University of California, Davis
Department of Economics
Macroeconomics

Date: September 2, 2010
Time: 5 hours
Reading Time: 20 minutes

PRELIMINARY EXAMINATION FOR THE Ph.D. DEGREE

Directions: Answer all questions. Feel free to impose additional structure on the problems below, but please state your assumptions clearly.

Short Answer Questions. *Keep your answers short and concise. (Each question is worth 7 points.)*

1. Consider the following optimization problem

$$V(k) = \max_c \{u(c) + e^{-\gamma} V(f(k) + (1-\delta)k - c)\}$$

   (a) What assumptions do we need on u, f and γ to guarantee that there exists a unique $V(\cdot)$ satisfying the above Bellman equation?

   (b) In the equation above, one time-period is equal to one unit of time. Reformulate the equation such that one time-period equals Δ units of time.

   (c) What is the continuous-time formulation of the above equation? *Hint: remember that* $\lim_{h\to 0}(f(x+ha) - f(x))/h = f'(x)a$.

2. Using words, *briefly* define the following concepts, and explain how we have put them to use in the course (Rendahl):

   (a) A complete metric space.

   (b) A norm, and in particular, the sup-norm.

   (c) The contraction mapping theorem.

   (d) The envelope theorem.

3. Consider a Lucas-tree type economy in which the endowment is growing stochastically and assume that agents have CRRA preferences. For tractability, assume that the endowment growth rate follows a two-state Markov process. Suppose one- and two-period bonds are traded in this economy and define the term premium as the difference between the expected return from selling a two-period bond after holding it for one period and the return from a one-period bond. Provide an intuitive explanation for why the term premium is positive if and only if consumption growth exhibits negative serial correlation.

Longer Answer Questions. *(Each question is worth 20 points.)*

4. Consider the following (very simple) consumption-savings problem with habits

$$V(a_0, c_{-1}) = \max_{\{a_{t+1}, c_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u(c_t - \gamma c_{t-1}) \tag{1}$$

$$subject\ to\ c_t + a_{t+1} = w + (1+r)a_t, \quad t = 0, 1, \ldots \tag{2}$$

with a_0 and c_{-1} given, and $\gamma \in [0, 1)$.

(a) Provide the Bellman equation to the above problem (no proof needed).

(b) Derive the first order conditions, apply the envelope theorem, and derive the Euler equation.

Now, let us modify the problem in the following way,

$$V(a_0, \tilde{a}_0, \tilde{c}_{-1}) = \max_{\{a_{t+1}, c_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u(c_t - \gamma \tilde{c}_{t-1}) \tag{3}$$

$$\text{subject to} \quad c_t + a_{t+1} = w + (1+r)a_t, \quad t = 0, 1, \ldots \tag{4}$$

$$\tilde{c}_t = H(\tilde{a}_t, \tilde{c}_{t-1}), \quad and \quad \tilde{a}_{t+1} = G(\tilde{a}_t, \tilde{c}_{t-1}) \tag{5}$$

with a_0, $\tilde{a}_0$, and $\tilde{c}_{-1}$ given, and $\gamma \in [0, 1)$. $G(\cdot)$ and $H(\cdot)$ are, simply, exogenously provided functions.

(c) What is the Bellman equation associated with the problem above (no proof needed).

(d) Derive the first order conditions, apply the envelope theorem, and derive the Euler equation.

Let us now assume that the economy is populated by a *continuum* of individuals, all starting their lives with the same level of $a_0, \tilde{a}_0, \tilde{c}_{-1}$. Moreover, $\tilde{a}_t$ and $\tilde{c}_t$ now happens to be *average* – or, as everyone is identical, *representative* – savings and consumption levels, respectively.

(e) What is the relationship between the functions $G(\cdot)$, $H(\cdot)$, and the individual's policy functions for c_t and a_{t+1}? (For simplicity, denote the individual's policy functions as $a' = g(a, \tilde{a}, \tilde{c}_{-1})$ and $c = h(a, \tilde{a}, \tilde{c}_{-1})$.)

(f) The economy above is commonly referred to as "catching up with the Jones's". Can you (very briefly) explain why? Why is the Euler equation associated with this economy so different from the one with standard habits? Which one of these two economies ("habits" and "keeping up with the Jones's") do you believe is Pareto-optimal, and which one is not?

5. Let z be a random variable that takes on values in $Z = \{0, 1\}$. Here, 1 denotes employment and 0 unemployment. Consider an arbitrary process of consumption $\{c_t(z^t)\}_{t=0}^{\infty}$ where $c_t : Z^{t+1} \to \mathbb{R}_+$. For this question, the probability of an unemployed individual finding a job is endogenous and depends on her search effort. For simplicity we will assume that the agent directly can influence, and therefore choose, the probability of finding a job, denoted by p_t; that is $P(z_{t+1} = 1 | z_t = 0) = p_t$. Let us, for simplicity, assume that once a job is found, it lasts for perpetuity and is unaffected by the agent's choice of p; that is $P(z_{t+1} = 1 | z_t = 1) = 1$, for all values of p_t.

We can summarize these assumptions concisely by

$$\lambda(z^{t+1}) = p_t(z^t)\lambda(z^t), \text{ if } z_t = 0, \text{ and } \lambda(z^{t+1}) = \lambda(z^t), \text{ if } z_t = 1$$

where $p_t(z^t)$ is an agent's choice of search effort in period t, after having observed history z^t. $\lambda(z^{t+1})$ denotes as usual *the probability of history* z^{t+1} *occurring.*

Lastly, an agent's preferences are given by

$$\sum_{t=0}^{\infty} \sum_{z^t \in Z^{t+1}} \beta^t \{u(c_t(z^t)) - v(p_t(z^t))\}\lambda(z^t)$$

Notice that the agent gains disutility $v(p_t(z^t))$ of searching at "intensity" $p_t(z^t)$. For simplicity we assume that $v(0) = 0$.

(a) After any history z^t, define a *continuation value* in this economy. Denote this $\vec{V}(z^t)$.

(b) If $z_t = 0$, derive a necessary condition for the agents' search effort in period t. *Hint: Notice that if* $z_t = 0$, $\vec{V}(z^t) = u(c_t(z^t)) - v(p_t(z^t)) + \beta(p_t(z^t)\vec{V}((z^t, 1)) + (1 - p_t(z^t))\vec{V}((z^t, 0)))$

Now, let us assume there is a government providing unemployment insurance and taxing labor income. More precisely, the government will collect *all* labor income once the agent is employed, and always provide some sort of benefits equal to $c_t(z^t)$. The government does so in order to maximize her own (expected present value) revenue, and such that the agent is given some minimum (expected present value) utility, V_0. However, the government must also recognize that the agent will freely decide how much she searches. The government's optimization problem is then given by

$$J(V_0, z_0) = \max_{\{c_t(z^t)\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \sum_{z^t \in Z^{t+1}} \beta^t \{z_t w - c_t(z^t)\}\lambda(z^t) \tag{6}$$

$$\textit{subject to} \quad V_0 = \sum_{t=0}^{\infty} \sum_{z^t \in Z^{t+1}} \beta^t \{u(c_t(z^t)) - v(p_t(z^t))\}\lambda(z^t) \tag{7}$$

and also subject to the necessary condition derived in (b).

(c) Provide the Bellman equation associated with the optimization problem above (no need for a proof). *Hint: A good starting point is to derive the Bellman equation for* $z_0 = 1$, *and then for* $z_0 = 0$.

6. Consider a simple, representative agent RBC model in which output, y_t, is produced via a standard Cobb-Douglas production function:

$$y_t = z_t k_t^{\alpha} h_t^{1-\alpha}$$

where k_t denotes beginning-of-period capital, h_t is labor, and z_t is an $i.i.d.$technology shock. The depreciation rate of capital is 100%. In each period, agents make consumption and labor decisions in order to maximize lifetime expected utility:

$$E_0 \left[\sum_{t=0}^{\infty} \beta U(c_t, h_t) \right]$$

Within this environment, consider two variations defined by the functional form for $U(\cdot)$.

(a) In Economy A, agents have preferences given by:

$$U(c_t, h_t) = \ln c_t - \frac{1}{2} h_t^2$$

In this economy, do the following

i. Express the maximization problem as social planner problem and write down the associated Bellman equation.

ii. Solve for the equilibrium policy functions describing consumption, investment and labor.

(b) In Economy B, agents have preferences given by:

$$U(c_t, h_t) = \ln \left(c_t - \frac{1}{2} h_t^2 \right)$$

i. Express the maximization problem as social planner problem and write down the associated Bellman equation.

ii. Solve for the equilibrium policy functions describing consumption, investment and labor.

(c) Compare the equilibrium behavior in both economies and provide an explanation for the differences.

7. Consider the Mehra-Prescott model in which the economy is populated by infinitely lived representative agents that have time-separable preferences characterized by constant relative risk aversion. At birth, each agent is given a share of equity which provides ownership to the endowment process. The endowment, x_t, grows over time; the endowment growth rate follows a two-state Markov process with possible realizations (λ_1, λ_2) and a symmetric transition probability matrix with diagonal elements π. Assume that $\lambda_1 < \lambda_2$ and $\pi = 1/2$. In this economy, agents trade equity (with price of q_t) and one-period bonds (with net interest rate of r_t). Consumption and portfolio decisions are made in order to maximize:

$$E_0 \left[\sum_{t=0}^{\infty} \beta U(c_t) \right]$$

Given this environment, do the following:

(a) Define a recursive competitive equilibrium.

(b) Characterize the equilibrium behavior of equity prices and interest rates.

(c) Prove that the equity premium in this economy is positive. In general (i.e. for all parameter values consistent with equilibrium), is the equity premium in the Mehra-Prescott economy always positive? Explain.