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Time: 3 hours
Reading Time: 20 minutes

PRELIMINARY EXAM FOR THE Ph.D. DEGREE

Answer 4 questions, at least one from each part. Closed Book exam.

PART I

Question 1. Point allocation: (a) 10%; (b) 20%; (c) 20%; (d) 10%; (e) 20%; (f) 20%.

Consider the density

$$f(y) = \frac{\frac{1}{\gamma}\exp(-y/\gamma)}{[1+\exp(-y/\gamma)]^2},$$

where $-\infty < y < \infty$, $\gamma > \infty$ and it can be shown that

$$\begin{aligned} \mathrm{E}[y] &= 0 \\ \mathrm{V}[y] &= \frac{\gamma^2\pi^2}{3}. \end{aligned}$$

Suppose we have a random sample $\{(y_i, \mathbf{x}_i),\ i = 1, ...N\}$, where $\mathbf{x}_i$ is a $k \times 1$ nonstochastic regressor vector and y_i has the above density with

$$\gamma_i = \exp(\mathbf{x}_i'\boldsymbol{\beta}),$$

where $\boldsymbol{\beta} = \boldsymbol{\beta}_0$ in the data generating process.

(a) Give the log-likelihood function.

(b) Give the first-order conditions for the MLE $\widehat{\boldsymbol{\beta}}$ of $\boldsymbol{\beta}_0$.

(c) Obtain the limit distribution of $\sqrt{N}(\widehat{\boldsymbol{\beta}} - \boldsymbol{\beta}_0)$.
Obtain this as easily as you can. You need not formally verify any LLN and CLT used here, but state any assumptions made.

(d) Does consistency of $\widehat{\boldsymbol{\beta}}$ require correct specification of the density of y, or can consistency be obtained under weaker assumptions on y? Explain your answer. A brief explanation will do and there is no need to apply a LLN or CLT.

(e) Given the above information, provide the objective function for a consistent method of moments or generalized method of moments estimator for $\boldsymbol{\beta}$, that is not the MLE. Your answer must be specific to this question.

(f) Give the expression for the asymptotic variance for your specific estimator in part (e). Obtain this as easily as you can.

Question 2. Point allocation: (a) 20%; (b) 20%; (c) 10%; (d) 20%; (e) 20%; (f) 10%.

2. Consider the classic Tobit model with

$$\begin{aligned} y_i^* &= \mathbf{x}_i'\boldsymbol{\beta} + u_i \\ u_i &\sim \mathcal{N}[0, \sigma^2], \end{aligned}$$

where we observe $\mathbf{x}_i$ and $y_i = \min(y_i^*, 0)$.

(a) Give the log-likelihood function for the MLE of $\boldsymbol{\beta}$ and σ^2.

(b) The statistical package Shazam is unusual in that it reports estimates and standard errors for $\boldsymbol{\alpha} = \boldsymbol{\beta}/\sigma$ and $\tau = 1/\sigma$, rather than directly for $\boldsymbol{\beta}$ and σ.
State how to obtain both estimates and standard errors of $\boldsymbol{\beta}$ given knowledge of $\widehat{\boldsymbol{\alpha}}$, $\widehat{\tau}$, and $\widehat{\mathrm{V}}$, where $\widehat{\mathrm{V}}$ is the estimated variance matrix of $[\widehat{\boldsymbol{\alpha}}' \ \widehat{\tau}]'$.

(c) Consider the same setting as part (b), where standard errors of $\widehat{\boldsymbol{\beta}}$ are not directly reported. State how to use the bootstrap to obtain the standard error of $\widehat{\boldsymbol{\beta}}$.

(d) Show that for $z \sim \mathcal{N}[0, \sigma^2]$, $\mathrm{E}[z|z > c] = \phi(c)/[1 - \phi(c)]$, where $\phi(\cdot)$ and $\Phi(\cdot)$ denote the standard normal pdf and cdf .

(e) Return to the original Tobit model at the start of this question. Give an expression for $\mathrm{E}[y|\mathbf{x}]$, with complete derivation. [Hint: use the result given in part(d)].

(f) Hence state how to obtain an estimate of $\boldsymbol{\beta}$ in the Tobit model by NLS estimation.

(g) This part is unrelated to the previous parts.
Show that a three-choice multinomial model can be estimated using a binary logit package. A brief answer will do.

PART II

Question 3. Point allocation: (a) 15%; (b) 5%; (c) 5%; (d) 35%; (e) 20%; (f) 20%.

Consider the model

$$y_t = \phi_1 y_{t-1} + \phi_2 y_{t-2} + \varepsilon_t,$$

where y_t is scalar and ε_t is a martingale difference sequence and a sample $t = 1, ..., T$ is available.

(a) Express this AR(2) in terms of its Beveridge-Nelson decomposition form. Be explicit in terms of the correspondence between the AR(2) parameters ϕ_1 and ϕ_2 and the parameters of the B-N form, say δ and ρ.

(b) Derive under what conditions is the AR(2) model stationary in terms of an expression for its two roots, say λ_1 and λ_2. You do not need to solve the resulting two equation system.

(c) Suppose $\lambda_1 = 1$. Solve the system of two equations found in part (b).

(d) Given your answer in part (a), derive the asymptotic distribution of β in the regression

$$\Delta y_t = \delta \Delta y_{t-1} + \beta y_{t-1} + \varepsilon_t$$

Hints: let $u_t \equiv (1-\delta L)^{-1}\varepsilon_t$ with $|\delta| < 1$, $\gamma_j \equiv E(u_t u_{t-j}) = \sigma^2 \delta^j (1-\delta)^{-1}; \alpha = \sigma/(1-\delta); \xi_t = u_1 + ... + u_t$

$$T^{-1/2} \sum_{t=1}^{T} u_t \xrightarrow{d} \beta W(1)$$

$$T^{-1/2} \sum_{t=1}^{T} u_{t-j}\varepsilon_t \xrightarrow{d} N\left(0, \sigma^2 \gamma_j\right)$$

$$T^{-2} \sum_{t=1}^{T} \xi_{t-1}^2 \xrightarrow{d} \alpha^2 \int_0^1 [W(r)]^2 \, dr$$

$$T^{-1} \sum_{t=1}^{T} u_t u_{t-j} \xrightarrow{p} \gamma_j$$

$$T^{-1} \sum_{t=1}^{T} \xi_{t-1}\varepsilon_t \xrightarrow{d} \frac{1}{2}\sigma\alpha \left\{W(1)^2 - 1\right\}$$

(e) Suppose instead that you estimate the AR(2) in its original form by least-squares. Without formal derivation and based on your findings in part (d), explain what is the asymptotic distribution of the parameter ϕ_1.

(f) Given your answer in part (e), which of the following hypothesis can be tested using the usual OLS results:

(a) $H_0 : \phi_2 = 1$

(b) $H_0 : \phi_1 + 2\phi_2 = 3$

(c) $H_0 : 2\phi_1 = \phi_2$.

Question 4. Point allocation: (a) 10%; (b) 5%; (c) 20%; (d) 15%; (e) 20%; (f) 30%.

Consider the following stationary and ergodic data generating process for the random variable y_t :

$$y_t = \rho y_{t-1} + \varepsilon_t,$$

where ε_t is a martingale difference sequence with $E(\varepsilon_t^2) = \sigma^2 > 0$ and $\sup_t E\left[|\varepsilon_t|^{2+\delta}\right] < \infty$ for $\delta > 0$; $|\rho| < 1$.

(a) Obtain $E\left[y_t^2\right]$.

(b) Given that y_t is weakly stationary and ergodic, invoke an appropriate theorem to show that,

$$\text{plim}_{T\to\infty}\frac{1}{T}\sum_{t=1}^{T} y_t^2 \to E\left[y_t^2\right].$$

(c) Given (a) and (b) and knowledge that ε_t is a M.D.S., derive the limiting distribution of the least squares estimator for ρ. Please be precise in invoking all the relevant theorems that you require for your derivation.

(d) Now suppose you are interested in the impulse response for period two, ρ^2. Given the asymptotic distribution that you just derived for $\widehat{\rho}_{LS}$, derive an asymptotic approximation for the distribution of ρ^2.

(e) Since the previous derivation is only a first order approximation to the true distribution, you decide to use the bootstrap to obtain a better approximation for the standard error of ρ^2. Describe the steps involved in calculating an appropriate bootstrap estimator for this moment.

(f) Instead, suppose you estimate ρ^2 with the projection of y_{t+1} on y_{t-1}. Derive the asymptotic distribution of the resulting least squares estimate and compare with your answer in (e). Briefly, is there anything you can do to improve the efficiency of this estimator?

PART III

Question 5. Point allocation: (a) 25%; (b) 25%; (c) 25%; (d) 25%.

Suppose that as a part of your dissertation you are estimating a linear regression model. You are concerned about parameter stability in your model and perform the F-tests suggested by Bai and Perron (Econometrica, 1998). Your results are printed below:

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Output from the Bai and Perron testing procedures

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(i) supF tests against a fixed number of breaks

The supF test for 0 versus 1 breaks (scaled by q) is: 10.07
The supF test for 0 versus 2 breaks (scaled by q) is: 16.27
The supF test for 0 versus 3 breaks (scaled by q) is: 15.13

The critical values at the 5% level are (for k=1 to 3):
12.89 11.60 10.46

(ii) Dmax tests against an unknown number of breaks

The UDmax test is: 16.27
(the critical value at the 5% level is: 13.27)

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The WDmax test at the 5% level is: 16.01
(The critical value is: 14.19)

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(iii) supF(l+1|l) tests using global optimizers under the null

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The supF(2 | 1) test is : 15.25
The supF(3 | 2) test is : 6.91

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The critical values of supF(i+1|i) at the 5% level are (for i=1 to 3) are:
12.89 14.50 15.42

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(a) Draw conclusions about the number of breaks in your sample.

(b) Suppose you conclude from the Bai/Perron tests that there are some breaks in the parameters of your model and you want to incorporate these into your analysis. One of your options is to treat the breaks as deterministic as in the Bai/Perron approach. A second option is to model the breaks as random draws from some distribution using, for example, a Markov switching model. Outline the advantages and disadvantages of these two approaches.

Question 5 (continued)

(c) This part is distinct from parts (a), (b), and (d). Consider a model with a log-likelihood function given by $L(X;\theta)$. Suppose you only observe a subset Y of the data, i.e. you observe $Y \subset X$. Outline how you could use the EM algorithm to estimate θ.

(d) This part is distinct from parts (a)-(c). Consider the problem of estimating $E(y|x)$ vector using the Nadaraya-Watson kernel estimator. Suppose that x is a $q \times 1$ vector of binary random variables. Explain the curse of dimensionality and explain why it is much less of a curse for binary x variables than continuous variables.

Question 6. Point allocation: (a) 25%; (b) 25%; (c) 25%; (d) 25%.

Suppose that the solution of a representative agents portfolio allocation problem is given by the following Euler equation

$$1 = E_t\left(\beta \frac{U'(C_{t+1})}{U'(C_t)} \frac{(P_{t+1} + D_{t+1})}{P_t}\right),$$

where C_t is consumption, P_t is the price of a financial asset, and D_t is the dividend paid out by the financial asset. Suppose that the agents utility function is given by

$$U_t = -C_t^{-1},$$

and you have quarterly time series data on P_t and D_t for one financial asset, and on C_t. In addition, you have data on the risk-free rate of return, R_t.

(a) Write down the sample moment conditions that you would use to obtain the GMM estimates of the unknown parameter β. Be careful to justify your choice of moments and to explain how you would obtain the efficient GMM estimates.

(b) Suppose that you estimate β using more than one moment condition. Write down Hansen's J-statistic and explain how you would use it to make inference about the validity of the moment conditions and the model implied by the above Euler equation. If you were to reject the null hypothesis using Hansen's J-test, what would you do?

(c) Suppose that you estimate β using more than one moment condition. Explain how you would test for weak identification using the results of Stock and Wright (Econometrica, 2000).

(d) This part is distinct from parts (a)-(c). Suppose you are forecasting some variable y conditional on a large vector of explanatory variables x using a linear model. Three methods you could use to construct your forecasts are ordinary least squares, partial least squares, and principal components regression. List the advantages and disadvantages of each method.