ANSWER KEY

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PRELIMINARY EXAMINATION FOR THE Ph.D. DEGREE

Directions: Answer all questions. Feel free to impose additional structure on the problems below, but please state your assumptions clearly. Point totals for each question are given in parentheses.

1. (10) Consider a standard Solow growth model with constant labor growth which, in equilibrium, exhibits all of Kaldor's stylized facts of growth (the economy is on a balanced growth path). If a general, constant returns to scale production function is used (that is, we are not restricting technology to be a Cobb-Douglas production function), provide an intuitive explanation for why technological progress must be labor augmenting only. Or, more specifically, what features of a balanced growth path and the model combine to rule out capital augmenting technological progress?

   ANSWER: One can demonstrate this formally, but I will make an intuitive argument. In a balanced growth path that exhibits Kaldor's stylized facts of growth, the capital-labor ratio $\left(\frac{K_t}{N_t}\right)$ is growing at a constant rate while the capital-output ratio $\left(\frac{Y_t}{K_t}\right)$ is constant. Since production is assumed to exhibit constant returns to scale, we can express the level relationship: $Y_t = F\left(K_t e^{gt}, N_t\right)$ as $Y_t/K_t = e^{gt} f\left(\frac{N_t}{K_t e^{gt}}\right)$ where e^{gt} denotes the capital augmenting technological progress (i.e. the proposed idea). Since the left hand side is constant, we require that the two terms on the right-hand side (both changing) must cancel out. If f is iso-elastic, they indeed do -but this is precisely what a Cobb-Douglas production exhibits. In general, this will not be the case. Hence, technological progress must be labor augmenting only with a general CRS production function.

2. (20) Consider the following variant of Stockman's cash-in-advance model. The primary distinction is that asset and goods markets are assumed to meet sequentially with the asset market preceding the goods market. The assets in the economy are: capital (k_t), one-period nominal bonds (B_t), and money (M_t). Nominal bonds costs 1\$ in period t and return \$$N_t$ in next period's asset market. In the asset market, agents also receive the returns from capital (i.e. the revenue from the sale of output in last period's goods market and the sale of undepreciated capital) and the lump-sum monetary transfer. They combine this with any money left over from the previous goods market to buy new capital, bonds and money. Next agents visit the goods market where money is used to finance consumption (i.e. consumption is subject to the cash-in-advance constraint). The aggregate money stock grows at the constant rate $\mu > 0$ and capital depreciates at the rate $\delta \in (0, 1)$. (Note that, as in Stockman, agents' use capital to produce output via the production function $y_t = f(k_t)$. This output is sold in the goods market.) Households have standard time separable preferences as in Stockman.

   (a) Set up the agent's maximization problem as a dynamic programming problem. (Hint: Note that the budget constraint relevant to the asset market does not include consumption.)

   (b) Prove that, in this economy, the Lagrange multiplier associated with the budget constraint is equal to agent's marginal utility of consumption. Compare this result to that in Stockman's model.

   (c) Define a steady-state equilibrium. Prove that money is not superneutral. Again, compare this result to that in Stockman's paper.

   (d) Prove that $N_t = 1$ (i.e. net nominal interest rates are zero) if and only if the cash-in-advance constraint is not binding. Explain.

   ANSWER:This setup corresponds to the following budget and CIA constraints:

$$\begin{aligned} & P_{t-1} f\left(k_{t-1}\right) + P_t k_{t-1}(1-\delta) + B_{t-1} N_{t-1} + \left(M_{t-1} - P_{t-1} c_{t-1}\right) + T_t \qquad (1) \\ = \; & P_t k_t + B_t + M_t \end{aligned}$$

$$M_t \geq P_t c_t \tag{2}$$

The dynamic programming problem is

$$V(s_t) = \max \left\{ \begin{array}{c} U(c_t) + \beta V(s_{t+1}) \\ +\lambda_t \left(\begin{array}{c} \frac{P_{t-1} f(k_{t-1})}{P_t} + k_{t-1}(1-\delta) + \frac{B_{t-1} N_{t-1}}{P_t} + \\ \frac{(M_{t-1} - P_{t-1} c_{t-1})}{P_t} + \frac{T_t}{P_t} - k_t - \frac{B_t}{P_t} - \frac{M_t}{P_t} \end{array} \right) \\ +\gamma_t \left(\frac{M_t}{P_t} - c_t \right) \end{array} \right\} \tag{3}$$

where the state vector is $s_t = (M_{t-1}, c_{t-1}, k_{t-1}, B_{t-1})$. Note that consumption last period affects how much money is brought into period t hence that is why it is a state variable, along with M_{t-1}. (Note: it would be possible to introduce a new variable, real wealth as defined by the left hand side of eq.(1) and use this as the sole state variable.) Taking derivatives and applying the envelope theorem yields the following first order conditions:

$$c_t : U_t' = \gamma_t + \beta \lambda_{t+1} \frac{P_t}{P_{t+1}} \tag{4}$$

$$k_t : \lambda_t = \beta \lambda_{t+1} \left[\frac{P_t}{P_{t+1}} f'(k_t) + 1 - \delta \right] \tag{5}$$

$$M_t : \frac{\lambda_t}{P_t} = \frac{\gamma_t}{P_t} + \beta \lambda_{t+1} \frac{1}{P_{t+1}} \tag{6}$$

$$B_t : \frac{\lambda_t}{P_t} = \beta \lambda_{t+1} \frac{N_t}{P_{t+1}} \tag{7}$$

First combining eqs. (6) and (4) yields:

$$\lambda_t = U_t'$$

This is the answer to (b) - the reason is that wealth of any kind can be converted into money in the asset market which precedes the goods market. Using this in eq. (5) and solving for steady-state (imposing $\frac{P_t}{P_{t+1}} = \frac{1}{1+\mu}$) yields:

$$\beta^{-1} = \frac{f'(\bar{k})}{1+\mu} + 1 - \delta$$

This is the answer to part (c) - money is not superneutral in this economy since the inflation tax affects the revenues from capital. (Why is this different from Stockman's model?) Finally, manipulating the last two necessary conditions yields the Fisher relationship:

$$R_t = N_t \frac{P_t}{P_{t+1}}$$

For part (d), note that eq.(7) can be re-written as:

$$N_t^{-1} = \beta \left[\frac{\lambda_{t+1}/P_{t+1}}{\lambda_t / P_t} \right]$$

while eq.(6) can be rearranged to yield:

$$1 = \frac{\gamma_t}{\lambda_t} + \beta \left[\frac{\lambda_{t+1}/P_{t+1}}{\lambda_t / P_t} \right]$$

Hence if and only if $\gamma = 0$ (the CIA constraint is not binding) will $N_t = 1$.

3. (20) Consider a representative agent economy in which, every period, agents allocate income between consumption, capital, bonds, and equity. Capital acquired in period t is used to produce output in period $t+1$; the production function is given by

$$y_t = z_t k_t$$

where z_t denotes a random shock to technology. (The depreciation rate of capital is 100%.) It is assumed that z_t can take on two values $z_1 < z_2$. The evolution of z_t is governed by a symmetric transition probability matrix with diagonal elements $\pi > 1/2$. One and two period bonds are traded. Both bonds cost one unit of consumption in period t and return $R1_t$ and $R2_t^2$, respectively, upon maturity. The remaining asset, equity, sells at the price q_t and pays a dividend each period given by $d_t = \alpha y_t$ where $\alpha \in (0,1)$. Households choose consumption and assets in order to maximize expected lifetime utility given by:

$$E\left\{\sum_{t=0}^{\infty}\beta^t \ln c_t; \quad \beta \in (0,1)\right\}$$

Given this environment, answer the following questions:

(a) Express the household's maximization problem as a dynamic programming problem. (In setting up the budget constraint, assume that two-period bonds are sold after holding them for one period.)

(b) Define a recursive competitive equilibrium. Be precise in identifying the elements in your definition.

(c) Solve the economy as a social planner problem and derive the equilibrium policy functions for capital and consumption.

(d) Characterize the behavior of one-period interest rates in equilibrium. Explain your result.

(e) Compare the average return from selling a two-period bond after holding it for one period to that from holding a one-period bond to maturity. Explain.

(f) Prove that, if $d_t = c_t$, then $q_t = k_{t+1}$. Explain.

ANSWER: The budget constraint is:

$$\gamma_{t-1}(q_t + \alpha y_t) + R1_{t-1}b1_{-1} + \frac{R2_{t-1}^2}{R1_t}b2_{t-1} + z_t k_t = c_t + k_{t+1} + q_t\gamma_t + b1_t + b2_t \tag{8}$$

where γ_t denotes shares of equity, q_t is the price of equity and all other variables are standard.

(b) A recursive competitive equilibrium is defined by a set of functions: there will be optimal policy functions for capital and consumption, $c_t = c(a_t, k_t, z_t)$, $k_{t+1} = k(a_t, k_t, z_t)$ where a_t denotes the households capital stock and (k_t, z_t) denotes the aggregate state vector. There will also be three functions defining the asset prices: $q_t = q(k_t, z_t), R1_t = R1(k_t, z_t), R2_t = R2(k_t, z_t)$. Finally, equilibrium is also defined by the value function given by the standard Bellman equation. So, in total, there are 6 six functions defining equilibrium. In addition, we have market clearing: $\gamma_t = b1_t = b2_t = 0\ \forall\ t$.Also, the goods market clears: $c(k_t, z_t) + k(k_t, z_t) = z_t k_t$ and we require $a_t = k_t$. A good answer will mention all of these features.

(c) We have seen this environment repeatedly, the solution will be $c_t = (1-\beta) z_t k_t; k_{t+1} = \beta z_t k_t$.

(d) The Euler condition for one-period bonds is:

$$c_t^{-1} = R1_t \beta E_t\left[c_{t+1}^{-1}\right]$$

Using the policy function for consumption yields:

$$R1_t^{-1} = E_t\left[z_{t+1}^{-1}\right]$$

Given the assumption of positive serial correlation, it is easy to establish that $E_1\left[z_{t+1}^{-1}\right] > E_2\left[z_{t+1}^{-1}\right]$ implying $R1_2 > R1_1$.Because of the higher expected MPK in state 2, the interest rate will be higher in that state.

(e) The CCAPM says it is the covariance between the return of a risky asset and agents' MU of consumption that determines the risk premium on that asset - the variance is not the critical factor. In this model, it is easy to show that the expected return from selling a two-period bonds after one period will be LESS than the certain one-period yield. That is, the term premium (or risk premium associated with selling the two period bond after holding it for one period) is negative: the return from such a strategy covaries positively with agents' MU of consumption. To see this, suppose z_2 occurs in period $t+1$. Then, as just shown, the one-period interest rate will be high which means that the returns from selling a two period bond before maturity will be low. But, consumption will be high because of the high MPK which means that the MU of consumption will be low. So this asset pays poorly when it is not needed and pays out well when consumption is low (MU is high). So agents will take a lower average return relative to a riskless rate since it helps to smooth consumption.

(f) Using the Euler condition associated with equity under the assumption that $d_t = c_t$, we have:

$$\frac{q_t}{c_t} = \beta E_t\left[\frac{q_{t+1} + c_{t+1}}{c_{t+1}}\right] = \beta E_t\left[\frac{q_{t+1}}{c_{t+1}}\right] + \beta$$

Recursively solving this expression forward yields $q_t = \frac{\beta}{1-\beta}c_t$. But using the policy function for consumption yields: $q_t = \beta z_t k_t$ which is identical to the policy function for capital. Since k_t can produce the sequence of $\{c_t\}_{t=1}^{\infty}$, it must sell at the same price as this asset.

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PRELIMINARY EXAMINATION
ANSWER KEY FOR QUESTIONS 4-6

Question 4 Answer Key a) The typical households problem can be written recursively as

$$
\begin{aligned}
V(k,K) &= \max_{c,k'} \left\{ u(c) + \beta V(k',K') \right\} \\
s.t.\ \ c + k' &= (w + rk)(1-\tau) + (1-\delta)k + T, && (1)\\
K' &= H(K), && (2)\\
w = w(K) &= F_2(K,1), && (3)\\
r = r(K) &= F_1(K,1), && (4)\\
T = T(K) &= \tau[w(K) + r(K)K], && (5)
\end{aligned}
$$

where k is the individual capital, and K is the aggregate capital. Moreover, (1) is the household's budget constraint, (2) is the aggregate law of motion of capital, (3) and (4) follow directly from market clearing, and (5) is the government revenue as a function of the aggregate state.

The definition of a RCE is standard, and can be found in the lecture notes (or in problem 4 of PS 6). In the definition of RCE, the most important part is to clarify that consistency requires $g(K,K) = H(K)$, where H was defined above, and g is the typical household's policy function.

b) The Euler equation for the typical household is given by

$$u'(c) = \beta[(1-\tau)F_1(K',1) + 1 - \delta]\, u'(c').$$

But the objective here was to express everything as a function of the aggregate capital stock only. To that end, notice that we can use the budget constraint to write consumption as

$$
\begin{aligned}
c &= [F_2(K,1) + F_1(K,1)K](1-\tau) + (1-\delta)K + \tau[F_2(K,1) + F_1(K,1)K] - H(K) = \\
&= F_2(K,1) + F_1(K,1)K + (1-\delta)K - H(K)
\end{aligned}
$$

If we exploit the Euler theorem we can get rid of the partial derivatives and write

$$c = F(K,1) + (1-\delta)K - H(K),$$

and finally, using the definition of the function f, we have

$$c = f(K) - H(K).$$

Using this last formula, we can write the dynamic equation that aggregate capital stock follows as:

$$u'[f(K) - H(K)] = \beta[(1-\tau)F_1(H(K),1) + 1 - \delta]\, u'[f(H(K)) - H(H(K))].$$

As I mentioned in the hint, this expression contains only K, parameters of the model (including the given functions $u, f, F,$) and the function H, which is part of the definition of the RCE. Let me point out that here you were not expected to be 100% formal. For instance, if you wrote K'' instead of $H(H(K))$, this is perfectly fine.

c) We know that in the steady-state equilibrium $c = c'$ and $K = K' = K''$. Hence, the Euler equation simplifies to

$$1 = \beta[(1-\tau)F_1(K^*, 1) + 1 - \delta].$$

Using the definition of the function f one more time, we can obtain the definition of the steady-state aggregate capital stock. More precisely, it is easy to check that:

$$K^*(\tau) \equiv \left\{K : f'(K) = \frac{1 - \beta\tau(1-\delta)}{\beta(1-\tau)}\right\}.$$

d) When $\tau = 0$, it is easy to check that $K^*(0)$ solves $f'(K^*(0)) = 1/\beta$, which, of course, is exactly the same as the steady-state level of capital in the baseline model with no government and taxes.

On the other extreme, if $\tau = 1$, it is easy to check that $K^*(0) \to 0$ (follows directly from the fact that $f'(0) = \infty$). This result is extreme but very intuitive. In a world where the government imposes taxation equal to 100% of people's income (and even though it returns the tax revenue to the households), the free riding problem is so severe that everybody chooses to not work or accumulate any capital.

e) Simple application of the implicit function theorem yields:

$$\frac{dK^*(\tau)}{d\tau} = \frac{1 - \beta(1-\delta)}{\beta(1-\tau)^2}\,\frac{1}{f''(K^*)},$$

which is strictly negative, since it shares the same sign as the term f''.

f) Given that $\delta = 1$ and $F(K, N) = K^a N^{1-a}$, it is easy to check that the steady-state level of capital has a nice closed form solution, and, in particular

$$K^* = [a\beta(1-\tau)]^{\frac{1}{1-a}}.$$

Therefore, the total tax revenue is given by

$$T(\tau) = \tau F(K, 1) = \tau\,[a\beta(1-\tau)]^{\frac{a}{1-a}}.$$

It is relatively easy to check (but also very intuitive) that the function T is hump-shaped in τ, and it equals 0 at both $\tau = 0$ and $\tau = 1$ (not surprisingly, given our discussion in part (d)).

When $a = 1/2$, we have $T = a\beta\tau(1-\tau)$, and it is easy to check that this expression attains a unique global maximum at $\tau^* = 1/2$.

Question 5 a) As I explained in the hint, here the state space is not just m, but (m, d), where d stands for potential debt that the buyer might have as a result of trading in the day market. Of course, d must not exceed the exogenous credit limit D. The value function for a buyer is

$$W^B(m,d) = \max_{X,H,\hat{m}} \{U(X) - H + \beta V(\hat{m})\}$$
$$\text{s.t. } X + \phi\hat{m} = H + \phi(m + \mu M) - d,$$

that is, the term d enters as a variable that makes the buyer worse off (it reduces her resources), since this is a liability for the buyer.

It can be easily verified that, at the optimum, $X = X^*$. Using this fact and replacing H from the budget constraint into W^B yields

$$W^B(m) = U(X^*) - X^* + \phi(m + \mu M) - d + \max_{\hat{m}} \{-\phi\hat{m} + \beta V(\hat{m})\}. \tag{6}$$

As is standard in this model, the optimal choice of the agent does not depend on the current state (due to quasi-linearity), and so the value function is linear in both m and d. Also, we can collect all the terms in (6) that do not depend on the state variable and write

$$W^B(m) = \varphi m - d + \Lambda, \tag{7}$$

where the definition of Λ is obvious.

For the seller the state space is similar, but things are even simpler, since this agent will never want to buy any money in the Walrasian market. It is very easy to show that

$$W^S(m,d) = \varphi m + d + \Lambda^S,$$

where $\Lambda^S \equiv U(X^*) - X^*$. Here, the term d enters as a variable that makes the seller better off (it increases her resources), because this is an asset (as opposed to a liability) for this agent.

b) Consider a meeting between a buyer who carries m units of money, and a seller who, of course, carries nothing. Let d denote the amount of credit that the buyer uses in the transaction, and x the amount of money that changes hands. Since the buyer makes a TIOLI offer, we need to solve the problem:

$$\max_{x,d,q} \{u(q) + W^B(m - x, d) - W^B(m, 0)\}$$
$$\text{s.t. } \; -q + W^S(x,d) - W^S(0,0) = 0,$$

and subject to $x \leq m$, $d \leq D$. Notice that the outside option of the buyer is to walk away and enter the night market with her original money holdings m and zero debt.

Similarly, the seller's outside option is to enter the CM with no money or credit. Now, using the linearity of the W functions, we can re-write the bargaining problem as:

$$\max_{x,d,q} \{u(q) - \phi x - d\}$$
$$\text{s.t.} \quad -q + \phi x + d = 0,$$
$$x \leq m, \; d \leq D.$$

It follows that the agents should always set $q = q^*$ as long as the buyer has a large enough combination of money and credit to compensate the seller for the cost that she suffered. Define the real purchasing power of the buyer as $\pi = \phi m + D$. If $\pi \geq q^*$, then $q = q^*$, and the bargaining solution for x, d satisfies $\phi x + d = q^*$. On the other hand, if If $\pi < q^*$, then the buyer will be constrained: she will exhaust all her resources, i.e. $d = D$ and $x = m$, and she will buy as much good as her budget allows, i.e. $q = \pi$.

c) Using the standard method we can arrive at the following objective function:

$$J(\hat{m}) = (-\phi + \beta\hat{\phi})\hat{m} + \beta\sigma\Big\{u(q(\hat{m}, D)) - \hat{\phi}x(\hat{m}, D) - d(\hat{m}, D)\Big\}.$$

Here you can see how the intuition that I gave you in the question becomes relevant. The buyer has a cost of carrying money, but no cost of carrying credit. But, of course, the buyer has a limit (i.e. D) of how much he can borrow against credit, and this affects the bargaining outcomes $q(.), x(.), d(.)$.

Parts d,e of this question become very easy once one realizes that credit has an important advantage over money here. The cost of carrying money is positive, but there is no such thing as a "cost of carrying credit": once you meet a seller you are free to obtain a loan up to D units (in terms of the numeraire). This simply means that if the buyer could get the quantity of q she desires using only credit, then she would NEVER want to hold any cash, which is to say her demand for money would be zero, which is to say the real value of money would be zero, or, if you prefer, a monetary equilibrium would never exist.

d) Given the discussion above, when $D \geq q^*$, the buyer can obtain the first best just by using credit. This means that she never wants to hold money. An argument along these lines would suffice to get full credit, but if you want to see it more formally, for $D \geq q^*$, we know from our bargaining solution that $\pi = \phi m + D \geq q^*$, which implies that $q = q^*$. Imposing these conditions on the objective function allows us to write the latter as

$$J(\hat{m}) = (-\phi + \beta\hat{\phi})\hat{m} + \beta\sigma\left[u(q^*) - q^*\right].$$

But then, bringing more money has a positive cost and no benefit. Clearly, the rational agent will choose $\hat{m} = 0$, and no monetary equilibrium will exist.

Let me point out that there is one and only one case in which money could be held: this would require the cost of money to be zero, or, equivalently, the monetary authority to run the Friedman rule. However, notice that here we have excluded the Friedman rule by imposing the restriction $\mu > \beta - 1$.

e) Now that $D < q^*$, there is room for money to play some non-trivial role, i.e. to bring consumption of q closer to the first-best q^*. Since $D < q^*$, and since money is costly to carry, we know that we will always be in the binding region of the bargaining solution. Now, the objective function becomes

$$J(\hat{m}) = (-\phi + \beta\hat{\phi})\hat{m} + \beta\sigma\Big\{u(\hat{\phi}\hat{m} + D) - \hat{\phi}\hat{m} - D\Big\}.$$

Taking the FOC with respect to $\hat{m}$ yields the money demand:

$$\phi = \beta\hat{\phi}\Big\{1 + \sigma\Big[u'(\hat{\phi}\hat{m} + D) - 1\Big]\Big\}.$$

We can now focus on a steady-state equilibrium where $\phi M = \hat{\phi}\hat{M}$. The equilibrium q will satisfy

$$\frac{1 + \mu - \beta}{\beta} = i = \sigma[u'(q) - 1], \tag{8}$$

where i is the nominal interest rate. Moreover, the equilibrium real balances satisfy $q = z + D$ (this follows straight from the bargaining solution).

There is one last important detail to clarify: eq. (8) determines equilibrium q if the equilibrium IS MONETARY. But equilibrium need not be monetary. We know that q has to be bounded from below by D (this is true because $q = z + D$; z cannot be negative, but it could be zero, and this is when q will equal D). However, if μ (or i) is very large, there will be a point where the q determined by (8) will become less than D: to see this, just notice that q is a decreasing function of μ, and as $\mu \to \infty$, $q \to 0$. Of course, having $q < D$ is not possible.

What is going on here is that there exists an upper bound of monetary policies, call it μ^c, such that monetary equilibrium can be supported only if $\mu \leq \mu^c$.[1] For any $\mu > \mu^c$ (or $i > i^c$), the authorities have made the cost of holding money so high that agents simply choose to never hold any money and carry out all their transaction with credit. Of course, it is also easy to check (and very intuitive) that μ^c is a decreasing function of D: the lower D is, the more inflation people will be willing to tolerate. But if D is large there is very little room for money to improve welfare, and agents will not be willing to tolerate a high rate of inflation. As an extreme case, when $D \geq q^*$ (as in part (d)), $\mu^c = \beta - 1$, and the region of monetary equilibria has become

[1] It is easy to check that, in terms of the nominal interest rate, this critical value is given by $i^c = \sigma[u'(D) - 1]$. In terms of μ the critical value is given by $\mu^c = \beta\{1 + \sigma\,[u'(D) - 1]\} - 1$.

an empty set. This is consistent with our answer in (d), where we saw that agents get the first-best just by using credit, and money is never valued.

Summary: To be super formal, we can conclude that the equilibrium q (as a function of monetary policy) is given by $q(i) = \max\{D, \tilde{q}(i)\}$, where we have defined $\tilde{q}(i) \equiv \{q : i = \sigma[u'(q) - 1]\}$, and the equilibrium real balances are given by $z(i) = q(i) - D$.

Question 6 This question is very standard (with the exception that it assumes a specific functional form for m, it is exactly the same as in your lecture notes).

a) The social planner's problem is given by:

$$\begin{aligned} \max_{\theta} \quad & \int_0^{\infty} e^{-rt} \left[(1-u)p + uz - pc\theta u\right] dt \\ s.t. \quad & \dot{u} = (1-u)\lambda - u\theta q(\theta) \end{aligned}$$

b) Set up the Hamiltonian

$$H = e^{-rt} \left[(1-u)p + uz - pc\theta u\right] + \mu \left[(1-u)\lambda - u\theta q(\theta)\right]$$

$$\frac{\partial H}{\partial \theta} = 0 \Rightarrow pcue^{-rt} + \mu\theta^{-\alpha}\left[1-\alpha\right] = 0 \tag{9}$$

$$\frac{\partial H}{\partial u} = -\dot{\mu} \Rightarrow e^{-rt}[-p+z-pc\theta] - \mu(\lambda + \theta^{1-\alpha}) = -\dot{\mu} \tag{10}$$

From the last equation it is easy to see that

$$\dot{\mu} = -r\mu. \tag{11}$$

Combining eq.(9) and (11), and after some algebra, we obtain

$$(1-\alpha)\theta^{-\alpha}\left[\frac{p-z}{pc} + \theta\right] = r + \lambda + \theta^{1-\alpha},$$

or equivalently

$$(r+\lambda)\theta^{\alpha} + \alpha\theta = \frac{(1-\alpha)(p-z)}{pc}.$$

So the θ that is chosen by social planner must satisfy the last expression, and it is easy to verify that it is uniquely given. Notice that in the last expression the term a coincides with the elasticity of the arrival rate of workers to firms with respect to the market tightness (which in the lectures we defined as $\eta(\theta)$).

c) We know that the decentralized market equilibrium will, in general, not coinicde with the planner's allocation because this environment is characterized by a search externality. The only way to get the two to coincide is if the so-called Hosios condition is satisfied, which here in particular requires $\eta(\theta) = a = \beta$, where β is the bargaining power of the worker. Of course, since the terms of trade are determined through Nash bargaining, there is no reason to expect why this condition should be satisfied. As a result, the entry of firms will be generically suboptimal.