1. There are two firms that produce a homogeneous product. Let $p_i$ be the price of firm $i$ ($i = 1, 2$). Assume for simplicity that the firms have zero costs. There are $N$ consumers, each with the same reservation price for the product, denoted by $r$. Thus each consumer buys from one of the firms if and only if at least one of the prices is less than or equal to $r$.

When $p_1 = p_2 \leq r$, 50% of the consumers go to firm 1 and 50% to firm 2. What happens when the prices are different? Some consumers are very sensitive to price differences, while others are not. For example, if $p_1 = p_2 + 0.01$ then some consumers might prefer firm 2 because they save 1 penny, but probably most consumers would be indifferent between the two firms. Let $f: [0, r] \rightarrow \mathbb{R}^+$ be a density function (thus $f(x) \geq 0$, for all $x \in [0, r]$, and $\int_0^r f(x)dx = 1$) that measures the price-difference sensitivity of consumers. For example, suppose that $p_1 < p_2 < r$. Then $\int_0^{p_2-p_1} f(x)dx$ gives the fraction of consumers who prefer the cheaper firm (firm 1), while the others are indifferent between the two firms. Assume that 50% of the indifferent consumers go to one firm and 50% go to the other firm.

(i) Write down the demand function of each firm (list all the possibilities).
(ii) Give a necessary and sufficient condition for $p_1 = p_2 = r$ to be a Nash equilibrium (you may make all the continuity and differentiability assumptions you like on the function $f$).
(iii) Suppose that $r = 120$, $\int_0^{20} f(x)dx = 0.4$, $f(20) = 0.014$. Is $p_1 = p_2 = 120$ a Nash equilibrium?
(iv) Assume that $f$ is constant (uniform distribution). Is $p_1 = p_2 = r$ a Nash equilibrium?
(v) Assume that $f(0) > 0$ and $f$ is continuous. Is $p_1 = p_2 = 0$ a Nash equilibrium?
(vi) What assumptions on $f$ would yield the “Bertrand paradox”?
2. PART I. Consider a Cournot duopoly where both firms have a cost function of the form \( C(q) = F + cq \) (where \( F \) and \( c \) are positive constants). The following diagram shows the reaction curve of firm 1 and two reaction curves of firm 2, labeled A and B. Reaction curve A is the initial one, while B is the one that results after a change has occurred in firm 2’s marginal cost.

(I.a) Has firm 2's marginal cost gone up or down? [Prove your claim.]

(I.b) Are the profits of firm 1 at the new Nash equilibrium higher or lower than at the old Nash equilibrium? What about the profits of firm 2? [Prove your claims.]

![Diagram of reaction curves](image)

PART II. Consider a Cournot duopoly. The following diagram shows the reaction curve of firm 2 (\( q_i \) is the output of firm \( i, i = 1,2 \)). Show the shape of the profit function of firm 2 when \( q_1 = 1 \), when \( q_1 = 3 \), when \( q_1 = 5 \), when \( q_1 = 7 \) (draw four different graphs and in each of them measure \( q_2 \) on the horizontal axis).

![Graph of profit function](image)

Note: the heavy dots belong to the curve
3.

Consider an industry with two firms that produce a homogeneous product.

a) The prices charged by the two firms are highly correlated over time. An economist points to this fact as proof that the two firms are colluding. Do you agree? Why?

b) Suppose you observe marginal costs (or a good proxy for). How could you test if the firms are colluding?

c) If the firms are colluding, how could you test if the collusion follows a pattern predicted by any particular theory? (you can choose any reasonable theory of collusion you like) Your tests can rely on existing work, but be clear about what data you are using, specifically how you plan to use the data, precisely what you are testing, and what are the alternative hypotheses.

d) Now assume that you do not observe marginal costs. Without any additional assumptions can you empirically identify the degree of market power in this industry? Explain. (Hint: it may help to demonstrate the identification problem using a graph or using linear demand and cost functions and the associated profit maximization condition.)

e) List two possible sets of assumptions that would allow you to identify the degree of market power. Explain.

f) Now suppose you are trying to estimate the demand system for a differentiated products industry. What are the difficulties of estimating the own- and cross-price demand elasticities?

g) Briefly describe/outline methods available to solve two of the problems you named in (f).
4.


a) Describe the empirical context of the Leslie paper. In particular, describe the data, the variation in the data and the research question.

b) Describe, as fully as you can, the empirical model. Pay particular attention what identifies the parameters of the model.

c) Discuss the empirical results. Be as specific as you can and discuss those results that make sense and those that do not.

d) Describe the empirical context of the Busse and Rysman. In particular, describe the data, the variation in the data and the research question.

e) Describe, as fully as you can, the empirical model. Pay particular attention what identifies the parameters of the model.

f) Discuss the empirical results. Be as specific as you can.
Section C

5.

Consider an alternative to ROR regulation called Return on Sales regulation. Under ROS regulation, the firm must choose K, L, and output Q to maximize profit given the constraint that profit is less than a multiple of sales. Formally, the constraint is \( \pi \leq kPQ \) for some constant \( k \geq 0 \). Assume \( k \) is small enough so that the regulation binds. For this question, use the same notation we used for ROR regulation; mathematical derivations will receive the highest credit but graphical explanations are acceptable.

a) Is production efficient? I.e., does the firm minimize costs for the output produced? Compare your answer here with ROR regulation.

b) Will the output \( Q_R \) produced by the regulated firm be larger or smaller than the output \( Q_M \) produced by the unregulated firm?

c) Will the firm expand output into the elastic portion of demand? Explain.

d) How does the output produced change as \( k \) is lowered to (but does not reach) zero?

e) What happens if \( k = 0 \)?

f) If the second best occurs in the inelastic region of demand, can the regulator attain it by appropriate choice of \( k \)?
6.

Consider the following Becker-style model of competition between two interest groups for regulation. Regulation redistributes income from Group 1 to Group 2. The initial income of each group is $Z_i^0$, $i = 1, 2$. Final income is $Z_i = Z_i^0 + R_i - a_i$, where $R_i$ is redistribution ($R_1 < 0$ and $R_2 > 0$) and $a_i > 0$ is money spent by Group $i$. There is deadweight loss, measured by $x$, when income is redistributed, so that the amount collected from Group 1 is $T = -R_1/(1+x)$, where $x > 0$ is a constant. Similarly, there is deadweight loss when the tax is distributed to Group 2: $T = (1+x)R_2$. Groups spend on lobbying to influence redistribution. Lobbying pressure $p_i$ is a function of $a_i$: $p_i = p(a_i)$, with $p'(a_i) > 0$ and $p''(a_i) < 0$. The tax $T$ is determined by aggregate influence $I$, which is a function of pressure by both groups: $T = I(p_1, p_2)$, where $\partial I/\partial p_2 > 0$ (Group 2 wants more redistribution), $\partial^2 I/\partial p_2^2 < 0$, $\partial I/\partial p_1 < 0$ (Group 1 wants less redistribution), $\partial^2 I/\partial p_1^2 > 0$, and $\partial^2 I/\partial p_1 \partial p_2 < 0$.

a) Assuming that each group wants to maximize its final income, find the Nash equilibrium best response $a_i$ of each group to the other group’s $a_j$. You should derive the first-order condition that implicitly defines the best responses $a_1^*(a_2)$ and $a_2^*(a_1)$. Are dollars spent on pressure by the two groups strategic substitutes or complements?

b) Show that if deadweight loss increases (i.e., $x$ increases), then the best response $a_1^*$ of Group 1 increases for any given $a_2$.

c) Show that if deadweight loss increases (i.e., $x$ increases), then the best response $a_2^*$ of Group 2 decreases for any given $a_1$.

d) Using your results above, what happens to equilibrium pressure exerted by each group? What happens to the amount collected for redistribution? Assume that the best response curves cross (in $(a_1, a_2)$ space) and that the slopes are such that Nash equilibrium is stable.

e) From the above, argue for or against this proposition: regulatory policies that improve welfare are more likely to be implemented than ones that do not.