Ph. D. Preliminary examination in Industrial Organization, June 2006

Answers to questions 1 and 2

1. (a) Suppose the Incumbent is passive. Then the entrant will enter iff $D_E \geq k$. Thus *ex ante* the probability of entry, if the Incumbent is passive, is $\text{Prob}\{k \leq D_E\} = F(D_E)$. Thus the incumbent's expected profits if he chooses to be passive are:

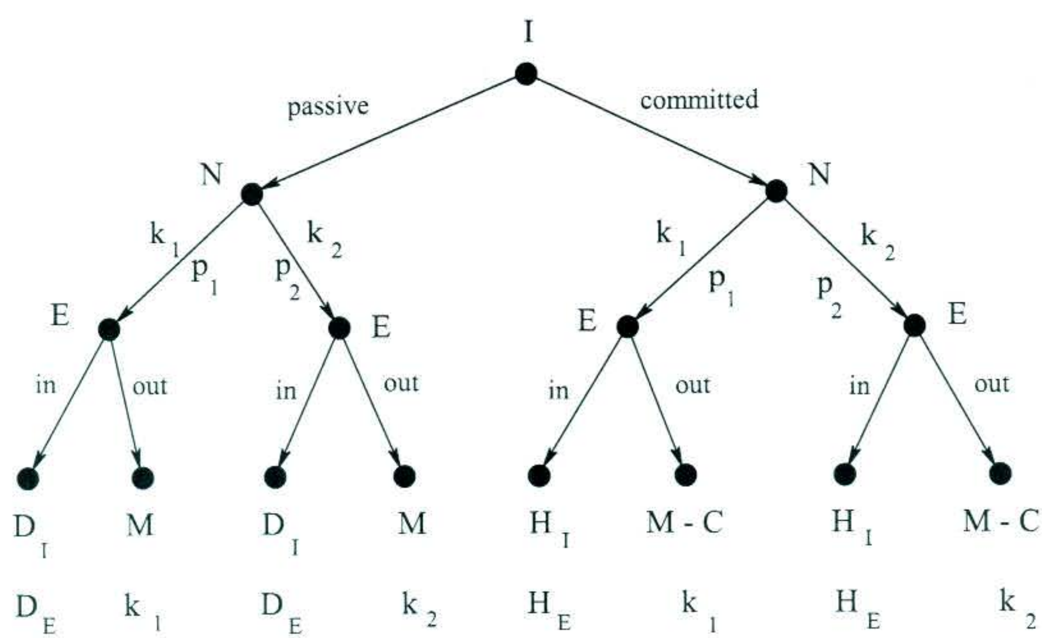
$$F(D_E)\, D_I + [1 - F(D_E)]\, M \qquad (1).$$

Similarly, if the incumbent is committed, entry occurs with probability $F(H_E)$ and the Incumbent's expected profits are:

$$F(H_E)\, H_I + [1 - F(H_E)]\, (M - C) \qquad (2).$$

Thus the incumbent will choose to be passive if (1) > (2), and will choose to commit if (2) > (1).

(b) Let I = incumbent, N = Nature, E = entrant, $p_1 = \text{Prob}\{k_1\}$, $p_2 = \text{Prob}\{k_2\}$.



(c) In this case we have that F(1) = 1/4, F(4) = 2/4, F(6) = 3/4 and F(12) = 1. Thus $F(D_E) = F(5) = F(4) = 2/4$ and $F(H_E) = F(2) = F(1) = 1/4$. Thus (1) above becomes:

$$(2/4)\,(5) + [1 - 2/4]\; 12 = 17/2 = 8.5$$

while (2) above becomes:

$$(1/4)\,(2) + [1 - 1/4]\,(12 - 1) = 35/4 = 8.75$$

Hence the Incumbent will choose **commitment**.

(d) In this case we have that F(1)=1/5, F(2)=2/5, F(4)=3/5 and F(7)=1. Thus $F(D_E) = F(7/2) = F(2) = 2/5$ and $F(H_E) = F(3/2) = F(1) = 1/5$. Thus (1) above becomes:

$$(2/5)\,(7/2) + [1 - 2/5]\;8 = 17/2 = 31/5 = 6.2$$

while (2) above becomes:

$$(1/5)\,(3/2) + [1 - 1/5]\,(8 - 2) = 51/10 = 5.1$$

Hence the Incumbent will choose **to be passive**.

2. If both firms choose the same quality k and charge the same price p then the indifferent consumer is given by the solution to $t = U(k)(t-p)$ which is $t = \dfrac{U(k)}{U(k)-1}p$. Thus demand is $Q = \left(1 - \dfrac{U(k)}{U(k)-1}p\right)N$ and inverse demand is $p = \dfrac{U(k)-1}{U(k)}\left(1-\dfrac{Q}{N}\right)$.

If both choose low quality, the BNE is given by both prices and both profits equal to zero. To find the CNE write the profit functions: $\pi_A = q_A \dfrac{1}{2}\left(1 - \dfrac{q_A+q_B}{1,200}\right)$ and $\pi_B = q_B \dfrac{1}{2}\left(1 - \dfrac{q_A+q_B}{1,200}\right)$. Solving the F.O.C. we get $q_A = q_B = 400$ with corresponding profits of $\pi_A = \pi_B = \dfrac{1,799}{10,800} = 0.167$.

If both choose high quality, the BNE is given by $p_A = p_B = 0.4$ (or slightly less than that) so that only firm A is active. Firm A's profits are (0.4 – 0.3) 666.67 = 66.67, while firm 2's profits are zero. To find the CNE write the profit functions: $\pi_A = q_A\left(\dfrac{9}{10}\left(1 - \dfrac{q_A+q_B}{1,200}\right) - 0.3\right)$ and $\pi_B = q_B\left(\dfrac{9}{10}\left(1 - \dfrac{q_A+q_B}{1,200}\right) - 0.4\right)$. Solving the F.O.C. we get $q_A = 311.1,\ q_B = 177.7$ with corresponding profits of $\pi_A = 72.6,\ \pi_B = 23.7$.

If one chooses high quality and the other chooses low quality then the consumer who is indifferent between high quality and low quality is given by the solution to $10(t-p_H) = 2(t-p_L)$ which is $t_{LH} = \dfrac{5}{4}p_H - \dfrac{1}{4}p_L$ and the consumer who is indifferent between buying low quality and nothing is given by the solution to $2(t-p_L) = t$ which is $t = 2p_L$. Thus demand is given by $D_H = \left(1 - \dfrac{5}{4}p_H + \dfrac{1}{4}p_L\right)1,200$ and $D_L = \left(\dfrac{5}{4}p_H - \dfrac{1}{4}p_L - 2p_L\right)1,200$. Inverse demand is $p_H = \dfrac{9}{10} - \dfrac{3}{4,000}q_H - \dfrac{1}{12,000}q_L$ and $p_L = \dfrac{1}{2} - \dfrac{1}{2,400}q_H - \dfrac{1}{2,400}q_L$

Thus, **if firm A chooses high quality and firm B low quality** in the Bertrand game the profit functions are $\pi_A = (p_A - 0.3)\left(1 - \frac{5}{4}p_A + \frac{1}{4}p_B\right)1,200$ and $\pi_B = p_B\left(\frac{5}{4}p_A - \frac{9}{4}p_B\right)1,200$. The BNE is $p_A = 0.5657,\ p_B = 0.1571$ with corresponding profits $\pi_A = 105.9,\ \pi_B = 66.67$. In the Cournot game the profit functions are $\pi_A = q_A\left(\frac{9}{10} - \frac{3}{4,000}q_A - \frac{1}{12,000}q_B - 0.3\right)$ and $\pi_B = q_B\left(\frac{1}{2} - \frac{1}{2,400}q_A - \frac{1}{2,400}q_B\right)$. The CNE is $q_A = 377.143,\ q_B = 411.428$ with corresponding profits of $\pi_A = 106.678,\ \pi_B = 70.531$.

If firm A chooses low quality and firm B high quality in the Bertrand game the profit functions are $\pi_A = p_A\left(\frac{5}{4}p_B - \frac{9}{4}p_A\right)1,200$ and $\pi_B = (p_B - 0.4)\left(1 - \frac{5}{4}p_B + \frac{1}{4}p_A\right)1,200$. The BNE is $p_A = 0.171,\ p_B = 0.617$ with corresponding profits $\pi_A = 79.31,\ \pi_B = 70.7$. In the Cournot game the profit functions are $\pi_A = q_A\left(\frac{1}{2} - \frac{1}{2,400}q_B - \frac{1}{2,400}q_A\right)$ and $\pi_B = q_B\left(\frac{9}{10} - \frac{3}{4,000}q_B - \frac{1}{12,000}q_A - 0.4\right)$. The CNE is $q_A = 445.71,\ q_B = 308.57$ with corresponding profits of $\pi_A = 82.776,\ \pi_B = 71.412$.

Thus we can simplify the game to:

GAME1:BERTRAND

FIRM B

		L	H
FIRM	L	0 , 0	79.31 , 70.7
A	H	105.9 , 66.67	66.67 , 0

Hence game 1 has two SPEs: (H,L) and (L,H).

GAME 2 : *COURNOT*

FIRM B

		L	H
FIRM	L	0.167 , 0.167	82.78 , 71.41
A	H	106.68 , 70.53	72.6 , 23.7

Hence game 2 also has two SPEs: (H,L) and (L,H).

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Answers to Questions 5 and 6

5. Price discrimination

(a) If the price for each unit is up to the demand curve, then the problem for the firm is

$$\max_{Q} \pi = \int_0^Q p(q) - MC(q)dq - F \tag{1}$$

Note that $p(q)$ is the demand curve and q is just a dummy variable for the integration. The FOC from Leibnitz' Rule is:

$$\frac{d\pi}{dQ} = 0 \Rightarrow p(Q) - MC(Q) = 0 \Rightarrow p(Q) = MC(Q) \tag{2}$$

And so the firm picks the quantity Q that sets MC equal to the price of the last unit sold.

(b) Now the margin on a unit q will be $r(q) - MC(q)$, where $r(q)$ is the max price the firm is allowed to charge, $r(q) \equiv MC(q) + k(q)\left[p(q) - MC(q)\right]$ where again $p(q)$ is the demand curve. The margin simplifies to

$$k(q)\left[p(q) - MC(q)\right] \tag{3}$$

of course. The problem for the firm is

$$\max_{Q} \pi = \int_0^Q k(q)\left[p(q) - MC(q)\right]dq - F \tag{4}$$

The FOC from Leibnitz' Rule is:

$$\frac{d\pi}{dQ} = 0 \Rightarrow k(Q)\left[p(Q) - MC(Q)\right] = 0 \Rightarrow p(Q) = MC(Q) \tag{5}$$

The key thing here is that $k(q)$ is a predetermined function, so there is no term like $k'(Q)$ in the FOC. A special case of this is where k is a constant; then it is a simple rule "firm keeps fraction k of surplus created on each unit sold". It must be the case that the surplus the firm is allowed to capture is greater than the FC, otherwise the firm won't produce.

(c) This is a variation of "firm keeps fraction k of surplus created on each unit sold" where the fraction kept is endogenous to the total quantity produced. The assumption that $m'(Q) > 0$ might come about because a regulator is trying to encourage the firm to produce more output (the more the firm produces, the large the margin it keep on all units). Now the margin on an inframarginal unit q will be $m(Q)\,[p(q) - MC(q)]$, where the level m is set based on the marginal unit Q. The problem for the firm is

$$\max_Q \ \pi = \int_0^Q m(Q)\,[p(q) - MC(q)]\,dq - F \tag{6}$$

The FOC from Leibnitz' Rule is:

$$\frac{d\pi}{dQ} = 0 \Rightarrow m(Q)\,[p(Q) - MC(Q)] + \int_0^Q m'(Q)\,[p(q) - MC(q)]\,dq = 0 \tag{7}$$

$$\Rightarrow p(Q) - MC(Q) = -\int_0^Q \frac{m'(Q)}{m(Q)}\,[p(q) - MC(q)]\,dq \tag{8}$$

You are given that $m'(Q) > 0$, so the implication is that $p(Q) < MC(Q)$; *the firm is induced to produce too much* (from the social point of view). The key thing here is that $m(Q)$ depends on the final output chosen, and affects all the inframarginal units too, so there is an extra term in the derivative.

6. a. and b. This question is based on section 2.4 of Laffont and Tirole's textbook, *A Theory of Incentives in Produrement and Regulation*. See the text for answers.

c. The main distinction between the models is the Laffont and Tirole allow for ex post cost observability and B&M do not.