed form solution for) the equilibrium allocation of the capital stock. (Hint: Derive the Euler equation using any method you like. Then guess and verify a "policy rule" of the form $k_{t+1} = g k_t^a$, where g is an unknown to be determined.)
e) What is the capital stock equal to as $t \to \infty$? What is the ADE value of the rental rate of capital as $t \to \infty$?
f) What is the ADE price of the consumption good in $t = 1$?

[1] The capital stock depreciates no matter whether it is rented out to a firm or not.
[2] You will not explicitly need them, so there is no need to be more precise.
[3] Here firms face a static problem. I am not asking you to explicitly spell it out, but it will be critical for a correct definition of the RCE.

5. (20) Consider the Mortensen-Pissarides model with endogenous job destruction. Labor force is normalized to 1. The matching function $m(u, v) = u^a v^{1-a}$ brings together unemployed workers and vacant firms. A large measure of firms decide whether to enter the labor market with exactly one vacancy. When a firm meets an unemployed worker a job is formed. The output of a job is px per unit of time, where p is the general productivity and x the idiosyncratic. When a vacancy is unfilled, firms have to pay a search cost, given by pc per unit of time.

The parameter p is constant, but x is not. Every new job is created at $x = 1$, which will be the maximum possible value of x. However, there exists an exogenous Poison rate λ, which can "hit" every existing match. If this happens, the new idiosyncratic productivity of that match will be a random draw from a uniform distribution with support in the interval $[0, 1]$. Once such a shock arrives, the worker and the firm can either terminate the match or renegotiate a wage, which will be specific to the new value of x. All job negotiations are based on Nash bargaining, where β represents the worker's bargaining power. Whenever a worker is in the state of unemployment, she enjoys a benefit of $z < p$ per unit of time. Focus on steady state equilibria of this model. Let the discount rate of agents be given by r.

a) What is the arrival rate of workers to vacant firms as a function of market tightness, $\theta = v/u$? What is the arrival rate of jobs to unemployed workers?
b) Write down the value functions for a filled job and a vacancy. Assume that, after a productivity shock has hit, firms terminate the match if and only if $x \leq R$. Refer to R as the reservation idiosyncratic productivity.
c) Write down the value functions of an unemployed and an employed worker. Make sure your answers are in accordance with the assumption in part (b).
d) What is the value of a filled job when we evaluate it at $x = 1$ and $x = R$?
e) Take the following result as given: since the firm and worker can always negotiate a new wage after a productivity shock has hit, the wage paid to a worker who is employed by a firm with idiosyncratic productivity x, is given by $w(x) = (1 - \beta)z + \beta p(x + c\theta)$. Refer to this equation as the wage curve (WC). Explain the WC intuitively.
f) Use the WC, and parts (b) and (d) in order to provide a closed form solution for the value function of a filled job.
g) Use your answer in part (f) in order to derive the job creation (JC) curve for this labor market. Plot the JC curve in a graph with θ on the horizontal and R on the vertical axis and explain its slope, both algebraically and intuitively.
h) Derive the job destruction (JD) curve for this labor market. Plot the JD curve in a graph with θ on the horizontal and R on the vertical axis and explain its slope, both algebraically and intuitively.
i) Use your answers in (g) and (h) in order to claim that there exists a unique equilibrium pair (θ^*, R^*). Finally, close the model by describing equilibrium unemployment.

6. (10) Consider the "cash-goods vs credit goods" model discussed in class. The representative agent maximizes

$$\sum_{t=0}^{\infty} \beta^t u(c_{1t}, c_{2t}).$$

Assume that $u_1, u_2 > 0$, $u_{11}, u_{22} < 0$, $\lim_{c_1 \to 0} u_1(c_1, c_2) = +\infty$, $\lim_{c_2 \to 0} u_2(c_1, c_2) = +\infty$. There is no production. Every period the representative agent has a constant endowment given by e, and we assume that there is a linear technology for converting the endowment good into either of the consumption goods, $e = c_{1t} + c_{2t}$. What makes things interesting is that the consumption of the first good should be financed out of cash on hand, i.e. c_1 is subject to a Cash-in-Advance constraint.
Now imagine that in this economy there are two types of money, red (R) and blue (B). These assets are perfect substitutes, i.e. the agents can finance c_1 using any combination of the two types of money. The supply of money of type $i = B, R$ in period t is $M_{i,t}$. The supply of each money is controlled by an individual monetary authority. M_{it} evolves according to $M_{i,t+1} = (1 + \mu_i)M_{i,t}$. The new money (of both types) is injected into the economy through lump-sum transfers. Throughout this question focus on steady states.

a) Write down the problem of the representative agent (including the CIA constraint) recursively. (Hint: It will be useful to write the budget and CIA constraints in real terms).
b) For any given μ_R, what should the value of μ_B be so that agents can have a positive demand for blue money? For full credit make your statement formal by deriving the demand for money. Partial credit will be given for correct intuition.
c) Now assume that in the beggining of time, the blue and red money authorities play a non-cooperative game. Their objective is to choose the largest value of μ_i possible, subject to the constraint that their money circulates in the economy (i.e. agents use it in order to buy c_1).[4] Describe the set of Nash equilibria for this game.

[4] Although the model here is dynamic, for simplicity assume that the monetary authorities play this game just once in $t = 0$. Whatever μ_i they choose will stick around forever.