University of California, Davis
Department of Economics
Advanced Economic Theory

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Time: 3 hours
Reading Time: 20 minutes

PRELIMINARY EXAMINATION FOR THE Ph.D. DEGREE

You need to answer four questions, at least one from each part.

Part 1.

1. Assume that the qualification of job candidates is assessed in terms of three criteria measured in terms of real-valued scores. For the purposes of this question, a hypothetical candidate can be identified with a vector of scores, i.e. with an element in $\mathbb{R}^3$. *Of course, here and throughout the exam, you need to justify/explain your answers in appropriate detail.*

Let $\succ$ on $\mathbb{R}^3$ be defined by $x \succ y$ if and only if $x_1 + x_2 + x_3 > y_1 + y_2 + y_3$ and $x_i > y_i + 1$ for at least one $i \in \{1, 2, 3\}$.

i) Is $\succ$ acyclic?

ii) Is $\succ$ transitive?

iii) Consider $\mathcal{A}(., \succ)$ on the set of finite choice-sets $\mathcal{F}(\mathbb{R}^3)$. Which of the important choice-consistency conditions ($\alpha, \beta, \gamma, \eta$ or η^*) are satisfied by $\mathcal{A}(., \succ)$?.

iv) Determine the transitive hull of $\succ$.

2. (Subjective Probability)

a) State Savage's representation theorem, restricting your attention to acts with a finite number of consequences. State each axiom in a formally precise manner, and interpret its content intuitively.

b) Savage's theorem contains a representation of “qualitative probability" relations with particular structure; state this representation, and point out what additional structure is added.

c) Let $\mathbb{N}$ denote the set of natural numbers and Σ denote the family of all sets $A \subseteq \mathbb{N}$ such that A or its complement A^c is finite. Define a relation $\trianglerighteq_{fin}$ on the family of *finite* subsets of Σ that orders sets lexicographically according to their greatest elements: $A \equiv B$ iff $A = B$, and $A \triangleright B$ iff, for some $k \geq 0$, the k largest elements of A and B agree, and i) the $k+1$st element of A is larger than the $k+1$st element of B, or ii) if A has more than k and B has only k elements. For example, $\{5\} \triangleright \{4,3,2\}$, $\{5,4,3\} \triangleright \{5,4,2\}$, and $\{5,4,3\} \triangleright \{5,4\}$.

i) Show that the ordering $\trianglerighteq_{fin}$ can be extended to a qualitative probability $\trianglerighteq$ on all of Σ.

ii) Can the qualitative probability $\trianglerighteq$ be represented by a finitely additive probability measure? (Hint: to answer this question, it suffices to focus on the subrelation $\trianglerighteq_{fin}$)

Part 2

3. Consider a two-period finance ($t = 0, 1$) economy with I agents, S states of nature at date 1 and one good at each date/state. Agent $i (i = 1, \ldots, I)$ has preferences for risky income streams represented by a concave increasing utility function $u^i : \mathbb{R}_+^{S+1} \rightarrow \mathbb{R}$, and a stream of endowment $\omega^i \in \mathbb{R}_+^{S+1}$. There are J financial securities traded at date 0 with a payoff matrix $V \in \mathbb{R}_+^{SJ}$. In such an economy, if $\text{rank}(V) = S$, then we say that the financial markets are "complete".

(a) Argue that this definition is justified by the fact that agents can do (at a cost) any income transfers that they wish (or need) to do.

(b) Show that with complete markets a financial market equilibrium is Pareto optimal

(c) The following example shows that this terminology is justified only in the absence of restrictions on the portfolios. Suppose that $I = 2$, $S = 2$, the two agents have the same utility function

$$u^i(x_0^i, x_1^i, x_2^i) = x_0^i + \beta(\log(x_1^i) + \log(x_2^i)), \quad i = 1, 2$$

and endowments ω^i such that $\omega_0^i > 2\beta$, $\omega_1^1 > \omega_1^2$. There are two Arrow securities, security s ($s = 1, 2$) paying one unit of good in state s and nothing in the other state. The trade in security 1 is restricted to $z_1^i \geq -A$, with $A < \frac{\omega_1^1 - \omega_1^2}{2}$. Show that the agents cannot do the transfers that "they would like to do", and that the equilibrium is not Pareto optimal, so that actually the "markets are incomplete" despite the rank of V.

4. Consider a two-period ($t = 0, 1$) monetary economy $\mathcal{E}(u, \omega, M)$ with L goods at each period, I agents, and $S > 1$ possible states of nature at date 1. $u = (u^1, \ldots, u^I)$ denotes the vector of utility functions of the agents, $\omega = (\omega^i)_{i=1}^I \in \mathbb{R}_+^{L(S+1)I}$ denote the vector of endowments, and $M = (M_0, M_1, \ldots M_S)$ denotes the money supply at each date in each state. There is only one security traded at date 0, a nominal bond with price q which pays one unit of money in each state a date 1. Goods are traded on spot markets, and $p = (p_0, p_1, \ldots, p_S) \in \mathbb{R}_+^{(S+1)L}$ denotes the vector of spot prices at each date in each state. The price level in each state is determined by a quantity theory equation $p_s \sum_{i=1}^I \omega_s^i = M_s$, $s = 0, 1, \ldots, S$. We want to show that generically a change in monetary policy, which leaves the average growth of the money supply unchanged but changes its variability, has a real effect on the equilibrium allocation. Since the result does not depend on separability of utility functions we do not introduce the probabilities of the states explicitly and consider changes M to $\widetilde{M}$ such that $\sum_{s=1}^S M_s = \sum_{s=1}^S \widetilde{M}_s$ and $M_s \neq \widetilde{M}_s$ for some $s \geq 1$. Any other normalization of the date 1 money supply would give the same result.

(a) Let $(\bar{p}, \bar{x}; \bar{q}, \bar{z})$ be a spot-financial equilibrium of $\mathcal{E}(u, \omega, M)$, where $\bar{x} = (\bar{x}^I)_{i=1}^I \in \mathbb{R}_+^{L(S+1)I}$ describes the agents' consumption and $\bar{z} = (\bar{z}^I)_{i=1}^I \in \mathbb{R}^I$ their bond holdings . Show that if $(\tilde{p}, \bar{x}; \tilde{q}, \tilde{z})$ is an equilibrium of $\mathcal{E}(u, \omega, \widetilde{M})$ then $\tilde{p}_s$ must be collinear to $\bar{p}_s$ for $s = 0, 1, \ldots, S$.

(b) Deduce from (a) that if $(\bar{p}, \bar{x}; \bar{q}, \bar{z})$ is an equilibrium of $\mathcal{E}(u, \omega, M)$ such that $\bar{z} \neq 0$ then $\bar{x}$ can not be part of an equilibrium of $\mathcal{E}(u, \omega, \widetilde{M})$.

(c) Show that, generically in $(\omega, M) \in \mathbb{R}_{++}^{L(S+1)I} \times \mathbb{R}_{++}^{(S+1)}$, at an equilibrium $(\bar{p}, \bar{x}; \bar{q}, \bar{z})$, $\bar{z} \neq 0$. Given time constraint, restrict the proof to the case where $L = 1$.

(d) Accepting that the result of (c) is true for any number of goods, argue that we have proved the property that we wanted, and explain the intuition for the result.

Part 3

5. Selten (1975) defined the Trembling Hand Perfect Equilibrium of a finite extensive-form game with perfect recall to be the Trembling Hand Perfect Equilibrium of its associated agent-normal form game. As you recall, the agent-normal form is the normal-form in which for each information set of a player in the extensive-form there is a player (the "agent") in the agent-normal form. Every agent of an extensive-form player has the same payoffs as the player. Why did he use the agent-normal form game and not the associated strategic-form game?

a. Find an extensive-form game for which any Trembling Hand Perfect Equilibrium of its associated strategic-form game differs from any Trembling Hand Perfect Equilibrium of its associated agent-normal form game.

b. Find an extensive-form game for which there is a Trembling Hand Perfect Equilibrium of its associated strategic-form game that is not subgame perfect in the extensive-form game.

(Recall the definition: A Trembling Hand Perfect equilibrium of a finite strategic strategic game is a mixed strategy profile σ with the property that there exists a sequence $(\sigma^k)_{k=0}^{\infty}$ of completely mixed strategy profiles that converges to σ such that for each player i the strategy σ_i is a best response to σ_{-i}^k for all values of k.)

6. Here is something we never discussed in class. So we may want to learn about it in this exam: Okada (1981) was bothered by the fact that Selten's definition of Trembling Hand Perfection requires just **a** sequence $(\sigma^k)_{k=0}^{\infty}$. He strengthened the definition as follows: A *Strict* Trembling Hand Perfect equilibrium of a finite strategic strategic game is a mixed strategy profile σ with the property that **for all** sequences $(\sigma^k)_{k=0}^{\infty}$ of completely mixed strategy profiles that converge to σ such that for each player i the strategy σ_i is a best response to σ_{-i}^k for all values of k.

Does Strict Trembling Hand Perfect Equilibrium always exist? If yes, prove existence. If no, show a counter example.