

PRELIMINARY EXAMINATION FOR THE PH.D. DEGREE

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*Answer all questions.*

1. Consider the following limited participation monetary model. Firms hire labor ( $h_t$ ) to produce output ( $y_t$ ) using the linear production function:

$$y_t = \gamma h_t ; \gamma > 0$$

Output is sold at the nominal price of  $P_t$  per unit of output.

Firms pay labor in advance of production – they borrow the wage bill from financial intermediaries. Hence the cost of labor inputs is:

$$Cost = R_t W_t h_t$$

Where  $R_t$  denotes the (gross) nominal interest rate and  $W_t$  is the nominal wage. Financial intermediaries receive funds from two sources: Households make deposits ( $I_t$ ) at the beginning of the period before the current state of the world is known and the current monetary transfer (which determines the current state) is received from the government. The financial intermediary inelastically provides the funds in the form of loans to businesses to finance their wage bill. That is, financial intermediaries make no profits by assumption:

$$I_t + g_t \bar{M}_{t-1} = W_t h_t$$

The income from the loans is distributed entirely to the households.

Households make deposit, consumption, and labor decisions in order to maximize lifetime expected utility. Preferences are given by:

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t [\ln c_t + A(1 - h_t)] \right\}$$

As mentioned above, before knowing the current realization of the monetary growth rate, they allocate part of their nominal wealth to the banking sector. Then, after observing  $g_t$ , they make consumption and labor decisions. It is assumed that the consumption is subject to a cash-in-advance constraint. This scenario implies the following budget and cash-in-advance constraints:

$$M_t = W_t h_t + M_{t-1} - I_t - P_t c_t + R_t (I_t + g_t \bar{M}_{t-1})$$
$$M_{t-1} + W_t h_t - I_t > P_t c_t$$

Note that, as implied by the discussion above, current labor income can be used to finance current consumption. The only source of uncertainty in the economy is due to the monetary growth rate; this random variable is assumed to be independently and identically distributed with  $E(g_t) > 0$ . Given this environment, do the following:

- a. Set up the firm's and household's maximization problem. Derive and interpret the associated necessary condition. In your answer, demonstrate how this model is distinguished from a typical cash-in-advance model
- b. Define a stationary monetary equilibrium in this economy.
- c. Demonstrate that the liquidity effect is present – i.e. that the correlation of interest rates and money growth is negative. Show that this implies that money growth is procyclical in the economy. Again, compare this result to a standard CIA model.
- d. Suppose 2-period bonds were traded in a market that opened after the realization of the monetary growth rate. Write down the Euler condition that would determine the equilibrium behavior of the two-period interest rate. (It is not necessary to write down the maximization problem.) What would you predict the sign of the term premium to be in this economy? Provide an **intuitive** argument.

2. Write down a simple CIA model (i.e. no uncertainty) which demonstrates that the optimal quantity of money is associated with a zero nominal interest rate.

3. The basic Taylor model of price-level adjustment was derived under the assumption that the nominal wage set in period  $t$  remained unchanged for periods  $t$  and  $t+1$ . Suppose instead that each period  $t$  contract specifies a nominal wage  $x_t^1$  for period  $t$  and  $x_t^2$  for period  $t+1$ . Assume these are given by  $x_t^1 = p_t + \kappa y_t$  and  $x_t^2 = E_t p_{t+1} + \kappa E_t y_{t+1}$ . The aggregate price level at time  $t$  is then equal to  $p_t = \frac{1}{2}(x_t^1 + x_{t-1}^2)$ . If aggregate demand is given by  $y_t = m_t - p_t$  and  $m_t = m_0 + \omega_t$ , answer the following questions:

- a. Derive the rational expectations solution for the price level.
- b. What are the dynamics of  $y_t$ ?
- c. Compare the dynamics you just obtained with those in the usual Taylor model. Explain your results.
- d. How would the dynamics of  $p_t$  and  $y_t$  have changed if the contracting equation took place over three periods, so that  $p_t = \frac{1}{3}(x_t^1 + x_{t-1}^2 + x_{t-2}^3)$  and  $x_t^3 = E_t p_{t+2} + \kappa E_t y_{t+2}$ ? You need not solve the model again, just discuss the answer with the appropriate conjectured solution.

4. Consider the following simplified version of the backward-looking, new-Keynesian model in Rudebusch and Svensson (1999):

$$\begin{aligned}x_t &= \varphi x_{t-1} + \alpha(i_t - x_t) + u_t \\ \pi_t &= \beta \pi_{t-1} + \kappa x_t + e_t \\ i_t &= \rho i_{t-1} + \gamma_1 x_{t-1} + \gamma_2 \pi_{t-1} + \omega_t\end{aligned}\tag{1}$$

where  $x_t$  is the output gap;  $\pi_t$  is inflation (say in deviations from a target, to simplify the constants); and  $i_t$  is the nominal interest rate (in deviations from the real rate, say). Answer the following questions (most of them are rather brief):

- a. Write down the structural VAR(1) representation of this system
- b. Hence, what Cholesky ordering would allow one to identify this model from a reduced-form VAR(1)?
- c. Instead, suppose the policy rule is specified as a function of the contemporaneous realizations of  $\pi_t$  and  $x_t$ . What information, if any, would allow you to recover the structural form from the reduced-form VAR? Explain.
- d. Briefly discuss one method that would allow you to recover the structural parameters of (1).
- e. Write down the more common, forward-looking version of the model in (1).
- f. What advantages/disadvantages does this formulation have with respect to the formulation in (1)?
- g. Suppose the central bank is unsure whether a forward-looking or a backward-looking model are the best representation of the actual economy. What elements of the policy rule would ensure that it is robust to either model?