PRELIMINARY EXAMINATION FOR THE Ph.D. DEGREE

MACROECONOMICS

August 23, 2019

Directions: The exam consists of six questions. Questions 1, 2 concern ECN 200D (Geromichalos), questions 3, 4 concern ECN 200E (Oloyne), and questions 5, 6 concern ECN 200F (Caramp). You only need to answer five out of the six questions. If you prefer (and have time), you can answer all six questions, and your grade will be based upon the best five scores. Feel free to impose additional structure on the problems below, but please state your assumptions clearly. You have 5 hours to complete the exam and an additional 20 minutes of reading time.
Question 1 (20 points)

Consider the standard Mortensen-Pissarides model in continuous time. Labor force is normalized to 1. Unemployed workers with measure \( u \) search for jobs, and firms with measure \( v \) search for unemployed workers. The matching technology is given by the function \( m(u,v) \), which is increasing in both arguments and exhibits constant returns to scale. It is convenient to define the market tightness \( \theta \equiv v/u \). A large measure of firms decide whether to enter the labor market with exactly one vacancy. When a firm meets an unemployed worker a job is formed. The output of a job is \( p \) per unit of time. However, while the vacancy is unfilled, firms have to pay a search or recruiting cost equal to \( pc \) per unit of time. Upon a successful match, the worker’s wage is negotiated through Nash bargaining. Let \( \beta \) represent the worker’s bargaining power.

The destruction rate of existing jobs is exogenous and given by the Poisson rate \( \lambda \). Once a shock arrives, the firm closes the job down. Subsequently, the worker goes back to the pool of unemployment, and the firm exits the labor market.

So far this is just the model we described in class. What is new here is that unemployed workers are not (necessarily) eligible for unemployment benefits for the whole duration of their unemployment spell. More precisely, a worker who just lost her job starts receiving a benefit \( z \) per unit of time right away, but she will stop being eligible for this benefit at a Poisson rate \( \delta \). Thus, at any point in time, there are two types of unemployed workers: those who are still eligible for \( z \), and those whose unemployment benefit eligibility has expired. This, of course, will be important for the negotiation process between a firm and an unemployed worker.\(^1\)

Throughout this question focus on steady state equilibria and let the discount rate of agents be given by \( r \). Also, assume that \( \delta \geq \lambda \), which will simplify some of the derivations later on.

a) Draw a figure that describes the flows in and out of the various states a worker may be in. Denote these states by \( E \), for employed, \( U \), for unemployed with benefits, and \( U_n \), for unemployed without benefits.

b) Let \( u \) denote the equilibrium measure of all unemployed workers, and \( u_n \) the measure of unemployed workers who do not receive benefits. Equating the inflows and outflows for the states \( E \) and \( U_n \) (defined in part a), derive the analogue of the Beveridge curve for this economy. That is, find two equations that describe the equilibrium variables \( u, u_n \) as functions of the market tightness \( \theta \).

In what follows let \( w \) denote the wage of a worker who got a job while still eligible for unemployment benefits, and let \( w_n \) denote the wage of a worker who was not eligible for unemployment benefits when she negotiated with the firm.

\(^1\) It should be obvious why: even though the two types of unemployed workers have the exact same bargaining power (\( \beta \)), they do not have the same outside option.
c) Describe the value functions for a worker in the various states.

d) Describe the value functions for a firm in the various states.

e) Exploiting the free entry of firms into the labor market, describe the Job Creation (JC) condition for this economy.\(^2\)

f) Solve the bargaining problem between a firm and an unemployed worker in state \( U \) and \( U_n \) and derive the analogue of the Wage Curve (WC) for this economy.\(^3\)

g) Using your findings in part (f) show that, for any given \( \theta \), we have \( w > w_n \). What is the economic meaning of this result?

h) Without going into detail, provide a strategy to solve for the steady state equilibrium. More precisely, explain how you would combine the various equilibrium conditions derived so far in order to characterize the five equilibrium variables.

\(^2\)Hint: This equation should involve only the equilibrium variables \( w, w_n, \theta \) and parameters of the model, i.e., it should not involve any value functions.

\(^3\)Hint: Since here we have two wages, you should provide two wage equations. Like before, these equations should involve the equilibrium variables \( w, w_n, \theta \), but no value functions.
Question 2 (20 points)

Consider an economy that consists of two islands, \( i = \{1, 2\} \). Each island has a large population of infinitely-lived, identical agents, normalized to the unit. There is a unique consumption good, say, coconuts, which is not storable across periods. Although within each island agents have identical preferences over consumption, across islands there is a difference: Agents in island 2 are more patient. More precisely, the lifetime utility for the typical agent in island \( i \) is given by

\[
U_i \left( \{c^i_t\}_{t=0}^\infty \right) = \sum_{t=0}^\infty \beta^i_t \ln(c^i_t),
\]

where \( \beta_i \in (0, 1) \), for all \( i \), and \( \beta_2 > \beta_1 \).

Due to weather conditions in this economy, island 1 has a production of \( e > 0 \) units of coconuts in even periods and zero otherwise, and island 2 has a production of \( e \) units of coconuts in odd periods and zero otherwise. Agents cannot do anything to boost this production, but they can trade coconuts, so that the consumption of the typical agent in island \( i \), in period \( t \), is not necessarily equal to the production of coconuts on that island in that period (which may very well be zero). Assume that shipping coconuts across islands is costless.

a) Describe the Arrow-Debreu equilibrium (ADE) allocations in this economy. You can use any method you like, but I strongly recommend that you exploit Negishi’s method.

b) Describe the ADE prices in this economy.

c) Plot the equilibrium allocation for the typical agent in island \( i \), i.e., \( \{\tilde{c}^i_t\}_{t=0}^\infty \), \( i = \{1, 2\} \), against \( t \). Is there any period \( t \) in which \( \tilde{c}^1_t = \tilde{c}^2_t \)? If yes, please provide a closed form solution for that value of \( t \).\(^4\)

\(^4\) Please provide the formula for the \( t \) that solves \( \tilde{c}^1_t = \tilde{c}^2_t \), without worrying whether it is an integer.
**Question 3 (20 points)**

Consider the following decentralized real business cycle model. The representative household has preferences over consumption and leisure. Expected lifetime utility is:

\[
E_0 \sum_{t=0}^{\infty} \beta^t \left( \frac{(C_t H_t^{-h})^{1-\sigma}}{1 - \sigma} - \frac{N_t^{1+\eta}}{1 + \eta} \right)
\]

where \( C \) is consumption and \( N \) is hours worked. These preferences include a form of habit formation and the "habit stock", \( H_t \), is simply equal to consumption in \( t - 1 \): \( H_t = C_{t-1} \). \( h \) governs the importance of habits and \( 0 \leq h < 1 \).

The household maximizes lifetime utility subject to their budget constraint:

\[
C_t + I_t = w_t N_t + (1 + r^K - \delta) K_t + \Pi_t
\]

where \( w \) is the real wage, \( N \) is hours worked, \( K \) is capital, \( I \) is investment, \( r^K \) is the rental price of capital and \( \Pi \) are profits from firms. Capital evolves as follows:

\[
K_{t+1} = (1 - \delta) K_t + I_t
\]

but there is full depreciation each period and \( \delta = 1 \).

Competitive firms produce output using capital and labor. The production function is

\[
Y_t = Z_t K_t^\alpha (X_t N_t)^{1-\alpha}
\]

where TFP, \( Z \), is stochastic and follows the following Markov process

\[
\log Z_t = \rho \log Z_{t-1} + \epsilon_t
\]

\( X_t / X_{t-1} = \gamma \) is the deterministic growth rate of labor augmenting technological change but assume \( \gamma = 1 \) and \( X = 1 \). You do not need to de-trend the model.

a) Write down the household’s problem in recursive form and write down the firm’s maximization problem. Derive the household’s first order conditions and the firm’s optimal hiring rules.

b) Carefully define a recursive competitive equilibrium.
c) Assume $\sigma = 1$. Using guess and verify, find the policy functions for investment, consumption, hours worked and output and show that these are independent of $C_{t-1}$ (Hints: Guess that consumption and investment are a constant share of output. You will also find it easier to combine various equilibrium conditions from part (a) before applying the guess and verify method). How, and why, do TFP shocks affect output, consumption, investment and hours worked in this model?

d) Now consider the possibility that $\sigma > 1$ (rather than $\sigma = 1$). By inspecting the relevant household equilibrium conditions from part (a), discuss how this might affect the dynamics of consumption following a TFP shock. You do not need to resolve the model, just provide the relevant economic intuition based on the equilibrium conditions.

e) Briefly explain how you would solve this model using Value Function Iteration. Make sure you mention any additional challenges that are specific to this model and how you might deal with them. For simplicity, assume labor supply is inelastic.
Question 4 (20 points)

This question considers a variant of the standard New Keynesian model where the government can now purchase a basket of goods $G_t$, which is completely funded by lump sum taxes. Assume that $G_t$ is not productive and does not provide utility.

The linearized conditions are given below. In percentage deviations from steady state: $\hat{c}_t$ is consumption, $\hat{w}_t$ is the real wage, $\hat{n}_t$ is hours worked, $\hat{y}_t$ is output, $\hat{\phi}_t$ is real marginal cost and $\hat{\gamma}_t$ is government spending. In deviations from steady state: $\hat{i}_t$ is the nominal interest rate, $\hat{\pi}_t$ is inflation. $\hat{\gamma}_t$ is the output gap (relative to the model with flexible prices): $\hat{\gamma}_t = \hat{y}_t - \hat{y}_t^n$.

### Households

$$E_t\hat{c}_{t+1} - \hat{c}_t = \frac{1}{\sigma}(\hat{i}_t - E_t\hat{\pi}_{t+1})$$

$$\hat{\hat{w}}_t = \sigma\hat{c}_t + \psi\hat{n}_t$$

### Firms

$$\hat{\hat{y}}_t = \hat{n}_t$$

$$\hat{\hat{w}}_t = \hat{\phi}_t$$

$$\hat{\hat{\pi}}_t = \beta E_t(\hat{\pi}_{t+1}) + \kappa\hat{\gamma}_t$$

$\sigma$ and $\psi$ come from household preferences. $1/\sigma$ is the elasticity of intertemporal substitution and $\psi$ is the inverse of the Frisch elasticity. $\kappa$ is inversely related to the degree of price stickiness.\footnote{$\kappa = (\sigma + \psi)^{\frac{1-\theta}{\theta}}$ where $\theta$ is the probability that a firm cannot adjust its price.} $0 < \beta < 1$.

### Resource constraint

$$\hat{\hat{y}}_t = \gamma_c\hat{\hat{c}}_t + \gamma_g\hat{\hat{g}}_t$$

$\gamma_c$ is the steady state share of consumption in output and $\gamma_g$ is the steady state share of government spending in output.

### Policy:

$$\hat{i}_t = \phi_\pi\hat{\pi}_t$$

where $\phi_\pi > 1$. Government spending, $\hat{\gamma}_t$, follows an AR(1) process

$$\hat{\gamma}_t = \rho\hat{\gamma}_{t-1} + e_t$$

where $e_t$ is i.i.d. and $0 \leq \rho < 1$
a) Show that, with flexible prices, the natural level of output can be written as

\[ \hat{y}_t^o = \Gamma \gamma \hat{g}_t \]

\[ \Gamma \equiv \frac{\sigma}{\sigma + \psi \gamma_c} \]

Briefly explain how and why a reduction in government spending causes a fall in output in this flexible price model. (Hints: start by combining equations 4, 5, 6 and 8. Also note that the real wage is constant in the flexible price model given the constant marginal product of labor).

b) The full sticky price model can be simplified to 3 equations (and equation (10)):

\[ E_t \hat{y}_{t+1} - \hat{y}_t = \gamma (\hat{z}_t - E_t \hat{p}_{t+1}) - \gamma \hat{g}_t (1 - \Gamma)(1 - \rho) \hat{g}_t \] (11)

\[ \hat{p}_t = \beta E_t (\hat{p}_{t+1}) + \kappa \hat{y}_t \] (12)

\[ \hat{i}_t = \phi \hat{p}_t \] (13)

Using the method of undetermined coefficients, find the response of the output gap and inflation to an exogenous decrease in \( \hat{g}_t \) when prices are sticky and monetary policy follows the Taylor Rule above. Guess that the solution for each variable is a linear function of the shock \( \hat{g}_t \). Is the fall in output larger or smaller in this sticky price model (than in part (a))? Explain.

c) Instead of following the Taylor Rule above, monetary policy is now set optimally. Derive the optimal monetary policy rule under discretion. Assume the steady state is efficient. (Hint: As in class, assume that the loss function has quadratic terms for the output gap and inflation, with a relative weight \( \theta \) on the output gap). What is the optimal path for output and inflation following the reduction in government spending?

d) Suppose the monetary policymaker wants to implement the optimal policy from part (c) using an interest rate rule for \( \hat{i}_t \). The policymaker is considering a rule which sets \( \hat{i}_t \) equal to \( \frac{\gamma_c}{\gamma_c} (1 - \Gamma)(1 - \rho) \hat{g}_t \), the natural real interest rate in this model. Briefly explain why this will not work. Furthermore, suggest a modification to the proposed rule that would successfully generate the outcomes in part (c). (Hint: you do not need to derive anything. You should be able to answer this from your knowledge of the model)

e) Now consider two modifications to the model: (i) government spending provides utility where the utility function is: \( \frac{G_t^{1-\sigma}}{1-\sigma} + \chi \log G_t - \frac{N_t^{1+\psi}}{1+\psi} \) (ii) the decrease in government spending is initially used to lend money to households rather than to cut taxes (although, over time, the government reverses this policy and cuts lump sum taxes later). Discuss how these changes might affect your answers in parts (a) and (b). You do not need to derive anything, just explain the economic intuition.
Question 5 (20 points)

Consider the following three period economy, with time denoted by \( t = 0, 1, 2 \). The economy is populated by a continuum of measure 1 of individuals, each endowed with one unit of a storable consumption good. At \( t = 0 \), individuals have two options with regards to how they can invest their endowment. They can either stuff it in their mattress, where it gets a gross return equal to 1 (i.e., \( 1 + r = 1 \)), or they can invest it in a long-term project that yields a gross return \( R > 1 \) in period two. For example, an individual that invests an amount \( I \) will receive \( RI \) in period two, and has \( 1 - I \) stuffed under the mattress. In \( t = 1 \), individuals have the option of liquidating the long-term project at a penalty. If they liquidate, they only receive a return \( L \leq 1 \) (per unit invested) in period 1, rather than the return \( R \) in period 2.

At time \( t = 1 \), a fraction \( \pi = 1/2 \) of the individuals receive a liquidity shock. These individuals are “impatient” and only value consumption in period one. The fraction \( 1 - \pi \) individuals that do not receive a liquidity shock are “patient” and only value consumption in period two. At time \( t = 0 \), all individuals have the same chance of being hit by the liquidity shock. Assume that individuals do not discount the future, so that their ex-ante expected utility is given by

\[
U = \pi u(c_1) + (1 - \pi) u(c_2),
\]

where \( c_1 \) and \( c_2 \) are the consumption in period 1 and 2, respectively, and \( u(c) = \frac{c^{1-\sigma}}{1-\sigma} \), with \( \sigma > 0 \).

a) Assume there are no financial markets available, so that individuals must simply invest on their own. Given that an individual has invested an amount \( I \) at time \( t = 0 \), what will be the optimal levels of consumption, \( c_1, c_2 \), if:

(i) the individual receives a liquidity shock (i.e. is impatient);

(ii) the individual does not receive a liquidity shock (i.e. is patient).

Let \( \hat{c}_1 \) and \( \hat{c}_2 \) denote the consumption of an impatient individual in period 1 and of a patient individual in period 2, respectively.

b) What is the optimal level of investment when individuals have to invest on their own? Denote this level by \( \hat{I} \). **Hint:** Show that there exists \( L, \bar{L} \in [0, 1] \) such that if \( L \geq \bar{L} \), the optimal level of investment is equal to 1, and if \( L \leq \bar{L} \), the optimal level of investment is zero.

c) Suppose that when types are realized in period 1, this information is publicly observable. Suppose there exists a social planner that individuals entrust all of their endowment to at time 0. The social planner will pay impatient individuals \( c_1^* \) in period 1 and patient individuals \( c_2^* \) in period 2 (and zero otherwise). Solving the social planner's
problem, what is \( c_1^* \) and \( c_2^* \)? How much does the social planner invest? That is, what is \( I^* \)?

d) Assume that \( L = 1 \). Show that \( I^* < \hat{I} \), but that, if \( \sigma < 1 \), \( c_1^* < \hat{c}_1 \) and \( c_2^* > \hat{c}_2 \). In other words, show that the planner invests less than the individuals but it makes them face more consumption risk.

e) Assume that \( L = 1 \) and \( \sigma > 0 \). Now suppose an agent’s type is private information, and the social planner can only offer a contract contingent on an individual’s announcement of her type at time 1. Furthermore, at time 1, she meets each agent only once, with the meeting order randomly determined. If individuals report honestly, can the social planner achieve the same allocation as in question c)? Is it optimal for an individual to report honestly when everyone else does?

f) Suppose that \( L = 1 \) and \( \sigma < 1 \). Suppose a fraction \( 1 - \varepsilon \in (0, 1 - \pi) \) of agents (all of whom are patient) fear a run. In particular, these agents believe that a fraction \( \varepsilon > \pi \) are claiming to the planner that they are impatient. Is it optimal for these agents to lie as well? Given your answer to this question, are runs possible when \( \sigma < 1 \)?
Question 6 (20 points)

Consider an economy with 2-period lived overlapping generations of agents. Population is constant and normalized to one. When young, agents have a unit endowment of labor, which they supply inelastically on the labor market at the wage \( w_t \). They consume \( c_{t,t} \) and save \( w_t - c_{t,t} \). For the moment, assume all their savings go into physical capital \( k_{t+1} \), which fully depreciates after use. When old, they rent capital at the rate \( r_{t+1} \) and consume \( c_{t,t+1} = r_{t+1}k_{t+1} \). Their preferences are

\[
    u(c_{t,t}) + \beta u(c_{t,t+1})
\]

where \( u(c) = \log c \). The production function is Cobb-Douglas

\[
y_t = k_t^\alpha l_t^{1-\alpha}
\]

a) Find the optimal savings decision of the consumer born at time \( t \), taking as given the prices \( w_t \) and \( r_{t+1} \).

b) Solve the problem of the representative firm and use market clearing in the labor market to derive expressions for \( w_t \) and \( r_t \) as functions of \( k_t \).

c) Obtain a law of motion for equilibrium \( k_{t+1} \).

d) Find a steady state with constant capital stock \( k_t = k_{SS} \). Show that if

\[
    \frac{\alpha}{1 - \alpha} \frac{1 + \beta}{\beta} < 1
\]

then \( r_{SS} < 1 \).

c) Let \( \bar{k} \) be the level of capital that maximizes the per-period net output (that is, it maximizes \( y_t - k_t \)). Show that if \( r_{SS} < 1 \), then \( k_{SS} > \bar{k} \), and hence a planner would be able to make all agents better off by reducing the capital stock in all periods.

f) Suppose now that the agents are allowed to trade a useless, non-reproducible asset in fixed unit supply, which trades at the price \( p_t \). We call this asset a "bubble". Argue that if \( p_t > 0 \) and \( k_{t+1} > 0 \) the agent must be indifferent between holding capital and the bubble asset, and derive the associated arbitrage condition.

g) Show that if (14) holds, there exists a steady state equilibrium with \( p_t = p_{SS} > 0 \).