

PRELIMINARY EXAMINATION FOR THE Ph.D. DEGREE

You need to answer four questions, at least one from each part.

Part 1.

1. Assume that the qualification of job candidates is assessed in terms of three criteria measured in terms of real-valued scores. For the purposes of this question, a hypothetical candidate can be identified with a vector of scores, i.e. with an element in \mathbb{R}^3 : *Of course, here and throughout the exam, you need to justify/explain your answers in appropriate detail.*

Let \hat{A} on \mathbb{R}^3 be defined by $x \hat{A} y$ if and only if $x_1 > y_1$ and $x_i > y_i$ for at least one $i \in \{2, 3\}$:

- a) Is \hat{A} acyclic?
- b) Is \hat{A} transitive?
- c) Consider $A(\cdot; \hat{A})$ on the set of finite choice-sets $F(\mathbb{R}^3)$: Which of the important choice-consistency conditions (A ; B ; C ; D or E) are satisfied by $A(\cdot; \hat{A})$?
- d) Determine the transitive hull of \hat{A} .

2. Let X denote a finite set of prizes, and $L(X)$ the set of all probability distributions on X .

Let \succsim be a weak order on $L(X)$:

a) State the von Neumann-Morgenstern Theorem (existence and uniqueness parts).

b) Prove the uniqueness part of the Theorem.

c) Now suppose that \succsim is defined on the set of binary lotteries $L_2(X)$: The von Neumann-Morgenstern Theorem is still true on this restricted domain, but its proof needs to be modified. Why?

d) How?

Part 2

3. Consider an exchange economy with two periods ($t = 0, 1$) and uncertainty described by S states of nature at date 1. There are L goods at each date and agent i has the initial endowment $\omega^i \in \mathbb{R}_+^{L(S+1)}$. The preferences of each agent are characterized by a utility function $u^i : \mathbb{R}_+^{L(S+1)} \rightarrow \mathbb{R}$ which is smooth, strictly quasi-concave and has indifference surfaces which do not intersect the axes.

- Define a spot-financial market equilibrium for the economy where $p = (p_0, \dots, p_S)$ is the vector of spot prices, $q = (q_1, \dots, q_J)$ the vector of security prices and V is the $S \times J$ matrix of payoffs of the securities (assume that the payoff of the securities is in units of good 1 whose price in each state is normalized to 1).
- Suppose the financial markets are complete. Using the agents' first-order conditions at an equilibrium and the first-order conditions for Pareto optimality, show that the equilibrium is Pareto optimal.
- Suppose as in (b) that the financial markets are complete, that there is only one good (income) at each date, that the first security is the riskless bond ($V^1 = (1, \dots, 1)$) and that agents have VNM utility functions

$$u^i(x^i) = v_0^i(x_0^i) + \sum_{s=1}^S \rho_s v^i(x_s^i)$$

where $\rho_s > 0$ is the probability of state s ($s = 1, \dots, S$) and $v^{i'} > 0$, $v^{i''} < 0$. Show that the equilibrium price of security j can be written as

$$\bar{q}_j = \frac{E(V^j)}{1 + \bar{r}} + \text{cov}(V^j, \frac{\bar{\pi}}{\rho})$$

where \bar{r} is the equilibrium interest rate and $\bar{\pi}$ the vector of state prices implicit in the prices of the securities. Explain the economics of the risk premium.

- Let $w_1 = (w_1, \dots, w_S)$ where $w_s = \sum_{i=1}^I \omega_s^i$ denotes aggregate output in state s . Show that there exists a strictly decreasing function g such that the equilibrium risk premium can be written as

$$\bar{q}_j = \frac{E(V^j)}{1 + \bar{r}} + \text{cov}(V^j, g(w_1))$$

Interpret this formula.

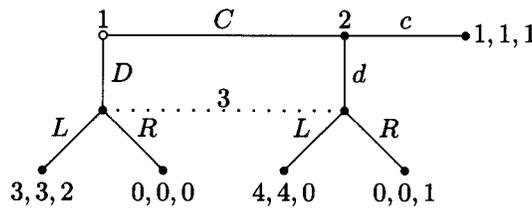
4. Consider an exchange economy with two periods ($t = 0, 1$) and uncertainty described by S states of nature at date 1. There is one good at each date and agent i has the initial endowment $\omega^i \in \mathbb{R}_+^{(S+1)}$. The preferences of each agent are characterized by a utility function $u^i : \mathbb{R}_+^{L(S+1)} \rightarrow \mathbb{R}$ which is smooth, differentiably strictly quasi-concave and has indifference surfaces which do not intersect the axes. There are J securities with a payoff matrix V at date 1. Unlike the previous question, the financial markets are incomplete ($J < S$).

- (a) Write out the definition of a financial market equilibrium and the first-order conditions which are satisfied at an equilibrium.
- (b) Explain from the first-order conditions why an equilibrium is not likely to be Pareto optimal.
- (c) Suppose now that you want to prove formally that an equilibrium with incomplete markets is typically suboptimal. Explain what you are able to prove and give a (precise) outline of the proof.
- (d) What are the non-generic cases that you know where a financial market equilibrium is Pareto optimal despite the fact that the financial markets are incomplete?

Part 3

5. [Selten's horse, Trembling hand perfection] Consider the game depicted in the figure.

- (a) Show¹ the set of all Nash equilibria of this game.
- (b) Show the set of all Subgame perfect equilibria of this game.
- (c) Show the set of all Sequential equilibria of this game.
- (d) Show the set of all Trembling hand perfect equilibria of this game.
- (e) If the outcomes reached in (c) and (d) are different, argue why they are different. If the outcomes reached in (c) and (d) coincide, show an example of a game in which Sequential equilibria and Trembling hand equilibria do not coincide. If you think that there is no such game in which Sequential equilibrium and Trembling hand equilibrium lead to different outcomes, argue why there shouldn't be such game.
- (f) Prove the following proposition: Every finite strategic game has a Trembling hand perfect equilibrium.
- (g) Does the proposition in (f) imply that every finite extensive game has a Trembling hand perfect equilibrium? Explain your answer.
- (h) Does the proposition in (f) imply that every finite game has a Sequential equilibrium? Explain your answer.
- (i) On a conceptual level, what do you think about the attractiveness and soundness of the trembling hand story (not necessarily only in the context of Selten's horse)?



¹When I write “show the set of ... equilibria”, this should also include a verification that indeed the outcomes are equilibria.

6. We continue with Trembling hand perfection, but direct our focus to evolution.

- (a) Prove the following proposition: Let σ be a mixed strategy in a finite symmetric 2-player strategic game. If σ is an Evolutionary stable strategy, then (σ, σ) is a Trembling hand perfect equilibrium.

(Hint: You are allowed to make use of the fact that if a Nash equilibrium of a two-player game is undominated (neither strongly nor weakly dominated), then it is a Trembling hand perfect equilibrium. All other steps you need to prove.)

- (b) Consider a large population of players who are randomly matched to play a finite 2-player symmetric strategic game. If σ is an asymptotically stable strategy in the replicator dynamics, is (σ, σ) a Trembling hand perfect equilibrium? Explain your answer.