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PRELIMINARY EXAMINATION FOR THE Ph.D. DEGREE

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*Directions: Answer all questions. Feel free to impose additional structure on the problems below, but please state your assumptions clearly. Point totals for each question are given in parentheses.*

1. (10) Consider a consumer with preferences

$$U = \int_0^{\infty} e^{-\rho t} u(c_t) dt,$$

where  $\rho$  is the subjective discount rate,  $c$  is consumption, and  $u(c) = \ln c$ . She receives an exogenous flow of income  $y$  and can borrow or lend freely at a constant interest rate  $r$ , subject to a no-Ponzi-game condition that rules out infinite debt. Her flow budget constraint is

$$\dot{a} = ra + y - c,$$

where  $a$  represents financial wealth and  $a_0$  is the initial value.

- (a) Derive the first-order conditions for optimal consumption.
- (b) At what rate does consumption change? Interpret its sign.
- (c) Derive the optimal decision rule for consumption. Provide an interpretation for the case  $\rho = r$ .

2. (20) Consider the following optimal growth model. Agents' preferences are given by

$$E_0 \left[ \sum_{t=0}^{\infty} \beta^t A_t \ln c_t \right]$$

The term  $A_t$  denotes an *i.i.d.* demand shock. Output is a concave function of beginning of period capital and a technology shock. That is,

$$y_t = z_t k_t$$

It is assumed that  $z_t$  is *i.i.d.* both over time and with respect to  $A_t$ . (That is, the two shocks are independently distributed.) The depreciation rate of capital is 100%. Within this environment, do the following

- (a) Set up the social planner problem as a dynamic programming problem and derive the necessary conditions.
- (b) Conjecture a solution to the policy functions for consumption and savings. Derive the equations that determine these optimal policy functions and characterize their qualitative behavior.
- (c) Suppose one period (real) bonds were introduced into this economy. How do demand shocks affect interest rates in this economy? Explain.

3. (20) Consider the basic real business cycle model in which the representative agent maximizes the expected value of discounted utility given by:

$$E_0 \left[ \sum_{t=0}^{\infty} \beta^t (\ln c_t - \gamma h_t) \right]$$

Technology is given by a standard Cobb-Douglas production function, i.e.  $y_t = z_t k_t^\alpha h_t^{1-\alpha}$  in which  $z_t$  is an *i.i.d.* technology shock with *c.d.f.* given by  $G(z_t)$ . The depreciation rate of capital is 100%. Do the following:

- (a) Solve the model as a social planner problem and derive the optimal policy functions.
- (b) Discuss how well the implied time series characteristics of the model match those seen in the data. (Restrict your discussion to those features discussed in class.) Discuss a minimal set of changes in the model environment that could produce better consistency between the model and business cycle data.

4. (20) Consider the standard growth model in discrete time. There is a large number of identical households (normalized to 1). Each household wants to maximize life-time discounted utility

$$U(\{c_t\}_{t=0}^{\infty}) = \sum_{t=0}^{\infty} \beta^t u(c_t).$$

Each household has the following endowments: a) an initial capital stock  $x_0$  at time 0, b) one unit of productive time per period that can be devoted to work, and c) one unit of land. Final output is produced according to

$$y_t = F(k_t, n_t, l_t),$$

where  $F$  is a CRS production function, and  $k_t, n_t, l_t$  denote capital, labor, and land services, respectively. This technology is owned by firms whose number will be determined in equilibrium. Output can be consumed ( $c_t$ ) or invested ( $i_t$ ). We assume that households own the capital stock (so they make the investment decision) and the land, and they rent out capital, labor, and land services to the firms. The depreciation rate of the capital stock ( $x_t$ ) is denoted by  $\delta$ .<sup>1</sup> The land does not depreciate, i.e. it is in fixed supply. Finally, we assume that households own the firms, i.e. they are claimants to the firms' profits. The functions  $u$  and  $F$  have the usual properties that we have described in class.<sup>2</sup>

- (a) Consider an Arrow-Debreu world. Describe the households' and firms' problems and carefully define an AD equilibrium. How many firms operate in this equilibrium?
- (b) In this economy, why is it a good idea to describe the AD equilibrium capital stock allocation by solving the (easier) Social Planner's Problem?

From now on assume that  $F(k_t, n_t, l_t) = A k_t^{\alpha_1} n_t^{\alpha_2} l_t^{1-\alpha_1-\alpha_2}$  and  $\delta = 1$ , with  $A > 0$  and  $\alpha_1, \alpha_2 \in (0, 1)$ . The household's instantaneous utility function is given by  $u(c) = \ln(c)$ .

- (c) Fully characterize (i.e. find a closed form solution for) the equilibrium allocation of the capital stock. (**Hint:** Guess and verify a "policy rule" of the form  $k_{t+1} = gk_t^\alpha$ , where  $g$  is an unknown to be determined.)

In the remaining questions, for full credit, your answers should be functions only of the parameters of the model, i.e.  $\alpha_1, \alpha_2, \beta, A, x_0$  etc.

- (d) What is the ADE value of the rental rate of capital as  $t \rightarrow \infty$ ?
- (e) What is the ADE price of the consumption good in  $t = 1$ ?
- (f) What is the ADE price of land services in  $t = 2$ ?

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<sup>1</sup> The capital stock depreciates no matter whether it is rented out to a firm or not.

<sup>2</sup> You will not explicitly need them, so there is no need to be more precise.

5. (20) Consider the following extension of the Mortensen-Pissarides model in continuous time. Labor force is normalized to 1, but there are two types of workers, Type 1, with measure  $\pi \in (0, 1)$ , and Type 2, with measure  $1 - \pi$ . The two types of workers are identical with only one exception: when a Type  $i$  worker is matched with a firm, that match is terminated at an exogenous (Poisson) rate  $\lambda_i$ , with  $\lambda_2 > \lambda_1$ .<sup>3</sup> Although firms would typically prefer to match with Type 1 workers, it is illegal to discriminate against Type 2 workers. We will capture this idea by assuming that the matching technology is *unbiased*, i.e., when a firm matches with a worker, the probability that this worker is Type 1, depends only on the relative measure of Type 1 workers in the pool of unemployed.<sup>4</sup> On the flip side, this assumption means that the arrival rate of jobs to a worker does not depend on her type. Once a match has been formed, the worker's type is immediately revealed.

Let the measure of unemployed workers of Type  $i$  be  $u_i$ , and let the total measure of unemployed workers be  $u = u_1 + u_2$ . A large measure of (identical) firms can enter the market and open exactly one vacancy (the equilibrium measure of vacancies,  $v$ , will be determined by a free entry condition). A CRS matching function brings together unemployed workers and vacant firms, and, due to the "unbiased" matching technology, the total number of matches depends only on  $v$  and  $u$ , i.e.,  $m = m(u, v)$ , and it is increasing in both arguments. As is standard, let  $\theta \equiv v/u$  denote the market tightness.

The output of a job is  $p > 0$  per unit of time, and while a firm is searching for a worker it has to pay a recruiting cost,  $pc > 0$ , per unit of time. Notice that, while searching, the firm has an *expectation* about which type of worker it will match with, and this expectation depends on the relative measure of types in the pool of unemployed. All agents discount future at the rate  $r > 0$ , and all unemployed workers enjoy a benefit  $z > 0$  per unit of time. When a firm and a worker meet, the wage, which is contingent on the worker's type, is determined through Nash bargaining, with  $\beta \in (0, 1)$  denoting the worker's power.

Throughout this question focus on steady states.

- (a) Describe the arrival rate of workers of Type  $i$ ,  $i = 1, 2$ , for the typical firm. What is the arrival rate of a firm to the typical worker of Type  $i$ ,  $i = 1, 2$ ?
- (b) Using your findings in part (a), describe the Beveridge curve (i.e., the relationship between unemployment,  $u_i$ , and market tightness,  $\theta$ ) for each type of worker. **Hint:** Equate the flows in and out of the pool of unemployment for each type. Don't forget to use the exogenous  $\pi$  here.
- (c) Describe the unemployment rate within the Type  $i$  population. Denoting this term by  $\gamma_i$ , show that  $\gamma_1 < \gamma_2$ , for any  $\theta$ .
- (d) Let  $V, J_1, J_2$  denote the value functions of a firm that is vacant, matched with a Type 1 worker, and matched with a Type 2 worker, respectively. Also, let  $U_i, W_i$ ,  $i = 1, 2$ , denote the value functions of a Type  $i$  worker who is unemployed or employed, respectively. Describe these value functions.
- (e) Exploit the free entry condition to provide the job creation curve for this economy. **Hint:** The JC curve should relate the terms  $\theta, w_1$ , and  $w_2$ . Everything else in that equation should be a parameter.
- (f) For  $i = 1, 2$ , describe the wage curve ( $WC_i$ ) for type  $i$  workers. This should be a relationship between  $\theta$  and  $w_i$  (only).
- (g) Prove that the equilibrium wages satisfy  $w_1 > w_2$ .
- (h) What are the endogenous variables that need to be characterized, and how many equations are available? (There is no need to characterize the equilibrium, since it will be a very time-consuming task).

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<sup>3</sup> You can think that Type 2 workers are immigrants who are as productive as local workers, but their Visa may be revoked, thus leading to a higher job destruction rate.

<sup>4</sup> For instance, suppose that there are  $u$  unemployed workers looking for jobs, and 75% of them are of Type 1. Then, conditional on meeting a worker, the probability that this worker is a Type 1 is 75%.

6. (10) Consider the discrete time monetary-search model we saw in class. At day time, trade takes place in a decentralized market characterized by anonymity and bilateral meetings (call it the DM), while, at night, trade takes place in a Walrasian or centralized market (call it the CM). There are two types of agents, buyers and sellers, and the measure of both is normalized to the unit. The per period utility is  $u(q) + U(X) - H$ , for buyers, and  $-q + U(X) - H$ , for sellers;  $q$  is the quantity of DM good,  $X$  is consumption of the CM good (the numeraire), and  $H$  is hours worked in the CM. In the CM, one hour of work delivers one unit of the numeraire. The functions  $u, U$  satisfy standard properties, and there exists  $X^* > 0$  such that  $U'(X^*) = 1$ . All goods are non storable.

In this economy there are two assets. The first is fiat money, which is storable and recognizable, hence it will serve as a medium of exchange in the DM. The supply of money follows the process  $M_{t+1} = (1 + \mu)M_t$ , and new money is introduced via lump-sum transfers to buyers in the CM. The second asset is a **real, one-period bond**. Agents can buy any amount of this asset in the CM at the ongoing price  $\psi_t$ , and each unit of the asset will deliver  $d$  (for dividend) units of the numeraire good in the next period's CM. A new set of these assets is introduced (or issued) in every period's CM (recall that the asset exists for one period only). For reasons that we will not analyze here, this asset cannot serve as a medium of exchange in the DM, i.e., it is illiquid.

In every period, a fraction  $\pi \in (0, 1)$  of buyers, call them the C-types, have a desire to consume the DM good, while the remaining  $1 - \pi$  buyers (the N-types) do not. Buyers learn their types after all CM trade has concluded but before the DM opens, and the shock that determines their types is *iid*. Between the CM and the DM there is a third market, where C-types can sell the real (and illiquid) asset to N-types for cash (which the N-types will not need in the current period). Refer to this market as the secondary financial market (SFM).<sup>5</sup>

The SFM is a bilateral asset market. A CRS matching function  $f(\pi, 1 - \pi)$  brings C and N-types together. When an N-type and a C-type meet they mutually benefit from a trade in which the N-type transfers  $\tau$  units of money to the C-type in exchange for  $\chi$  units of the real asset. After the SFM trades have concluded C-types proceed to the DM, where they use money to purchase goods from sellers. Assume that all C-type buyers match with a seller.

Let  $W(\cdot)$  be the CM value function of a buyer, and  $V(\cdot)$  the DM value function of a C-type buyer (only these buyers visit the DM). Also, let  $\Omega^i(\cdot)$  be the SFM value function of a type- $i$  buyer,  $i \in \{C, N\}$ .

- (a) Describe the function  $W(\cdot)$ , and show that it is linear in all its arguments/state variables (what these arguments are, however, is for you to determine).
- (b) Letting  $(q, p)$  denote the quantity of good and the units of money exchanged in a typical DM meeting, describe the function  $V(\cdot)$ , where, again, determining the state variables is your task.
- (c) Describe the functions  $\Omega^i(m, a)$ ,  $i \in \{C, N\}$ , for a buyer who enters the SFM with  $m$  units of money and  $a$  units of the real asset. **Hint:** Recall that some buyers will match in the SFM and some will not, and the outcome of the matching process will critically affect a buyer's continuation value.

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<sup>5</sup> Clearly, this question is identical in spirit to Question 6 of the June 2015 Prelim. The only difference is that here the allocation of money into the hands of the C-types takes place through selling assets, and not through a loan market.