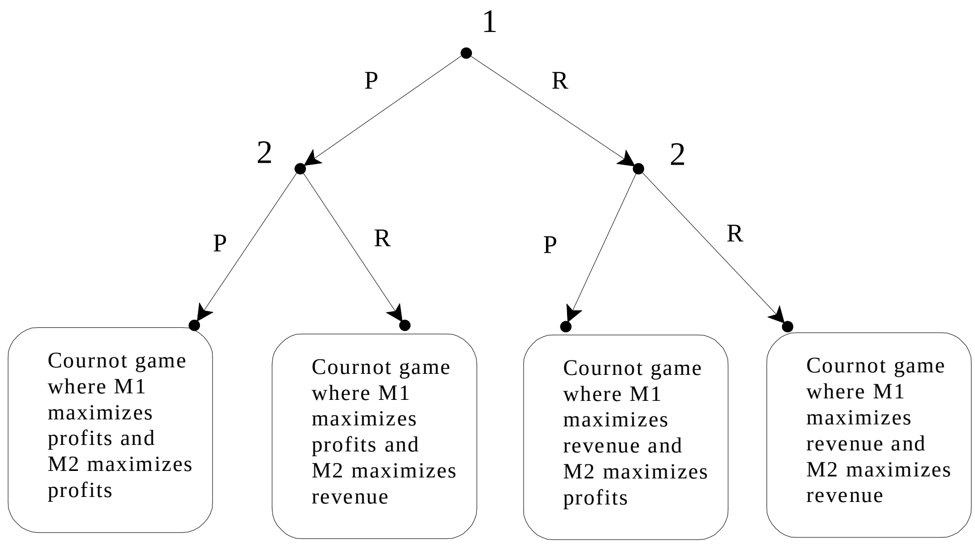
Ph. D. Preliminary examination in Industrial Organization, July 2007

Answers to questions 1 and 2

1. (a) The extensive form is as follows



(b) Number the subgames 1 to 4 from left to right.

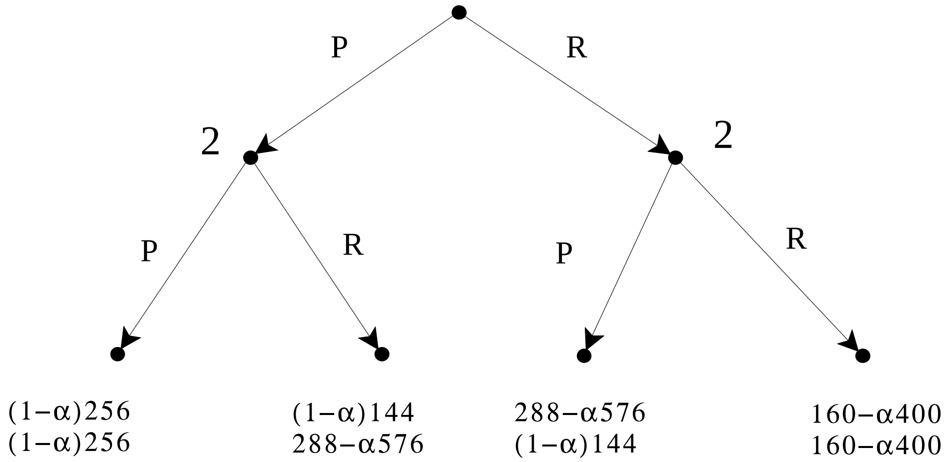
Subgame 1: q_1 is chosen to maximize $\alpha\Pi_1(q_1,q_2)=\alpha\left(q_1(60-q_1-q_2)-12q_1\right)$ and q_2 is chosen to maximize $\alpha\Pi_2(q_1,q_2)=\alpha\left(q_2(60-q_1-q_2)-12q_2\right)$. Solving $\frac{\partial\Pi_1}{\partial q_1}=0$ and $\frac{\partial\Pi_2}{\partial q_2}=0$ gives $q_1=q_2=16$. Player 1's payoff is $(1-\alpha)\Pi_2(16,16)=(1-\alpha)256$ and the same is true for player 2.

Subgame 2: q_1 is chosen to maximize $\alpha\Pi_1(q_1,q_2)=\alpha\left(q_1(60-q_1-q_2)-12q_1\right)$ and q_2 is chosen to maximize $\alpha R_2(q_1,q_2)=\alpha q_2(60-q_1-q_2)$. Solving $\frac{\partial\Pi_1}{\partial q_1}=0$ and $\frac{\partial R_2}{\partial q_2}=0$ gives $q_1=12$ and $q_2=24$. Player 1's payoff is $(1-\alpha)\Pi_1(12,24)=(1-\alpha)144$ and player 2's payoff is $\Pi_2(12,24)-\alpha R_2(12,24)=288-\alpha 576$.

Subgame 3: q_1 is chosen to maximize $\alpha R_1(q_1,q_2)=\alpha q_1(60-q_1-q_2)$ and q_2 is chosen to maximize $\alpha\Pi_2(q_1,q_2)=\alpha\left(q_2(60-q_1-q_2)-12q_2\right)$. Solving $\frac{\partial R_1}{\partial q_1}=0$ and $\frac{\partial\Pi_2}{\partial q_2}=0$ gives $q_1=24$ and $q_2=12$. Player 1's is $\Pi_1(24,12)-\alpha R_1(24,12)=288-\alpha 576$ and player 2's payoff is $(1-\alpha)\Pi_2(24,12)=(1-\alpha)144$.

Subgame 4: q_1 is chosen to maximize $\alpha R_1(q_1,q_2)=\alpha q_1(60-q_1-q_2)$ and q_2 is chosen to maximize $\alpha R_2(q_1,q_2)=\alpha q_2(60-q_1-q_2)$. Solving $\frac{\partial R_1}{\partial q_1}=0$ and $\frac{\partial R_2}{\partial q_2}=0$ gives $q_1=20$ and $q_2=20$. Player 1's is $\Pi_1(20,20)-\alpha R_1(20,20)=160-\alpha 400$.

Thus the game reduces to:



Now, $(1-\alpha)256 \le 288-\alpha 576$ if and only if $\alpha \le \frac{1}{10}$ and

$(1-\alpha)144 \le 160-\alpha 400$ if and only if $\alpha \le \frac{1}{16}$. Thus,

Case 1: $\alpha < \frac{1}{16}$**.** Then 2 will offer a revenue contract at both nodes and there is a unique subgame-perfect equilibrium where both offer a revenue contract.

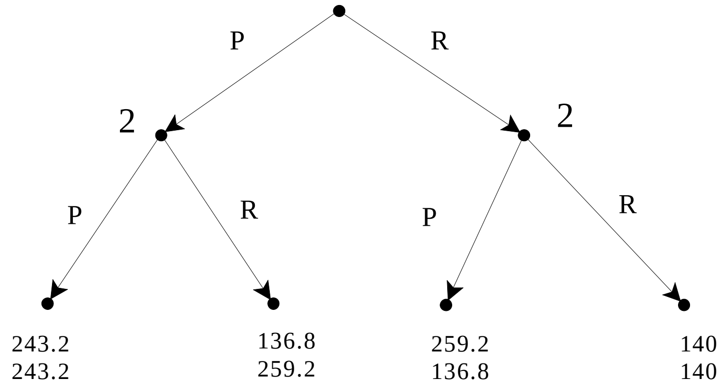
Case 2: $\alpha = \frac{1}{16}$**.** Then 2 offers a revenue contract if 1 offered a profit contract and is indifferent between revenue and profit contracts if 2 offered a revenue contract. There are three subgame-perfect equilibria: (P, RR), (R, RR) and (R, RP).

Case 3: $\frac{1}{16} < \alpha < \frac{1}{10}$**.** In this case 2 offers a revenue contract if 1 offers a profit contract and a profit contract if 1 offers a revenue contract. There is a unique subgame-perfect equilibrium: (R, P).

Case 4: $\alpha = \frac{1}{10}$**.** In this case 2 offers a profit contract if 1 offers a revenue contract and is indifferent between profit and revenue contracts if 1 offered a profit contract. There are two subgame-perfect equilibria: (P,P) and (R,P).

Case 5: $\alpha > \frac{1}{10}$**.** In this case 2 offers a profit contract at both nodes and there is a unique subgame-perfect equilibrium where both offer a profit contract: (P,P).

(c) When $\alpha = \frac{1}{20}$ we are in case 1. The reduced game is



The subgame-perfect equilibrium is (R,RR) with payoffs of 140 for each player.

(d) This is an instance of the advantages of delegating choices to somebody with different incentives from your own. A revenue-maximizing manager expands output relative to a profit-maximizing manager and the reaction of the competitor is to reduce output (output levels are strategic substitutes). However, this situation ends up being a prisoners' dilemma situation: both player would be better off if they were to run the firms themselves.

2. If the merger is allowed, HAL-Entil is a single firm with unit cost of production equal to 3 like HAL.

The Cournot equilibrium is therefore:

$$q_1 = q_2 = \frac{997}{6} = 166.17, \quad Q = \frac{1994}{6} \cong 332.33, \quad P = \frac{1006}{3} \cong 335.33.$$

If the merger is not allowed, let w be the price that Entil charges HAL. Then the latter has a unit cost of (1+w). The Cournot equilibrium is given by:

$$q_1 = \frac{1001 - 2w}{6}, \quad q_2 = \frac{995 + w}{6}, \quad Q = \frac{1996 - w}{6}, \quad P = \frac{1004 + w}{3}$$

Entil will choose w to maximize its profits given by $(w - 2)\frac{1001 - 2w}{6}$. Thus will choose

$$w = \frac{1005}{4} = 251.25.$$

The corresponding quantity and output will be (substituting in the above formulas):

$$Q = \frac{6979}{24} \cong 290.79, \quad P = \frac{5021}{12} \cong 418.41.$$

Since, by hypothesis, the government only cares about the welfare of consumers, the merger should be allowed because it will bring about a reduction in the price.