
PRELIMINARY EXAMINATION FOR THE Ph.D. DEGREE

Directions: Answer 4 of the 6 questions, including all 3 in part 1 and 1 of the 3 from part 2. Unless stated otherwise, the components of a question are equally weight. The exam is closed book.

PART 1

Question 1. An economy is populated by a representative household that makes labor, consumption and asset choices in order to maximize:

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[\ln c_t + \ln \left(\frac{M_t}{P_t} \right) + A(1 - h_t) \right] \right\}$$

where c_t denotes consumption, M_t is money chosen in period t , and h_t denotes labor. In addition to labor income, the household is supplied with one unit of capital which it rents to firms at the nominal price of R_t . The income from labor and capital is combined with the income from one period nominal bonds purchased in the previous period and beginning of period money holdings in order to finance consumption, new bond purchases and the acquisition of new money. (Note that the household does not acquire new capital - hence the aggregate capital stock is fixed at one.) Also, the household receives a lump sum monetary transfer from the government, denoted T_t .

Firms in the economy hire capital and labor in order to maximize profits. These inputs are used to produce output, y_t , using a constant returns to scale technology:

$$y_t = z_t k_t^{1-\alpha} h_t^\alpha$$

where z_t denotes a stochastic technology shock. The law of motion for z_t is given by

$$z_t = z_{t-1} \varepsilon_t \tag{1}$$

where ε_t is an i.i.d. shock with $E(\varepsilon_t) > 1$. Note that this implies that the economy is, on average, growing over time.

The government controls the money supply and adjusts the monetary growth rate according to the current innovation to the technology shock. That is, the money supply is given by:

$$M_t = (\gamma \varepsilon_t) M_{t-1} \tag{2}$$

If $\gamma > 1$, then the central bank “accommodates” a technology shock; if $\gamma < 1$, this can be interpreted as the central bank “leaning against the wind”.

Given this description of the economy, do the following:

- i. Write down the household’s maximization problem as a dynamic programming problem and derive the associated necessary conditions.
- ii. Derive the necessary conditions associated with the firm’s maximization problem.
- iii. Define a recursive monetary equilibrium. (Again, note that output in this economy is growing over time.)
- iv. Characterize the equilibrium behavior of consumption, labor, inflation, and nominal interest rates in this economy.
- v. How does the value of γ affect the equilibrium behavior of nominal interest rates? Explain.

Question 2. Point allocation: (a) 25%; (b) 25%; (c) 25%; (d) 25%

Suppose you are analyzing a tri-variate monetary VAR consisting of log output, log prices and money (y, p, m respectively). The reduced-form VAR is

$$X_t = AX_{t-1} + u_t, \quad E(u_t u_t') = \Omega_u = C_0 C_0'$$

the reduced-form, infinite-order *MA* representation is

$$X_t = u_t + B_1 u_{t-1} + \dots, \quad \sum_{j=0}^{\infty} B_j \equiv B,$$

and the structural infinite-order *MA* representation is

$$X_t = C_0 \varepsilon_t + C_1 \varepsilon_{t-1} + \dots, \quad \sum_{j=0}^{\infty} C_j \equiv C,$$

so that $X_t = (y_t, p_t, m_t)'$ and A is a 3×3 matrix, for example. You are interested in presenting the impulse response functions corresponding to the structural model. Answer the following questions:

(a) To achieve identification, you assume that money is neutral in the long-run and that in the long-run, the quantity theory of money holds. Write down the conditions that would incorporate these assumptions for the purposes of identification of the structural model.

(b) Briefly explain why your system is not identified and propose as many economically meaningful (and justified) restrictions as you would need to achieve identification.

(c) Instead, consider identification based on short-run restrictions. State the assumptions you make and their economic justification. How would you use these restrictions to achieve identification?

(d) Consider now a simpler system consisting of y and m only and suppose that Ω_u is a diagonal matrix. Show that C_0 must be also diagonal, regardless of whether we use short-run or long-run identification assumptions.

Question 3: Consider the following dynamic new Keynesian model,

$$\begin{aligned}x_t &= E_t x_{t+1} - \sigma^{-1}(i_t - E_t \pi_{t+1}) + \xi_{xt}, \\ \pi_t &= \beta E_t \pi_{t+1} + \kappa x_t + \xi_{\pi t}, \\ i_t &= i^* + \rho i_{t-1} + (1 - \rho)[\psi_x x_t + \psi_\pi \pi_t] + \xi_{it},\end{aligned}$$

where for simplicity all the shocks are assumed to be mean zero and identically and independently distributed.

(a) How would you go about checking whether this interest rate rule guarantees a unique nonexplosive equilibrium? (You don't have to do all the math, just set up the appropriate system and list all the steps you would follow.)

(b) What is the Taylor principle? Is it necessary for uniqueness in this model? Is it sufficient? Explain.

(c) Has the Fed always conformed to the Taylor principle? What do various economists say about this?

(d) Why is determinacy considered a desirable property for a monetary policy rule? What is so bad about sunspot solutions?

PART 2

Question 4. Write down a simple stochastic growth model with a basic cash-in-advance constraint – i.e. a model similar to that studied in Cooley and Hansen (AER, 1989) and in Walsh’s textbook. Assume that only monetary shocks are present. Derive the necessary conditions describing equilibrium and discuss (briefly) how you would calibrate the parameters in the model. Also, describe some of the equilibrium properties of the model (motivate your discussion on intuitive grounds since the actual solution of the model requires numerical methods.) In particular, is money growth procyclical?

Question 5. Point allocation: (a) 10%; (b) 10%; (c) 10%; (d) 15%; (e) 15%; (f) 5%; (g) 15%; (h) 5%; (i) 15%

Commitment vs. Discretion: Consider a central bank’s choice of policy between optimal commitment and discretion. To simplify the problem, we assume here that interest rates can be costlessly modified and hence, one can think of the output gap directly as the policy variable the central bank controls. In this way, we do not need to discuss the IS curve.

Therefore, suppose the inflation adjustment equation is given by

$$\begin{aligned}\pi_t &= \beta E_t \pi_{t+1} + \kappa x_t + e_t, \\ e_t &= \rho e_{t-1} + \varepsilon_t, \\ \varepsilon_t &\sim i.i.d. (0, \sigma),\end{aligned}\tag{1}$$

where x is the output gap; π is the inflation rate; and ε is an i.i.d. shock. All variables are in deviations from steady-state.

Under commitment, the central bank’s loss function can be characterized as:

$$L_t = E_t \sum_{i=0}^{\infty} \gamma^i (\pi_{t+i}^2 + \lambda x_{t+i}^2),$$

with first order conditions:

$$\pi_t = -\phi_t, \quad i = 0,\tag{2}$$

$$E_t(\pi_{t+i} - \phi_{t+i} - \phi_{t+i-1}) = 0, \quad i \geq 1,\tag{3}$$

$$E_t(\lambda x_{t+i} - \kappa \phi_{t+i}) = 0, \quad i \geq 0,\tag{4}$$

where ϕ_{t+i} is the Lagrange multiplier associated with the period $t+i$ inflation adjustment equation.

(a) Briefly discuss the dynamic inconsistency arising from the optimal pre-commitment strategy in expressions (2) and (3).

(b) To address the dynamic inconsistency problem, Woodford (1999) introduces an alternative definition of pre-commitment policy called the *timeless perspective*. Briefly explain the nature of this policy with respect to your answer in (a).

(c) Combining expressions (3) and (4) to eliminate the Lagrange multiplier from expression (3), express inflation as a function of the output gap only and plug-in the resulting expression in the inflation adjustment equation (1).

(d) Conjecturing a solution to the expression in part(c) of the form $x_t = a_x x_{t-1} + b_x e_t$, use the method of undetermined coefficients to find expressions for a_x and b_x as a function of deep parameters. You need not solve the quadratic expression resulting for a_x .

(e) Given the reduced form solution in part (d), express inflation from part(c) as a function of past values of the output gap and the shock e .

Under discretion, the central bank is unable to affect the private sector's expectations on future inflation (since today's policy choices do not bind the central bank to future policy actions) and chooses, each period, to minimize the loss function

$$L = \pi_t^2 + \lambda x_t^2$$

subject to the inflation expression in (1). First order conditions for this problem deliver the condition

$$\kappa \pi_t + \lambda x_t = 0 \quad \forall t. \quad (5)$$

(f) Use expression (5) to eliminate inflation from the inflation adjustment equation.

(g) Conjecturing a solution of the form $x_t = \delta e_t$, use the method of undetermined coefficients to find the equilibrium expression for inflation under optimal discretion.

(h) Briefly explain whether or not there is an inflation bias from the discretion strategy.

(i) Comment on the differences/similarities of the solution under commitment in part (e) and the solution under discretion in part (g).

Question 6. Bils and Klenow (2004) study microeconomic data on price adjustment and report 2 salient facts. One is that the median time between price adjustments is approximately 4.5 months, and the other is that roughly 75 percent of firms make no adjustment in a given month. Comment on what their facts say about the following models.

(a) The purely forward-looking new Keynesian model with Calvo price setting.

(b) The hybrid new Keynesian model with forward-looking price setting as well as indexation to past prices.

- (c) The Mankiw-Reis sticky-information model.
- (d) The state-dependent pricing model of Dotsey, King, and Wolman.