

## Answer Key Part A: Questions on ECN 200D (Rendahl)

1. (a) The employment duration is given by  $1/(1-p)$ .  
 (b) The unemployment rate is given by  $u = \frac{1-p}{1+(1-p)-s}$ .  
 (c) The unconditional probability of employment is  $1-u$ .
2. (a) A Cauchy sequence  $\{x_t\}$  is such that for each  $\varepsilon$  there exist a  $N$  such that  $|x_n - x_m| < \varepsilon$  for all  $n, m \geq N$ . We will show that  $v_n(x)$  is Cauchy.  
 (b) A complete metric space is a space in which every Cauchy sequence is a converging sequence. When we know that  $v_n(x)$  is Cauchy, we will show that  $v_n(x)$  exists in a complete metric space and therefore that  $v_n(x) \rightarrow v(x)$ .  
 (c) The theorem of the maximum states, in particular, that if  $v_n(x)$  is continuous, so is  $v_{n+1}(x)$ .  
 (d) The contraction mapping theorem states that if  $T$  is a contraction mapping, then there exist a unique  $v$  such that  $v = Tv$ .
3. (a) A stationary competitive equilibrium is an interest-rate  $r$  and wage  $w$  such that,
  - i.  $g(a, \theta)$  solves the households problem.
  - ii. The stationary distribution  $\psi$  satisfies

$$\psi(a', \theta') = \sum_{\theta \in \{0,1\}} \sum_{a: a'=g(a,\theta)} \psi(a, \theta) \lambda(\theta', \theta)$$

iii.

$$A = \int_{a,\theta} a \psi(a, \theta) da$$

iv. Markets clear, that is

$$w = F_N(A, (1-u)) \quad r = F_K(A, (1-u))$$

- (b) For the Hugget economy, we have to delete  $\delta$  in the agents' budget constraints. A stationary competitive equilibrium is an interest-rate  $r$  such that,
  - i.  $g(a, \theta)$  solves the households problem.

ii. The stationary distribution  $\psi$  satisfies

$$\psi(a', \theta') = \sum_{\theta \in \{0,1\}} \sum_{a: a' = g(a, \theta)} \psi(a, \theta) \lambda(\theta', \theta)$$

iii.

$$0 = \int_{a, \theta} a \psi(a, \theta) da$$

4. (a) The SP is,

$$v^*(s_0) = \max_{\{h_t(s^t)\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \sum_{s^t \in S^{t+1}} \beta^t u(h_t(s^t)w(s_t) + (1 - h_t(s^t))b) f(s^t)$$

s.t.  $h_t(s^t) \in \{0, 1\} \quad \forall t = 0, 1, \dots, \forall s^t \in S^{t+1}$

(b) There are a couple of ways to formulate the Bellman equation. I find the following the most elegant.

$$v(s) = \max\{u(w(s)), u(b)\} + \beta \sum_{s' \in S} v(s') \lambda(s', s)$$

(c) The SP is now

$$v^*(s_0) = \max_{\{h_t(s^t)\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \sum_{s^t \in S^{t+1}} \beta^t [u(h_t(s^t)w(s_t) + (1 - h_t(s^t))b) - h_t(s^t)\delta] f(s^t)$$

s.t.  $h_t(s^t) \in \{0, 1\} \quad \forall t = 0, 1, \dots, \forall s^t \in S^{t+1}$

and the FE

$$v(s) = \max\{u(w(s)) - \delta, u(b)\} + \beta \sum_{s' \in S} v(s') \lambda(s', s)$$