University of California, Davis
Department of Economics
Macroeconomics

Date: August 25, 2016
Time: 5 hours
Reading Time: 20 minutes

PRELIMINARY EXAMINATION FOR THE Ph.D. DEGREE

Directions: Answer all questions. Feel free to impose additional structure on the problems below, but please state your assumptions clearly. Point totals for each question are given in parentheses.

1. (10) Each period, an infinitely-lived agent divides his endowment of 1 unit of time between human capital production (a non-market activity) and work in order to maximize the present discounted value of lifetime earnings. Let h_t denote the human capital stock at time t and let $1 - l_t$ be time spent working. Income each period is given by $h_t (1 - l_t) w$ where w is the rental rate of human capital. New human capital (i.e. investment in human capital) is produced via the production function $(h_t l_t)^\alpha$; $\alpha \in (0, 1)$. Note that human capital depreciates at the rate δ and the real interest rate is given by the constant, r.

   (a) Express the agent's maximization problem as a dynamic programming problem and identify the states and controls.

   (b) Derive and interpret the Euler equations associated with this problem.

   (c) Assume that a steady-state exists so that $h_t = \bar{h}$ and $l_t = \bar{l}$. Solve for these steady-state values.

   (d) What is the impact of the two prices (w, r) on the steady-state values? Explain.

2. (20) Consider a representative agent economy in which preferences are given by:

$$E_0 \left(\sum_{t=0}^{\infty} \beta^t \left[\frac{c_t^{1-\gamma}}{1-\gamma} - \theta_t h_t \right] \right), \beta \in (0, 1), \gamma > 0$$

   where E denotes the expectations operator and θ_t is an *i.i.d.* random variable which affects the disutility of labor supply.

   Output in the economy is produced by firms that use labor and an inelastically supplied unit of non-depreciating capital (owned by firms - you can think of this as land). The choice of labor is made to maximize profits each period:

$$\begin{aligned} \max_{h_t} \pi_t &= y_t - w_t h_t \\ y_t &= h_t^{1-\alpha}, \ \alpha \in (0, 1) \end{aligned}$$

   where y_t denotes output and w_t is the wage. The profits are returned to the households.

   In addition to labor supply, agents also trade one period bonds (risk-free) that cost p_t at time t and return 1 unit of consumption in period $t + 1$. Given this environment, do the following:

   (a) Express the household's problem as a dynamic programming problem and derive the associated necessary conditions. Note that households take as given firm profits, the wage and the price of bonds (i.e. it is a standard competitive economy).

   (b) Find the competitive equilibrium allocation by solving the social planner's problem for this economy.

   (c) Solve for the policy functions which describe equilibrium consumption and labor. Provide an explanation for the implied behavior of these variables.

   (d) Determine the solution for the equilibrium price of bonds. Explain how the preference shock, θ_t, affects the price of bonds.

3. (20) Consider a standard optimal growth model in continuous time in which the aggregate production function is given by:

$$Y(t) = F[K(t), N(t)]$$

where $F(\cdot)$ has standard properties and $N(t)$ is growing at the rate $n > 0$.The depreciation rate of capital is given by $\delta > 0$.The single household inelastically supplies labor each period and then chooses consumption and savings in order to maximize:

$$\int_{t=0}^{\infty} e^{-\rho t} U(c(t)) N(t)\, dt$$

where $c(t) = \frac{C(t)}{N(t)}$ is per-capita consumption and $U(\cdot)$ has the functional form:

$$U(c(t)) = \begin{cases} \frac{c(t)^{1-\theta}}{1-\theta}; \theta \neq 1 \\ \ln c(t); \theta = 1 \end{cases}$$

It is assumed that all parameter values are such that a well-behaved equilibrium exists. In addition to output produced via the production function, output arrives exogenously every period at the rate of ϕ units per person. Given this environment, do the following:

(a) Express the social planner problem in intensive (i.e. per-capita) form - show your derivation.

(b) Write down the Hamiltonian for this problem and derive the necessary conditions; include the transversality condition.

(c) Derive the phase diagram for this economy - be sure to explain your derivation.

(d) Define the steady-state. What fraction of the exogenous output, ϕ, is consumed in steady-state? Why?

4. (20) Consider the standard growth model in discrete time. There is a large number of identical households normalized to 1. Each household wants to maximize life-time discounted utility

$$U(\{c_t\}_{t=0}^{\infty}) = \sum_{t=0}^{\infty} \beta^t u(c_t), \;\; \beta \in (0,1).$$

Each household has an initial capital $k_0 > 0$ at time 0, and one unit of productive time in each period that can be devoted to work. Final output is produced using capital and labor, according to a production function, F, which has the standard properties discussed in class, most notably, it is increasing in both arguments and exhibits CRS. This technology is owned by firms (whose measure does not really matter because of the CRS assumption). Output can be consumed (c_t) or invested (i_t). Households own the capital (so they make the investment decision), and they rent it out to firms. Let $\delta \in (0,1)$ denote the depreciation rate of capital. Households own the firms, i.e., they are claimants to the firms' profits, but these profits will be zero in equilibrium. The function u also has the usual nice properties, which I will not spell out here since you will not need them explicitly.

In this economy there is a government that collects taxes and (for simplicity) throws the tax revenues into the ocean. The government can implement one of the following two alternative taxation systems, let us call them System A and System B. System A is a proportional tax, $\tau \in [0,1]$, on agents' capital income. In other words, if the government implements System A, it collects a fraction τ of all the income that agents earn by renting out their capital to firms. System B is a proportional tax, $\tau \in [0,1]$, on agents' **investment**. In other words, if the government implements System B, it collects a fraction τ of all the resources that agents choose to allocate into investment.

(a) Write down the problem of the household recursively, under both taxation systems.[1] Pay special attention to the budget constraints. These constraints will not be the same under the two specifications. Also, notice that I am not asking you to define a RCE in detail; just state the representative agent's problem within a RCE environment.

(b) Describe the steady state equilibrium capital stock under taxation System A, for any given $\tau \in [0,1]$. Denote this object by $K_A^*(\tau)$.

(c) Describe the steady state equilibrium capital stock under taxation System B, for any given $\tau \in [0,1]$. Denote this object by $K_B^*(\tau)$.

(d) Assume that $F(K,N) = K^a N^{1-a}$, $a \in (0,1)$. Provide closed form solutions for the terms $K_A^*(\tau), K_B^*(\tau)$, described in parts (b),(c). **Hint:** Here, it is more convenient to work directly with F, i.e., do not work with the auxiliary function f that we introduced in the lectures.

(e) Plot the terms K_A^*, K_B^*, calculated in part (d), against $\tau \in [0,1]$ and in the same graph. Discuss briefly.

(f) Describe the government's total tax revenue in steady state under System B, T_B. Plot T_B as a function of the tax rate τ (this is the so-called Laffer curve). Discuss the shape (i.e., the monotonicity) of the Laffer curve for the various values of a and τ.

[1] Here, the firms face a static problem. I am not asking you to explicitly spell it out, but this problem is critical for the determination of the various prices.

5. (20) Consider the standard Mortensen-Pissarides model in continuous time. Labor force is normalized to 1. Unemployed workers, with measure $u \leq 1$, search for jobs, and firms with vacancies, with measure v, search for unemployed workers. The matching technology is given by $m(u, v) = u^a v^{1-a}$. It is convenient to define the market tightness $\theta \equiv v/u$. A large measure of firms decide whether to enter the labor market with exactly one vacancy. When a firm meets an unemployed worker a job is formed. The output of a job is p per unit of time. However, while the vacancy is unfilled, firms have to pay a search or recruiting cost equal to pc per unit of time. In an active match (job), the firm pays the worker a wage w per unit of time, which is determined through Nash bargaining when the two parties first match. Let β represent the worker's bargaining power.

   The destruction rate of existing jobs is exogenous and given by the Poisson rate λ. Once a shock arrives, the firm closes the job down. Subsequently, the worker goes back to the pool of unemployment, and the firm exits the labor market. Unemployed workers get a benefit of $z > 0$ per unit of time. Throughout this question focus on steady state equilibria and let the discount rate of agents be given by r.

   So far this is just the model we described in class. What is new here is that the unemployment benefit z does not stand for the utility of leisure or the value of home production, as is conveniently assumed in the baseline model. Here, z is a payment (in terms of the numeraire good) that the government delivers to the unemployed. Clearly, the government must tax someone in order to raise funds for the unemployment benefits, and we assume that it raises these funds by levying a lump-sum tax equal to T (per unit of time) on every employed worker. Hence, the government chooses both z and T, and must do so in a way that keeps the budget constraint satisfied at all times.

   (a) Describe the Beveridge curve (BC) of this economy.

   (b) Describe the value functions for a vacant firm (V) and a firm with a filled job (J).

   (c) What condition does J satisfy in equilibrium? Use your answer, together with your findings in part (b), to derive the job creation (JC) condition.

   (d) Describe the value functions for an unemployed (U) and an employed (W) worker.

   (e) Describe the wage curve (WC) in this economy. This will be a function of the usual terms and the new term T.

   (f) What is the relationship between T and z, u so that the government's budget constraint is satisfied at all times? Use this condition in order to get rid of T in the WC.

   (g) In the baseline model, it was easy to characterize the equilibrium values of (θ, w), since equations JC and WC contained exclusively these two variables. To do this here, we need a little more work, since at least one of these equations also contains the third endogenous variable, namely, u. Can you get rid of u and replace it with a term that contains θ (and other parameters)? **Hint:** Which equilibrium condition gives you u as a function of θ?

   (h) Plot the JC and WC curves in the w, θ space. Do they have the standard shape? Use your graph to discuss existence and uniqueness of equilibrium. What is the intuition behind your findings?

   (i) Assume that the government increases z. What is the effect of this policy change on equilibrium unemployment?

6. (10) Consider a standard growth model in discrete time. Throughout this question you can focus on the Social Planner's problem (vs the more complicated model with competitive markets). At $t = 0$ there is a large number of identical agents normalized to 1. The population grows at rate n per period, i.e., $N_t = (1+n)^t$. The representative agent's preferences are described by

$$U(\{c_t\}_{t=0}^{\infty}) = \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma}}{1-\sigma}.$$

The initial capital stock in this economy is K_0, and each agent can devote one unit of productive time (in each period) to work. Final output is produced using capital and labor, and production is characterized by the so-called labor-augmenting technology:

$$Y_t = F(K_t, N_t(1+g)^t),$$

where F is a CRS production function. Capital depreciates at rate $\delta \in (0,1)$. The Social Planner wishes to maximize per-capita life-time discounted utility.

(a) Describe the resource constraint of the Planner's problem. **Hint:** It will be useful express all the variables into "growth-adjusted per-capita variables", as we did in class.

(b) Characterize the optimal solution to the Planner's problem (i.e. derive the Euler equation).

(c) What happens to per-capita consumption in the long run? What happens to total consumption in the long run? (For full credit I expect you to derive the results carefully, and relate them to your work in part (b). However, partial credit will be given to correct, intuitive answers).