University of California, Davis
Department of Economics
Macroeconomics

Date: June 29, 2015
Time: 5 hours
Reading Time: 20 minutes

PRELIMINARY EXAMINATION FOR THE Ph.D. DEGREE

Directions: Answer all questions. Feel free to impose additional structure on the problems below, but please state your assumptions clearly. Point totals for each question are given in parentheses.

1. (10) Briefly discuss the following statements (keep your answers short and concise):

   (a) The consumption-based capital asset pricing model is inconsistent with high volatility of stock prices.

   (b) In standard real business cycle models, the MPK is highly procyclical. This implies that interest rates (i.e. real) will be as well.

2. (20) An economy is populated by identical, infinitely-lived agents (there is no population growth) that maximize the present discounted value of lifetime utility given by

$$\sum_{t=0}^{\infty} \beta^t \ln c_t;\ \beta \in (0,1)$$

where c_t denotes consumption. Output is produced via a standard Cobb-Douglas production function:

$$y_t = k_t^{\alpha}$$

where k_t denotes the beginning of period capital stock. (Implicitly it is assumed that labor supply is inelastically supplied by households to firms and that the labor input has been normalized to unity. Hence the labor market is ignored in this analysis.) In addition to consumption, households choose investment. This produces next period's capital stock using the following production function:

$$k_{t+1} = k_t^{\delta_t} i_t^{1-\delta_t}$$

where δ_t denotes a stochastic depreciation factor. It is assumed that this shock, the only source of uncertainty in the economy, is an *i.i.d.* random variable. Given this environment, do the following:

   (a) Solve for the recursive competitive equilibrium by solving the associated social planner's problem. In setting up the dynamic programming problem, use two constraints: the typical resource constraint and the capital production constraint. Denote the Lagrange multiplier on the budget constraint as λ_t while the Lagrange multiplier on the capital production constraint is given by the product of $\lambda_t q_t$ where q_t is the shadow price of capital (in terms of consumption).

   (b) Derive the associated Euler equations for the social planner problem. Give an intuitive explanation for the determination of q_t.

   (c) Define and solve for the recursive competitive equilibrium in this economy. (Note: This is simplified by first using output (y_t) as a state variable and then employing the guess and verify solution method.)

   (d) Give an intuitive explanation for the behavior of consumption and investment in this economy.

3. (20) Consider a standard Solow growth model that is augmented with labor migration. As is typical, the aggregate production function is given by $Y = (AL)^{\alpha} K^{1-\alpha}$ where Y is output, A is effectiveness of labor, L is labor, and K is capital. Also, as is typical, the law of motion for aggregate physical capital is given by $\dot{K} = sY - \delta K$ where $\dot{K} = dK/dt$, s is the savings rate and δ is the depreciation rate. The effectiveness of labor grows at the constant rate of $g : \dot{A} = gA$. In addition to population growth (given by the rate n), the country experiences migration M so that $\dot{L} = (nL + M)$. Migrants bring no physical capital and assume that the migration rate is positively related to the capital per worker. In particular, assume that the migration rate $m = M/L$ is given by $m = b \log(1+k)$ where $b > 0$ and $k = K/(AL)$ is the capital per effective units of labor. Given this, do the following:

   (a) Derive the expression for $\dot{k}$ in this economy. Compare this to the expression in the standard Solow model.

   (b) As in the standard Solow model, analyze graphically the behavior of the economy in a graph with k on the horizontal axis. Let k^*denote the balanced growth path level of k and compare k^*in the economy with migration to that in the standard Solow model.

   (c) Analyze using a phase diagram the stability properties of the balanced growth path for the case with migration.

   (d) Linearize the $\dot{k}$ function around the balanced growth path, define the speed of convergence of the economy to its steady-state and calculate it. Compare the speed of convergence in this economy to that in the standard Solow model.

4. (20) Consider the Mortensen-Pissarides model in continuous time. Labor force is normalized to 1. Unemployed workers, with measure $u \leq 1$, and firms with one vacancy each and total measure v search for each other, and v is determined endogenously by free entry. A CRS matching function, $m(u, v)$, brings together unemployed workers and vacant firms; m is increasing in both arguments. As is standard, let $\theta \equiv v/u$ denote the market tightness and $q(\theta) = m/v$ the arrival rate of unemployed workers to the typical firm.

   What is different here compared to the baseline model is that a "match" and a "productive job" are not equivalent by default. When a worker and a vacant firm meet, the firm must train the worker before she can start producing. A formed match turns into a productive job at a stochastic rate, $a \in (0, +\infty)$, so that $1/a$ can be thought of as the average time necessary for the training to be completed. Assume that the firm and the worker determine the wage level when they first meet (i.e., even before training starts), through Nash bargaining, with $\beta \in (0, 1)$ representing the worker's power. However, the wage upon which they have agreed will only be paid to the worker when she starts producing.[1]

   To close the model, we will make a few more standard assumptions. The output of a **productive job** is $p > 0$ per unit of time, and while a firm is searching for a worker it has to pay a search (or recruiting) cost, $pc > 0$, per unit of time. Firms that are training their workers do not pay this cost (they are done recruiting). Productive jobs are exogenously destroyed at rate $\lambda > 0$ (only productive jobs are subject to this shock; matches at the training stage cannot be terminated). All agents discount future at the rate $r > 0$, and unemployed workers enjoy a benefit $z > 0$ per unit of time. While at the training stage the worker does not receive an unemployment benefit (a trainee is not unemployed).[2]

   (a) Define the value functions of the typical firm at one of the three possible states: V (with an open vacancy), M (matched but still at the stage of training), and J (matched at the stage of production). Describe the steady state expressions for these value functions.

   (b) Define the value functions of the typical worker at one of the three possible states: U (unemployed), T (matched but still at the stage of training), and W (matched at the stage of production). Describe the steady state expressions for these value functions.

   (c) Combine the free entry condition (i.e., $V = 0$) with the expressions that you provided for V, M, J in order to derive the job creation curve of the economy.

   (d) Using the same methodology as in the lecture (adjusted only to accommodate the differences in the new environment), derive the wage curve for this economy.

   (e) Provide a restriction on parameter values such that a steady state equilibrium pair (w, θ) exists. Is it unique? (no need for a lengthy discussion)

   (f) What is the effect of a decrease in a on the equilibrium w and θ? Explain intuitively (but shortly).

   (g) Describe the Beveridge curve of this economy by looking at the flows of workers in and out of the various states. What effect will the decrease in a (discussed in the previous part) have on unemployment?

[1] Hence, two parties who met at time, say, t are negotiating over an object that will be paid in the future (at time $t + 1/a$, in expected terms). But, as is always the case, the Nash Bargaining problem is to split the generated surplus as of time t.

[2] **Suggestion:** Throughout this question you will be finding expressions that look familiar, but the (new) term a will show up. While you are not asked to do so (and you will not get extra points), you may want to let $a \to \infty$ and check that your answer coincides with the analogous expression in the baseline model. This should be the case because, after all, the baseline model is just the model described here as the training stage vanishes, i.e., as $a \to \infty$.

5. (20) Consider the standard growth model in discrete time. There is a large number of identical households normalized to 1. Each household wants to maximize life-time discounted utility

$$U(\{c_t\}_{t=0}^{\infty}) = \sum_{t=0}^{\infty} \beta^t u(c_t),\ \beta \in (0,1).$$

Each household has an initial capital k_0 at time 0, and one unit of productive time in each period that can be devoted to work. Final output is produced using capital and labor, according to a CRS production function F. This technology is owned by firms (whose measure does not really matter because of the CRS assumption). Output can be consumed (c_t) or invested (i_t). Households own the capital (so they make the investment decision), and they rent it out to firms. Let $\delta \in (0,1)$ denote the depreciation rate of capital. Households own the firms, i.e. they are claimants to the firms' profits, but these profits will be zero in equilibrium.

The function u is twice continuously differentiable and bounded, with $u'(c) > 0$, $u''(c) < 0$, $u'(0) = \infty$, and $u'(\infty) = 0$. Regarding the production technology, we will introduce the useful function $f(x) \equiv F(x,1) + (1-\delta)x$, $\forall x \in \mathbb{R}_+$. The function f is twice continuously differentiable with $f'(x) > 0$, $f''(x) < 0$, $f(0) = 0$, $f'(0) = \infty$, and $f'(\infty) = 1 - \delta$.

In this model the government taxes households' **investment** at the constant rate $\tau \in [0,1]$. The government returns all the tax revenues, T, to the households in the form of lump-sum transfers. Throughout this question focus on recursive competitive equilibrium (RCE).

(a) Write down the problem of the household recursively.[3] Carefully distinguish between aggregate and individual state variables. Then, define a RCE. **Hint:** Writing down the budget constraint correctly is essential for this question, so think carefully: the household can choose to allocate its wealth between consumption and investment in any way it likes, but for any unit of resources allocated into investment, a fraction τ of that amount will be subtracted from the household's budget (and it will be returned to them in the form of a lump-sum transfer).

(b) Write down the dynamic equation that the aggregate capital stock follows in this economy. **Hint:** Obtain the Euler equation for the typical household and impose the RCE conditions.

(c) Now focus on steady-states. Describe the steady-state equilibrium value of the aggregate capital stock in this economy, and denote it by $K^*(\tau)$. If the formula you arrived at involves the function F, I recommend that you replace it with the function f in order to answer the next parts.

(d) Describe the value of K^* when $\tau = 0$ and when $\tau = 1$.

(e) In class, we studied the RCE steady state level of capital in an economy where the government taxed the income from renting capital (as opposed to investment, which is the case here). In that model, we saw that for $\tau = 1$ the equilibrium capital stock reached zero. Based on your answer to part (d), does this also happen here? Provide an intuitive explanation of why (or why not).

(f) Let $F(K,N) = K^a N^{1-a}$, $a \in (0,1)$. Provide a closed-form solution for $K^*(\tau)$.

(g) Now focus on the special case where $F(K,N) = K^{\frac{1}{2}} N^{\frac{1}{2}}$. Calculate the government's total tax revenue, T, and plot it as a function of the tax rate τ (the so-called Laffer curve). Which value of τ maximizes tax revenues?

[3] Here firms face a static problem. I am not asking you to explicitly spell it out, but it will be critical for correctly defining a RCE.

6. (10) Consider the discrete time monetary-search model we saw in class. As in the baseline model, in the day time trade takes place in a decentralized market characterized by anonymity and bilateral meetings (call it the DM), and at night trade takes place in a Walrasian or centralized market (call it the CM). There are two types of agents, buyers and sellers, and the measure of both is normalized to the unit. The per period utility is $u(q)+U(X)-H$, for buyers, and $-q+U(X)-H$, for sellers; q is consumption of the DM good, X is consumption of the CM good (the numeraire), and H is hours worked in the CM. In the CM, one hour of work delivers one unit of the numeraire. The functions u, U satisfy standard properties. What is important here is that there exists $X^* > 0$ such that $U'(X^*) = 1$. Goods are non storable, but there exits a storable and recognizable object, called fiat money, that can serve as a means of payment. The supply of money, controlled by the monetary authority, follows the process $M_{t+1} = (1+\mu)M_t$, and new money is introduced via lump-sum transfers to buyers in the CM.

   So far, this is just a description of the model we saw in class. What is different here is that only a fraction π of buyers turn out to have a desire to consume the DM good in the current period; let us refer to these buyers as C-types (for consumption) and to the remaining $1-\pi$ buyers as N-types (for no-consumption). The shock that determines each buyer's type in every period is *iid*. A buyer learns her type **after all CM trade has concluded but before the DM opens**. To make things interesting we will assume that between the CM and the DM there is a third market, where C-types and N-types can meet and trade "liquidity", i.e., money. Let us refer to this market as the loan market (LM).[4]

   The LM is a bilateral market for loans, where N-types, who may carry some money that they do not need, meet C-types, who may need additional liquidity. A CRS matching function $f(\pi, 1-\pi)$ brings the two types together. Importantly, the LM **is not anonymous**, so that agents can make credible (and enforceable) promises. Hence, when an N-type and a C-type meet, they mutually benefit from a contract specifying that the N-type will give l units of money to the C-type right away, and the C-type will repay d (for debt) units of the numeraire good in the forthcoming CM.

   After the LM trades have concluded (for the agents who matched with someone), C-types proceed to the DM, where they use money to purchase goods from sellers. Assume that *all C-type buyers match* with a seller. Notice that I have not said anything about the splitting of the various surpluses (i.e., bargaining), because this information will not be necessary for what I am asking here.

   Let $W(.)$ be the CM value function of a buyer, and $V(.)$ the DM value function of a C-type buyer (since only these buyers visit the DM). Also, let $\Omega^i(.)$ be the LM value function of a type-i buyer, $i \in \{C, N\}$. Your task in this question is to describe these value functions. I am not asking you to analyze them. I recommend that you draw a graph summarizing the timing of the model.

   (a) Describe the function $W(.)$, and show that it is linear in all its arguments/state variables (what these arguments are, however, is for you to determine).

   (b) Let (q, p) be the quantity of good and the units of money exchanged in a typical DM meeting. Let (l, d) be the size of the loan (in dollars) and the promised repayment (in terms of the numeraire) specified in a typical LM meeting. What variables do the terms q, p, l, d depend on? **Hint:** Provide quick answers of the form "q is a function of the money holdings of the (C-type) buyer".

   (c) Describe the function $V(.)$, where, again, determining the state variables is your task.

   (d) Describe the functions $\Omega^i(m)$, $i \in \{C, N\}$, for a buyer who enters the LM with m units of money. **Hint:** Recall that some buyers (of either type) will match in the LM and some will not, and the outcome of the matching process will critically affect a buyer's continuation value. Make sure that this is reflected in the expression you provide.

[4] Since it is very important to understand the model setup, let me add an intuitive description of the environment. In the CM buyers choose their money holdings without knowing whether tomorrow they will really need this money. Then, once the CM trade has concluded and money-holding decisions are sunk, buyers learn their types. Thus, ex post, some buyers turn out to need the money that they brought with them (C-types), and some do not (N-types). Hence, the market that opens between the CM and the DM has a social value: it allows the reallocation of liquidity (money) into the hands of the agents who value it most (C-types).