University of California, Davis — Date: September 3, 2009
Department of Economics — Time: 5 hours
Macroeconomics — Reading Time: 20 minutes

PRELIMINARY EXAMINATION FOR THE Ph.D. DEGREE

Directions: Answer all questions. Feel free to impose additional structure on the problems below, but please state your assumptions clearly.

Short Answer Questions. *Keep your answers short and concise. (Each question is worth 7 points.)*

Question 1 Consider the sequence of function $\{v_n(x)\}_{n=0}^{\infty}$ defined by

$$v_{n+1}(x) = \max_{x' \in [0,x]} \{u(x - x') + \beta v_n(x')\}$$

with $v_0(x) = 0$. Under which conditions on u and β do we know that $v(x) = \lim_{n\to\infty} v_n(x)$ *exists* and is *unique* and *continuous*? Explain briefly.

Question 2 Consider a representative agent, optimal growth model with no population growth in which agents have logarithmic preferences and assume that agents' discount factor (β) is equal to 0.97. If the economy is growing at 3%, what will the equilibrium one-period real interest rate be in this economy? (All interest rates and growth rates are expressed on an annual basis.) Suppose consumption uncertainty is introduced into this economy. How will this affect the (average) equilibrium real interest rate?

Question 3 Let $\tilde{r}_{t+1}$ denote the realized gross one-period return on a risky asset purchased at time t and $r_{f,t}$ denote the gross one-period return on a risk-free asset (also purchased at time t). Assuming that consumption growth and returns are serially uncorrelated, then the risk premium associated with the risky asset can be written as:

$$E(\tilde{r}_{t+1}) - r_{f,t} = -\beta r_{f,t}\left[\rho(m_{t+1}, \tilde{r}_{t+1})\,\sigma(m_{t+1})\,\sigma(\tilde{r}_{t+1})\right]$$

where β is the discount factor, m_t represents agents' stochastic discount factor and $\rho(\cdot)$ and $\sigma(\cdot)$ represent correlation and standard deviation respectively. Derive the above expression and discuss its implications for the equity premium puzzle.

Longer Answer Questions. *(Each question is worth 20 points.)*

Question 4 In the first problem-set of the course (Rendahl) we considered the following "income-fluctuation problem"

$$\max_{\{c_t(y^t),a_{t+1}(y^t)\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t u(c_t(y^t)) \tag{1}$$

$$subject\ to \qquad c_t(y^t) + a_{t+1}(y^t) = (1+r)a_t(y^{t-1}) + y_t \tag{2}$$

where y_t is the individual's stochastic endowment in period t, and $y^t = (y_0, y_1, \ldots, y_t)$ is the corresponding history. Assume $\beta(1+r) = 1$.

a. Let $y_{t+1} = \rho y_t + \varepsilon_t$, where ε_t is a zero mean random shock. What is the Bellman equation corresponding to $(1)-(2)$ above? (Feel free to use the expectations operator instead of summing/integrating over possible events.)

b. In the problem-set, we showed that if $u(c) = ac - \frac{1}{2}bc^2$, the solution took the form

$$c_t = \frac{r}{1+r}[a_t(1+r) + E_t \sum_{s=0}^{\infty} \frac{1}{(1+r)^s} y_{t+s}] \tag{3}$$

That is, consumption in period t equals the annuity value of total assets plus "permanent income" (a term coined by Milton Friedman).

Find the consumption policy function that satisfies your Bellman equation in *(a)*, and show that your (recursive) solution coincides with (3) (Hint: Use the (recursive) Euler equation to derive the policy rule – the value function is very difficult to recover.).

c. What is the marginal propensity to consume, $\frac{\partial c_t}{\partial y_t}$? How does the MPC change with the parameter ρ? Interpret.

Question 5 Consider the following sequence problem

$$\max_{\{a_{t+1},c_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u(c_t) \tag{4}$$

$$subject\ to \qquad a_{t+1} + c_t = (1 + r_t)a_t + w_t, \quad t = 0, 1, \ldots \tag{5}$$

a. Assume that $r_t = r(x_t)$, $w_t = w(x_t)$, and that $x_{t+1} = (1 - \rho)\bar{x} + \rho x_t$. What is the Bellman equation corresponding to the above sequence problem?

b. Now assume for simplicity that $r_t = r$, (a constant), but that $w_{t+1} = \alpha w_t + \beta w_{t-2}$. What is the relevant (the smallest sufficient) state vector?

c. What is your answer to *(b)* if $w_{t+1} = \sum_{i=0}^{N} \gamma^i w_{t-i}$?

d. How does your answer to *(c)* change if $N = \infty$?

Note that in all problems associated with Question 5, there is an implicit assumption that there are sufficient ***givens*** *in (4)-(5) in order to accurately predict future prices. E.g. in (i) it is implicitly assumed that x_0 is given, and so on.*

Question 6 Consider the following simple RBC model with capital adjustment costs in which output is a function of beginning of period capital and an $i.i.d.$ stochastic technology shock:

$$y_t = z_t k_t^{\alpha}; \quad \alpha < 1$$

The representative agent has standard time-separable logarithmic preferences over current and future consumption and maximizes expected discounted utility:

$$E_0 \left[\sum_{t=0}^{\infty} \beta^t \ln c_t \right]; \quad 0 < \beta < 1$$

The law of motion for capital is given by:

$$k_{t+1} = k_t (1 - \delta) + F(i_t, k_t)$$

where the function $F(i_t, k_t)$ represents the adjustment costs associated with investment. Given this environment, do the following:

a. Express the agent's maximization problem as a dynamic programming problem. Interpret the resulting necessary conditions. In particular, show that the shadow price of capital, denote this as q_t, is related to the adjustment cost function, $F(\cdot)$. (Hint: It is recommended that you set up the law of motion as an additional constraint in the Lagrangian.)

b. To put more structure on the model, assume that the adjustment cost function takes the form:

$$F(i_t, k_t) = \left[1 - S\left(\frac{i_t}{k_t}\right)\right] i_t$$

It is assumed that $S(\delta) = S'(\delta) = 0$. What is the rationale for this assumption? What is the implication for q_t in the steady-state?

Question 7 An economy is populated by identical agents with expected lifetime utility given by:

$$E_0\left[\sum_{t=0}^{\infty} \beta^t \ln c_t\right]$$

Each period, agents rent their beginning-of-period capital stock (k_t) to (identical) firms (the rental rate of capital is denoted r_t) and also supply inelastically one unit of labor to firms (the wage rate is denoted w_t). The income generated by these factor supplies are used to acquire consumption (c_t) and new capital. The depreciation rate of capital is assumed to be 100%. Firms choose labor and capital every period in order to maximize profits where output is given by the technology:

$$y_t = z_t k_t^{\alpha} h_t^{1-\alpha}$$

where the law of motion for z_t is $z_t = z_{t-1}^{\rho} \varepsilon_t$ where $0 < \rho < 1$ and ε_t is an *i.i.d.* innovation. Assume that the unconditional mean of both z_t and ε_t is 1. Given this scenario, answer the following:

a. Write down the firm's and household's maximization problems and derive and interpret the associated necessary conditions. Define a recursive competitive equilibrium for this economy.

b. Rather than solve directly for the competitive equilibrium, one can instead solve an associated social planner problem. Express the relevant social planner problem and derive the associated necessary conditions. Define the solution to the social planner problem (Do not actually solve the model.)

c. One way to solve the social planner problem would be to log-linearize the expressions characterizing an equilibrium. Do this for the model described in part (b) and present the resulting set of linear expectational difference equations in matrix form. Describe how one would use this system to find the solution to the model; be precise in your description and identify the form that the solution will take.