

Macroeconomics Prelim
September 2011
Suggested Answer for Part B

Question 1 (15/50 points)

Consider the standard growth model in discrete time. There is a large number of identical households (normalized to 1). Each household wants to maximize life-time discounted utility

$$U(\{c_t\}_{t=0}^{\infty}) = \sum_{t=0}^{\infty} \beta^t u(c_t).$$

Each household has an initial capital stock x_0 at time 0, and one unit of productive time in each period, that can be devoted to work. Final output is produced using capital and labor services,

$$y_t = F(k_t, n_t),$$

where F is a CRS production function. This technology is owned by firms whose number will be determined in equilibrium. Output can be consumed (c_t) or invested (i_t). We assume that households own the capital stock (so they make the investment decision) and rent out capital services to the firms. The depreciation rate of the capital stock (x_t) is denoted by δ .¹ Finally, we assume that households own the firms, i.e. they are claimants to the firms' profits. The functions u and F have the usual nice properties.²

a) First consider an Arrow-Debreu world. Describe the households' and firms' problems and carefully define an AD equilibrium. How many firms operate in this equilibrium?

b) Write down the problem of the household recursively.³ Be sure to carefully define the state variables and distinguish between aggregate and individual states. Define a recursive competitive equilibrium (RCE).

For the rest of this question focus again on an Arrow-Debreu setting.

c) In this economy, why is it a good idea to describe the AD equilibrium capital stock allocation by solving the (easier) Social Planner's Problem?

¹ The capital stock depreciates no matter whether it is rented out to a firm or not.

² You will not explicitly need them, so there is no need to be more precise.

³ Here firms face a static problem. I am not asking you to explicitly spell it out, but it will be critical for a correct definition of the RCE.

d) From now on assume the following functional forms: $F(k_t, n_t) = Ak_t n_t$, $u(c_t) = c_t^{1-\sigma}/(1-\sigma)$. Fully characterize (i.e. find a closed form solution for) the equilibrium allocation of the capital stock. (Hint: Write the Planner's problem recursively. Then, in the Euler equation, guess and verify a "policy rule" of the form $k_{t+1} = \gamma k_t$, where γ is an unknown to be determined.) What happens in this economy in the long run?

Suggested Answer:

Parts a and b are taken word by word from the lecture notes, so I do not repeat them here to save some space. Lets move on to part c. c) The Planners problem is indeed much easier, because one only needs to describe the allocations, and not the prices of all commodities (which are infinite sequences). What allows us to use this technique here, is the fact that in this environment both Welfare Theorem hold. So we know that the competitive allocation and the Planners allocation will coincide. After characterizing the Planners allocation, we can construct the whole competitive equilibrium, like we did in class.

c) This is the well known Ak model. Let $B = A + 1 - \delta$. Notice that $c = Bk - k'$. The Euler's equation to the Planner's problem is given by

$$(Bk - k')^{-\sigma} = \beta B(Bk' - k'')^{-\sigma}.$$

Guess and verify that $k' = \gamma k$. After a little algebra, we can show that indeed the Euler equation can be solved under such a guess, and the unique solution to our unknown parameter is $\gamma = (\beta B)^{\frac{1}{\sigma}}$.

Hence, in this economy the stock of capital follows the difference equation $k' = (\beta(A + 1 - \delta))^{\frac{1}{\sigma}} k$. Given the initial stock k_0 , we can recover the whole sequence of capital (in fact the whole ADE). This economy either grows forever (if $\beta(A + 1 - \delta) > 1$, this is called a balanced growth path) or shrinks until it dies (if $\beta(A + 1 - \delta) < 1$). In the non-generic case where $\beta(A + 1 - \delta) = 1$, we have a steady state equilibrium.

Question 2 (17.5/50 points)

Consider the Mortensen-Pissarides model in continuous time. Labor force is normalized to 1. Unemployed workers, with measure $u \leq 1$, search for jobs. At the same time, firms with vacancies, with measure v , search for unemployed workers. The matching technology, which brings unemployed workers and vacant firms together, is described by the function $m(u, v)$. We assume that m is increasing in both arguments and homogeneous of degree 1 (i.e. it exhibits constant returns to scale). It is convenient to define the market tightness $\theta \equiv v/u$.

A large measure of firms decide whether to enter the labor market with exactly one vacancy. When a firm meets an unemployed worker a job is formed. The output of a job is p per unit of time. However, while the vacancy is unfilled, firms have to pay a search cost, given by pc , per unit of time (so this cost is proportional to productivity). In an active match (job), the firm pays the worker a wage w per unit of time, which is determined through Nash bargaining when the two parties first match. Let β represent the worker's bargaining power.

The destruction rate of existing jobs is exogenous and given by the Poisson rate λ . Once a shock arrives, the firm closes the job down. When this happens, the worker goes back to the pool of unemployment, and the firm exits the labor market. Unemployed workers get a benefit of z per unit of time.

Focus on steady state equilibria of this model. Let the discount rate of agents be given by r .

a) Express the arrival rate of workers to a vacant firm as a function of θ . Refer to this term as $q(\theta)$. Can you determine the sign of $q'(\theta)$?

b) Let $\eta(\theta)$ be the elasticity of $q(\theta)$ with respect to θ . Can you say anything about the range in which $\eta(\theta)$ belongs?

c) Express the arrival rate of jobs to workers as a function of θ . How does it relate to $q(\theta)$? How does it change as θ goes up?

d) Describe the Beveridge curve of this economy. More precisely, write down a formula that relates the steady state unemployment rate with the market tightness θ . Graph this relationship on the u, θ space.

e) Let V be the present-discounted value of a vacant job and J be the present-discounted value of a filled job. Write down the Bellman equations that these two values satisfy. Explain intuitively.

f) If there is free entry of firms into the labor market, what does J satisfy in equilibrium? Use your answer, together with your findings in part (e), to derive the job creation (JC) condition.⁴ Graph the JC condition in the w, θ space.

⁴ **Hint:** The JC condition should be a formula that relates w and θ and shows the wage firms are willing to pay as a function of market tightness.

g) Let U be the present-discounted value of the income stream of an unemployed worker, and W the analogous expression for an employed worker. Write down the Bellman equations that U and W satisfy. Explain intuitively.

h) Consider a typical match of a vacant firm and an unemployed worker. Write down the Bargaining game solved by these two parties.

After solving the bargaining problem, one can show that the equilibrium wage satisfies $w = z + \beta(p + pc\theta - z)$. Take this result as *given*.

i) Explain the intuition behind the wage curve given above. Also, graph this relationship in the w, θ space.

j) Which objects should a steady state equilibrium of the model define? Use your answers to parts (d), (f), and (i) in order to claim that the equilibrium is unique. You can do so either graphically or algebraically.

Suggested Answer:

a) Using total differentiation and the definition of θ , one can show that $q'(\theta) = -\frac{1}{\theta^2}m_1$, which is clearly strictly negative.

b) As we showed in the lecture, $\eta(\theta) \in (0, 1)$.

c) It is given by $\theta q(\theta)$. It is easy to show that this term is increasing in the market tightness.

d) From the law of motion of unemployment in steady state, we have

$$u = \frac{\lambda}{\lambda + \theta q(\theta)}.$$

e) We have

$$\begin{aligned} rJ &= p - w - \lambda J \\ rV &= -pc + q(\theta)(J - V). \end{aligned}$$

f) We have $J = pc/q(\theta)$. Combine this with the value function for J from part e yields the job creation curve:

$$w = p - (r + \lambda) \frac{pc}{q(\theta)}.$$

This is a decreasing function in the (w, θ) space.

g) We have

$$\begin{aligned} rU &= z + \theta q(\theta)(W - U), \\ rW &= w + \lambda(U - W). \end{aligned}$$

h) The typical i -th match solves:

$$\max_{w_i} (W_i - U)^\beta (J_i)^{1-\beta}.$$

You did not have to actually solve the problem. Just state it.

i) This is an increasing curve in the (w, θ) space. It means that the wage is equal to the outside option of the worker. Plus a percentage equal to β of the total surplus created by forming his match. The surplus s equal to production net of the unemployment benefit z , plus the savings in terms of recruiting costs, i.e. $pc\theta$.

j) We gave a detailed answer of that in the lecture.

Question 3 (12.5/50 points)

Consider a standard “Lucas trees” economy. There is a large number of identical households (normalized to 1) who wish to maximize expected life-time utility, given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \ln(c_t).$$

There is only one non-storable commodity that agents consume, call it coconuts. There are also infinitely lived objects, that we call trees, which yield coconuts. There is no production in this world. Agents can buy their shares at some price that they take as given. Let the supply of shares of the trees be normalized to 1. The holder of one share of the tree in period t is a claimant to the fruit d_t . We assume that d_t follows a Markov process. For any t , $d_t \in D \equiv \{d_1, \dots, d_N\}$. Let $\Gamma_{ij} = Pr(d_{t+1} = d_j | d_t = d_i)$. Assume that d_0 is given.

a) Characterize the AD equilibrium price of one coconut in period t after a certain history realization.

b) Set up the problem of the agent in a recursive form, and include the following asset: a claim, to be bought in period $t - 1$ after history \hat{h}_{t-1} , that pays one coconut in t if state j occurs. What is the equilibrium price of this asset?

c) What is the equilibrium price of a bond, bought in period $t - 1$ after history \hat{h}_{t-1} , which will deliver one coconut (with certainty) in period t ?

d) What is the equilibrium price of an option, bought in period $t - 1$ after history \hat{h}_{t-1} , that allows you to sell shares of the tree in period t , at the predetermined price x ?

Suggested Answer: Most questions are similar or even identical to the June 2011 prelim. See that answer key for more details.

Question 4 (5/50 points) ⁵

Consider again a “Lucas trees” economy as in Question 3. There are only two differences. First, the fruit process is deterministic: the tree yields d units of the fruit in each period with certainty. The second difference is the following: except from claims to the tree, there is another asset called fiat money. This is an intrinsically useless object (for example pieces of paper) that gives no fruit or utility to agents. What is special about money is that its supply is not exogenous, like the real asset. The supply of money in period t is given by M_t , and it is controlled by a monetary authority. This authority chooses the growth rate of money supply, μ , so that $M_{t+1} = (1 + \mu)M_t$. As in the case of claims to the tree, agents can purchase any amount of the new asset (fiat money) in a perfectly competitive market.

- a) What is the competitive equilibrium price of one share of the Lucas tree?
- b) If $\mu = 0$ (fixed supply of money), what is the equilibrium price of fiat money? Why?
- c) What is the range of monetary policies (i.e. of values of μ), that induce agents to hold fiat money?

Suggested Answer:

All the answers that I provide here can be shown more formally. However, I explicitly asked you (or at least encouraged you) to not show any work, and just provide an intuitive answer. This is what I will do as well. You can show these results more formally for practice.

- a) The Lucas tree will be priced at its “fundamental value”, i.e the stream of discounted consumption that it yields. Clearly, this is given by $\psi = \beta d / (1 - \beta)$.
- b) If $\mu = 0$, the rate of return on money is strictly negative. Hence, no one will want to hold money, when they can hold an asset (the real asset or, in other words, the shares to the Lucas tree) that yields a return equal to $R/\psi = (1 - \beta)/\beta > 0$.
- c) There is only one value of μ that will induce agents to hold fiat money, and that is the Friedman rule. i.e. $\mu = \beta - 1$. One can easily verify that only under that policy the rate of return on fiat money is exactly the same as the rate of return on the real asset (and, of course, strictly positive).

⁵ ***Suggestion:*** Answering this questions should take you 2 minutes. The answer has been discussed in class, but even if you do not remember it, you should be able to guess it, since it is very intuitive. Hence, in this question you do not need to show any work. Full credit will be given just for stating the correct result. In fact, I encourage you to just spell out the intuitive answer, rather than writing down any value functions.