

Delegations with Hidden Costs

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Abstract

Delegating future choices can involve various hidden costs for *planners* who need to select and motivate *doers* to execute these choices. Providing suitable incentives—such as advertising, direct monetary rewards, credible threats of litigation—should be costly for the planner. Even in the intrapersonal case where the planner and the doer are the same person, delegations may involve setting costly *commitments* or deadlines. When incentives and their costs are not observed directly, their presence can be still revealed through observable choice patterns. In general, the planner may exhibit a preference for commitment, and doers can violate the weak axiom of revealed preference when some of their incentives are hidden. In this paper, we accommodate such patterns via a delegation cost function h with a suitable mathematical structure that allows several distinct interpretations. First, the function h can be written in terms of a costly selection of doers with or without additional monetary rewards based on performance. Another equivalent interpretation for h assumes that the planner knows the doer’s preference, but finds it costly to *verify* his choices. Concerns about costly verifications are plausible in markets with adverse selection. Finally, our model can be identified even for *incomplete* datasets where ex post choices are observed only in some menus.

1 Introduction

When planning future choices, people can anticipate various costs that cannot be observed directly. In particular, some feasible alternatives can be costly to choose

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due to temptations (Gul and Pesendorfer [11], henceforth GP), contemplations (Ergin and Sarver [9]) and other cognitive reasons. To reduce such costs, planners can impose various *commitments* that constrain feasible choices (Strotz [19]). For example, people may keep only healthy foods and drinks at home, use self-exclusions from casino gambling, make promises and vows, set deadlines, etc (see the review of Bryan, Karlan, and Nelson [4]). Sunstein and Ulmann-Margalit [20] call such delegations *intrapersonal* because the planner is the same person as all future *doers* whose preferences can change over time.

In many other settings, choices must be delegated to doers who need not share the same physical identity with the planner or across themselves. Such delegations are called *interpersonal* as in Sunstein and Ulmann-Margalit.¹ In this case, the planner can select a doer from a relevant pool of available agents (e.g. employees on a payroll) and further influence the doer’s preference by advertising, persuasion, ethical codes, or direct monetary rewards for desirable actions. There is a vast literature that studies such monetary incentives in principal-agent problems (see Laffont and Martimort [13]).

Even if a planner has enough authority to order any feasible choice without additional incentives for a doer, it can be still costly—or maybe, impossible—to *verify* that such orders are accurately fulfilled. For example, an employer can make an employee sign a contract with a promise not to compete with the employer’s business after leaving. Yet such a promise is not verifiable in California courts. Fallick, Fleischman, and Rebitzer [10] provide empirical evidence that this non-verifiability enhances the mobility of talents between competing firms and stimulates agglomeration economies in Silicon Valley as opposed to other states where non-compete clauses can be enforced. More broadly, verifying any actions in courts can require evidence production, holding hearings, and other expenses. Several studies in the law literature (e.g. Choi and Triantis [7], Sanchiri and Triantis, [18]) model the effects of such costs on incomplete contracts.

When delegations costs are hidden and cannot be recognized directly, their presence can be still revealed through observable patterns both in planners’ and doers’ choices. To illustrate, consider a stylized example where the planner is a pharmaceutical company that produces two distinct drugs a and b for a particular diagnose and delegates prescription decisions to doctors. The company can have opaque pricing (e.g. because of complicated interactions with insurance policies) and also provide hidden incentives for doctors to prescribe a or b . However, the availability of drugs and prescription choices should be more readily observed. Let c and d be generic cheap versions of a and b that may be available in some markets. Then the rankings

$$\{a, b, d\} \succ \{a, b, c, d\} \quad \text{and} \quad \{a, b, c\} \succ \{a, b, c, d\} \quad (1)$$

¹Thaler and Shefrin [21] propose the terms *planners* and *doers* in their early model of commitments and self-control. We use this terminology in interpersonal settings as well.

are plausible for the planner and can be revealed through the efforts to delay the arrival of generic version c or d to the menus $\{a, b, d\}$ and $\{a, b, c\}$ respectively. Rankings (1) exhibit a preference for commitment, but do not conform to GP’s model of costly self-control because GP’s Set Betweenness is violated here.

Another plausible choice pattern is for doctors to prescribe

$$a \in \{a, b\} \quad \text{and} \quad b \in \{a, b, c\} \quad \text{and} \quad a \in \{a, b, c, d\}. \quad (2)$$

To motivate this pattern, assume that the planner’s delegation cost

- is high for a in the presence of its generic c , but low for a otherwise,
- is high for b in the presence of its generic d , but low for b otherwise.

Then (2) should hold if the planner moderately prefers to delegate a rather than b when delegations costs for a and b are similar. The pattern (2) violates the standard Weak Axiom of Revealed Preference (WARP), but also its weak version (weak WARP) proposed by Manzini and Mariotti [15]. Thus their sequential rationalizability model cannot capture some choice patterns that result from hidden delegation incentives.

Similarly, patterns (1)–(2) can arise when the planner finds it expensive (but not impossible) to verify a in the presence of c , or to verify b in the presence of d . For example, let a and b be two refurbished gadgets (e.g. smartphones) in perfect condition. Let c and d be the same kinds of phones in worse than perfect condition. Then the planner can reasonably expect to get c even if she orders a when c is also feasible in an online delivery service. In this case, she could buy a warranty or request a replacement, but such efforts are costly. Similarly, she can expect to get d if she orders b when d is also feasible. Then the rankings (1) are plausible because either menu allows to order a phone in perfect quality. The choices (2) also make sense for a planner who likes phone a slightly more than phone b . In this example, the motivation for (1)–(2) is reminiscent of Akerlof’s [2] classic *adverse selection* argument.

In this paper, we formulate and axiomatize a delegation model with a hidden cost function that captures both patterns (1) and (2). We also establish a general behavioral equivalence of several distinct delegation strategies that can result from selection of doers with or without monetary transfers, as well as costly verifications. Moreover, our results allow doers’ ex post choices to be incomplete so that observations are available only in some menus. This feature can make our model practical even for empirical datasets.

First, we take a finite consumption space X and model a planner’s preference \succeq over all menus $A \subset X$. Our main utility representation is

$$U(A) = \max_{x \in A} [u(x) - h(x, A)] \quad (3)$$

where u is the planner’s commitment utility and $h(x, A)$ is a *selective* cost function. The selective property for h agrees with several distinct delegation strategies that can be all captured by (3). Theorem 1 characterizes representation (3). Theorem 2 and 3 refine and identify the cost function h in (3).

Next, we translate representation (3) for a dataset Φ that consists of doers’ choices (x, A) such that x is feasible in A . The dataset can be incomplete and omit menus where choices are not observed. Our delegation model asserts

$$x = \arg \max_{x \in A} [u(x) - h(x, A)] \quad (4)$$

for all $(x, A) \in \Phi$. Theorem 4 characterizes this representation, and Theorem 5 simplifies this result for the case when Φ is complete.

Finally, we assert in Theorem 6 that any selective cost function h can be rewritten in several equivalent ways in terms of selection costs, monetary transfers, or verification costs. Accordingly, representations (3) and (4) can be combined with any of these interpretations.

Our findings can be related to several lines of research in the literature. GP’s [11, 12] models of *changing tastes* and *costly self-control* are special cases of representation (3) where the cost function $h(x, A)$ can be determined by a single function $t : X \rightarrow \mathbb{R}$ that represents the doer’s preference. In the case of changing tastes, $h(x, A) = 0$ if and only if x maximizes t in A . Otherwise, $h(x, A) = +\infty$. In the case of costly self-control, $h(x, A) = \max_{y \in A} t(y) - t(x)$. In contrast with our more general approach, either of GP’s models implies that ex post choices conform with utility maximization.

Another special case is the model of *informal commitments* proposed by Chandrasekher [5]. In his representation, $h(x, A) = 0$ if and only if x maximizes the doer’s utility t in $A \cap C$ for some informal commitment C that the planner can impose on the doer. The class of such commitments is derived endogenously. Chandrasekher’s result is obtained as corollary to our results in Theorem 7.

The ex post implications of our model are related to models of rationalization (Cherepanov, Feddersen, and Sandroni [6]), inattention (Masatlioglu, Nakajima, and Ozbay [16]), and their refinement by Lleras, Masatlioglu, Nakajima, and Ozbay [14]). In all of these models, each menu A has some unobserved consideration subset $\Psi(A) \subset A$ that can be constrained by rationalizations, inattention, and other cognitive issues. Our model relaxes the assumption that each choice must belong to some unobserved filter $\Psi(A) \subset A$. Instead, we postulate the existence of a cost function h that does not directly prohibit any elements in A from being chosen. Another distinct feature of our representation is its novel treatment of incomplete datasets.

2 Main Results

Consider the standard menu framework where choices are made sequentially at *ex ante* and *ex post* time periods.

Let $X = \{x, y, z \dots\}$ be a finite set of alternatives that may become feasible ex post. Let $\mathcal{M} = \{A, B, C \dots\}$ be the set of all *menus*—non-empty finite subsets of X . Interpret each menu $A \in \mathcal{M}$ as an action that, if taken ex ante makes the set $A \subset X$ feasible ex post. Singletons $\{x\}$ are written as x .

Let \mathcal{R} be the set of complete and transitive relations R on X . Such relations are called *weak orders*. For any $R \in \mathcal{R}$, let P be its asymmetric part.

A weak order $R \in \mathcal{R}$ is called *total* if for all $x, y \in X$, $xRyRx$ implies $x = y$. Let $\mathcal{T} \subset \mathcal{R}$ be the set of all total orders on X .

For any order $R \in \mathcal{R}$, function $u : X \rightarrow \mathbb{R}$, and menu $A \in \mathcal{M}$, let

$$u(A) = \max_{x \in A} u(x),$$

$$R(A) = \{x \in A : xRy \text{ for all } y \in A\}.$$

If R is total, then for each $A \in \mathcal{M}$, $R(A) \in X$ is a singleton. By convention, let $R(\emptyset) = \emptyset$ and $u(\emptyset) = -\infty$.

Consider a *planner*² with a preference \succeq over menus. Write its asymmetric and symmetric parts as \succ and \sim respectively.

Axiom 1 (Order). \succeq is complete and transitive.

Take any function $u \in \mathbb{R}^X$ that represents \succeq on X . Call u *commitment utility*.

Imagine that the planner must delegate ex post choices to *doers*—her future selves or other individuals. In general, delegations can involve various unobservable incentives. For example, the planner can motivate her future self by mental commitments, promises, cues etc. In the interpersonal case, doers can be stimulated by direct monetary transfers and/or persuaded by suitable information disclosures. Again, it can be problematic to observe such incentives directly. Therefore, we model delegation strategies with *hidden costs*.

Let $\mathcal{D} = \{(x, A) \in X \times \mathcal{M} : x \in A\}$ be the set of all pairs (x, A) where the option x is feasible in the menu A . For any $(x, A) \in \mathcal{D}$, interpret $h(x, A) \geq 0$ as the cost that the planner must incur to delegate x in A . Let $h(x, A) = +\infty$ when the doer is unwilling to choose x in A under any incentives that the planner can possibly provide. The costs $h(x, A)$ are hidden and hence, not taken as a primitive in our model. Instead, we use them to motivate axioms and representations for observable preferences.

Let \mathcal{H} be the set of all cost functions $h : \mathcal{D} \rightarrow [0, +\infty]$. Say that $h \in \mathcal{H}$ is *selective* if for all menus $A, B \in \mathcal{M}$ and alternatives $x \in A$ and $y \in X$,

²We model choices of both planners and doers. So the generic term *decision maker* would be confusing.

$$(H1) \quad h(x, x) = 0,$$

$$(H2) \quad h(x, A) \leq h(x, A \cup B),$$

$$(H3) \quad h(x, A) \geq \min\{h(x, y \cup A), h(y, y \cup A)\}.$$

Condition H1 normalizes delegation costs to zero in singleton menus. Monotonicity condition H2 is plausible because the delegation of x in A can adapt the same incentives as in $A \cup B$ and hence, should not cost more than $h(x, A \cup B)$. Turn to H3. The planner's cost $h(x, A)$ may combine a direct payment $p \geq 0$ to the doer with other expenses $q = h(x, A) - p$ that may be required for informational disclosures, advertising, selection efforts etc. Let the planner offer the same p for choosing either x or y in the menu $y \cup A$ and preserve all other aspects of her delegation strategy for A intact in $y \cup A$. This modified strategy should motivate the doer to choose either x or y in $y \cup A$ because all other feasible choices belong to A and hence, should be inferior to x in $y \cup A$. Thus the cost $h(x, A) = p + q$ should be sufficient to delegate one of the alternatives x or y in the menu $y \cup A$.

Assume that the planner evaluates any menu A by delegating a choice $x \in A$ that has an optimal combination of her commitment utility $u(x)$ and delegation cost $h(x, A)$. By H1 and H3, each menu A must contain some $x \in A$ such that $h(x, A) < +\infty$ and hence, the planner can always focus only on elements with bounded delegation costs. Assume that the aggregation of $u(x)$ and $h(x, A)$ is monotonic—strictly increasing in $u(x)$ and decreasing in $h(x, A)$ —but not necessarily additive. This assumption motivates several axioms for \succeq .

Axiom 2 (Positive Set-Betweenness (PSB)). *For all $A, B \in \mathcal{M}$,*

$$A \succeq B \quad \Rightarrow \quad A \succeq A \cup B.$$

Take any menus $A, B \in \mathcal{M}$. Let $x \in X$ be the planner's optimal delegation in $A \cup B$. Suppose that $x \in A$. By H2, $A \succeq A \cup B$ should hold because $h(x, A) \leq h(x, A \cup B)$. Similarly, if $x \in B$, then $B \succeq A \cup B$. PSB originally appears in Dekel, Lipman, and Rustichini's [8] model of cumulative temptations.

Axiom 3 (Dominance). *For all $y \in X$ and $A \in \mathcal{M}$,*

$$y \succeq x \quad \text{for all } x \in A \quad \Rightarrow \quad y \cup A \succeq A.$$

Take any $A \in \mathcal{M}$, and let $x \in A$ be the planner's optimal delegation in A . By H3, either x or y can be delegated in $y \cup A$ at a cost that does not exceed $h(x, A)$. If $y \succeq x$, then $y \cup A \succeq A$ should hold.

For any $A \in \mathcal{M}$, an element $x \in A$ is called *costly* in A if $x \succ x \cup A_x$ where

$$A_x = \{y \in A : x \succ y\}.$$

Indeed, the ranking $x \succ x \cup A_x$ implies that it should be costly to delegate x in $x \cup A_x$ and a fortiori, in A .

Axiom 4 (Reduction). *For any $A \in \mathcal{M}$ and $x, y \in X$, if x and y are both costly in $y \cup A$, then x is costly in A .*

If $y \succeq x$, then Reduction is trivial because $A_x = (y \cup A)_x$. Let $x \succ y$. Then $(y \cup A)_x = y \cup A_x$ and $A_y \subset A_x$. As y is costly in $y \cup A$, then $h(y, y \cup A_y) > 0$ and by H2, $h(y, x \cup y \cup A_x) > 0$. As x is costly in $y \cup A$, then $h(x, x \cup y \cup A_x) > 0$. By H3, $h(x, x \cup A_x) > 0$. Thus the ranking $x \succ x \cup A_x$ should hold because x cannot be delegated for free in the menu $x \cup A_x$, and any other feasible option $y \neq x$ in $x \cup A_x$ is strictly worse than x for the planner. Our Reduction axiom resembles Aizerman's property for choice functions (see Moulin [17]).

Say that $U : \mathcal{M} \rightarrow \mathbb{R}$ aggregates a cost function $h \in \mathcal{H}$ if for all $A \in \mathcal{M}$,

$$U(A) = \max_{x \in A} [u(x) - h(x, A)] \quad (5)$$

where $u(x) = U(x)$ for all $x \in X$. Say also that U is an aggregation of h , and h is aggregated by U . If $h \in \mathcal{H}$ is selective, then the aggregation formula (5) is well-defined for any $u \in \mathbb{R}^X$. In this case, $U(x) = u(x)$ because $h(x, x) = 0$, and each value $U(A) \geq \min_{x \in A} u(x)$ is bounded because by H1 and H3, there is $x \in A$ such that $h(x, A) = 0$.

Theorem 1. *\succeq satisfies Axioms 1–4 if and only if \succeq has a utility representation $U : \mathcal{M} \rightarrow \mathbb{R}$ that aggregates some selective function $h \in \mathcal{H}$.*

The proofs are in the appendix.

The planner as portrayed by (5) evaluates each menu $A \in \mathcal{M}$ via an alternative $x \in A$ that maximizes the difference between the commitment utility $u(x)$ and the hidden delegation cost $h(x, A)$. Here the cost function $h \in \mathcal{H}$ is selective, and the aggregation $u(x) - h(x, A)$ is additive. Note that Axioms 1–4 should still hold if $u(x)$ and $h(x, A)$ are aggregated via any function that is strictly increasing (decreasing) with respect to the first (second) variable. Thus the additive aggregation can be imposed without any loss of generality in (5). Our model has several other equivalent forms where the cost function $h \in \mathcal{H}$ is decomposed into more basic components.³

Given any domain Y , a non-negative function $f : Y \rightarrow \mathbb{R}_+$ is called *grounded* if its minimal value on Y is zero.

Then U aggregates some selective $h \in \mathcal{H}$ if and only if there is a set $\Theta \subset \mathcal{R}$ and a grounded function $f : \Theta \rightarrow \mathbb{R}_+$ such that

$$U(A) = \max_{R \in \Theta, x \in R(A)} [u(x) - f(R)] \quad \text{for all } A \in \mathcal{M}. \quad (6)$$

This equivalence follows from Theorem 6 below. Representation (6) portrays a planner who can delegate a choice $x \in A$ in a menu A by *selecting* a doer with a

³Formally, all of these equivalences follow from Theorems 6–?? below. To speed up the exposition of our main findings, we postpone the full statement of these auxiliary results.

ranking $R \in \Theta$ such that $x \in R(A)$ maximizes R in A . In this interpretation, the doer does not receive any transfers that are contingent on his choice rather than his identity.

Next, U aggregates some selective $h \in \mathcal{H}$ if and only if there is a finite set $\Gamma \subset \mathbb{R}^X$ and a grounded function $g : \Gamma \rightarrow \mathbb{R}_+$ such that

$$U(A) = \max_{t \in \Gamma, x \in A} [u(x) - g(t) - (t(A) - t(x))] \quad (7)$$

In contrast with (6), representation (7) recognizes the possibility of transfers that are based on ex post choices. Delegations proceed in two steps here. First, the planner incurs the cost $g(t)$ to select a baseline utility function $t \in \mathbb{R}^X$ for the doer. Second, she transfers $t(A) - t(x)$ to incentivize the choice of x in A . If the reward for any other choice $y \in A \setminus x$ is normalized to zero, then $t(A) - t(x)$ is the smallest compensation that is sufficient for the doer to select x rather than y such that $t(y) = t(A)$. Here all doers are assumed to have quasi-linear preferences with respect to monetary transfers from the planner.

In some applications, the domains Θ and Γ can be much smaller than \mathcal{D} and hence, functional forms (6)–(9) can be more parsimonious than (5). In particular, if $\Theta = \{R\}$ and $\Gamma = \{t\}$ are singletons, then representations (6) and (7) translate for all $A \in \mathcal{M}$ into

$$U(A) = u(R(A)) \quad (8a)$$

$$U(A) = \max_{x \in A} [u(x) - (t(A) - t(x))] \quad (8b)$$

because both f and g are grounded and hence, $f(R) = g(t) = 0$. These special cases are respectively the models of *changing tastes* and *costly self-control* proposed by Gul and Pesendorfer [11, 12] (GP). In the finite menu framework, GP characterize (8a) via two conditions, *Order* and *No Compromise*: for all $A, B \in \mathcal{M}$, either $A \sim A \cup B$ or $B \sim A \cup B$ must hold. Representation (8b) is also axiomatized by GP, but only for preferences over menus of lotteries. We do not attempt to characterize (8b) in the finite framework.⁴

2.1 Costly Verifications

Our delegation model has another equivalent form: U aggregates some selective $h \in \mathcal{H}$ if and only if there is a weak order $R^d \in \mathcal{R}$, a collection of menus $\Pi \subset \mathcal{M}$,

⁴GP's Independence Axiom does not apply without mixtures. In our model, Γ need not be a singleton even if \succeq satisfies *Set Betweenness*: for all $A, B \in \mathcal{M}$,

$$A \succeq B \quad \Rightarrow \quad A \succeq A \cup B \succeq B.$$

However, this condition does not guarantee that Γ is a singleton in our model. For example, if $X = \{a, b, c\}$, then the ranking $a \sim ab \succ b \sim abc \sim bc \succ ac \sim c$ satisfies Set Betweenness and Axioms 1–4, but cannot be represented by (7) with $\Gamma = \{t\}$. Indeed, the rankings $a \sim ab \succ b$ and $b \sim bc \succ c$ imply that $t(a) \geq t(b) \geq t(c)$, but $a \succ ac$ requires $t(c) > t(a)$.

and a function $v : \Pi \rightarrow \mathbb{R}_+$ such that $X \in \Pi$, $v(X) = 0$, and

$$U(A) = \max_{C \in \Pi, x \in R^d(A \cap C)} [u(x) - v(C)] \quad \text{for all } A \in \mathcal{M}. \quad (9)$$

This equivalence follows from Theorem ?? below. Representation (9) can be interpreted in terms of *costly verifications*. Imagine that the planner can request any choice $x \in A$ in the feasible menu A without any additional payments (e.g. via a threat of litigation or other credible penalties). However, it can be still costly to verify whether the doer accurately fulfils this request. The planner as portrayed by (9) can pick any set $C \in \Pi$ and then run a test at a cost $v(C)$ to determine whether the doer's choice in the feasible menu A belongs to C as well. If so, the doer's choice in $A \cap C$ is presumed to maximize his ranking R^d , which is known to the planner and is preserved across all feasible menus.

This interpretation motivates another property for the cost function h . Say that $h : \mathcal{M} \rightarrow \mathbb{R}$ is *subadditive* if for all $A, B \in \mathcal{M}$ and $x \in A \cap B$,

$$h(x, A \cup B) \leq h(x, A) + h(x, B).$$

This condition is plausible for verification costs. Assume that $h(x, A)$ and $h(x, B)$ are the costs of testing that $x \in C$ and $x \in C'$. Then the two tests and their costs can be combined in $A \cup B$ to verify that $x \in C \cap C'$. Obviously, if x maximizes R^d in $A \cap C$ and $B \cap C'$, then it also maximizes R^d in $(A \cup B) \cap (C \cap C')$.

Axiom 5 (Costless Subadditivity (CS)). *For all $A, B \in \mathcal{M}$ and $x \in A \cap B$,*

$$A \succ A \setminus x \quad \text{and} \quad B \succeq x \succ B \setminus x \quad \Rightarrow \quad A \cup B \succeq A.$$

Here the ranking $A \succ A \setminus x$ implies that x should be delegated in A . Otherwise, $A \succeq A \setminus x$ should hold. Similarly, $B \succ B \setminus x$ implies that x should be delegated in B as well. Moreover, $B \succeq x$ implies that $h(x, B) = 0$. By subadditivity, $h(x, A \cup B) = h(x, A)$. Thus $A \cup B \succeq A$ should hold because x can be delegated in $A \cup B$ at the same cost as in A .

Theorem 2. \succeq *satisfies Axioms 1–5 if and only if \succeq has a utility representation $U : \mathcal{M} \rightarrow \mathbb{R}$ that aggregates some selective, subadditive function $h \in \mathcal{H}$.*

This refinement imposes subadditivity on the delegation cost function $h \in \mathcal{H}$. The corresponding function v satisfies the dual condition $v(C \cap C') \leq v(C) + v(C')$ for all $C, C' \in \Pi$. See Theorem ?? below for details.

Note that Axioms 1–5 are logically independent. To illustrate, let $X = \{a, b, c\}$. Write menus in this three-element X without curly brackets and commas. For instance, a menu $\{a, b\}$ is abbreviated as ab .

- (1) The empty relation \succeq obeys Axioms 1–5, except for Order.
- (2) $U(A) = |A|$ represents \succeq that obeys Axioms 1–5, except for PSB.

- (3) $U(A) = -|A|$ represents \succeq that obeys Axioms 1–5, except for Dominance.
- (4) Preference $a \sim ac \succ ab \sim abc \succ b \succ bc \succ c$ obeys Axioms 1–5, except for Reduction. Indeed, $ac \succ c$ and $a \sim ab \succ b$, but $ac \succ abc$.
- (5) Preference $a \sim ac \succ ab \succ b \succ abc \succ bc \succ c$ obeys Axioms 1–4, but violates CM. Indeed, $ac \succ c$ and $a \sim ab \succ b$, but $ac \succ abc$.

2.2 Identifications of Hidden Costs

In the finite menu framework, \succeq does not determine the cost function h uniquely. However, we provide explicit formulas that derive h from \succeq without uniqueness guarantees. These formulas are an important part of our proofs, but can also serve to generate the required representations in practical examples.

A positive function $U : \mathcal{M} \rightarrow \mathbb{R}_{++}$ is called *regular* if for all $x \in X$ and $A \in \mathcal{M}$,

$$u(x) > U(A) \quad \Leftrightarrow \quad U(x) \geq 2U(A).$$

As \mathcal{M} is finite, then any complete and transitive \succeq has a regular utility representation. In particular, one can take $U(A) = 2^{V(A)}$ where $V : \mathcal{M} \rightarrow \mathbb{N}$ has natural values and represents \succeq .

Let \succeq be represented by a regular function $U : \mathcal{M} \rightarrow \mathbb{R}$. For all $(x, A) \in \mathcal{D}$, define two functions $e, e^* \in \mathcal{H}$ via

$$e(x, A) = \max_{B \in \mathcal{M}: x \in B \subset A} [u(x) - U(B)] \quad (10)$$

$$e^*(x, A) = \max_{B \in \mathcal{M}: x \in B \subset A \cup E^*(x)} [u(x) - U(B)] \quad (11)$$

where $E^*(x) = \{y \in X : x \sim \{x, y\} \succ y\}$.

Theorem 3. *If \succeq satisfies Axioms 1–4 and U is regular, then $e \in \mathcal{H}$ is selective and aggregated by U . If \succeq satisfies Axioms 1–5 and U is regular, then $e^* \in \mathcal{H}$ is selective, subadditive, and aggregated by U .*

This result provides the main steps in the proofs of both Theorems 1–2 and delivers the required representations (5) from the suitable lists of axioms. If $E^*(x) = \emptyset$, then identifications (10) and (11) coincide.

To illustrate, consider a preference \succeq with a regular utility function U written underneath the menus.

$$\begin{array}{cccccccc}
 c \prec & bc \prec & abc \prec & b \prec & ac \prec & ab \sim & a \\
 U & 1 & 2 & 3 & 6 & 7 & 14 & 14.
 \end{array}$$

Check that U aggregates the selective cost function

$$\begin{array}{cccccccccc}
 & (a, ab) & (b, ab) & (a, ac) & (c, ac) & (b, bc) & (c, bc) & (a, abc) & (b, abc) & (c, abc) \\
 e & 0 & 0 & 7 & 0 & 4 & 0 & 11 & 4 & 0.
 \end{array}$$

Accordingly, \succeq satisfies Axioms 1–4.

By contrast, \succeq violates CM because $ac \succ c$ and $a \sim ab \succ b$, but $ac \succ abc$. Thus e cannot be subadditive. Indeed, $e(a, abc) > e(a, ab) + e(a, ac)$. Note also that formula (10) need not work if U is not regular. In the above example, replace U with U' such that $U'(ab) = U'(a) = 8$, and $U'(A) = U(A)$ for all other A . Then U' represents \succeq , but it cannot be written in the required form (5). The formula (10) here delivers e' that violates H3 because $e'(a, ac) = 1 < 4 = e'(b, abc)$, and $e'(a, ac) < 5 = e'(a, abc)$.

To illustrate the subadditive case, consider another preference \succeq with a regular utility function U written underneath the menus.

$$\begin{array}{cccccccc}
 c < & bc < & abc < & b < & ac < & ab < & a \\
 U & 1 & 2 & 3 & 6 & 7 & 8 & 16.
 \end{array}$$

Check that U aggregates the selective, subadditive function

$$\begin{array}{cccccccccc}
 & (a, ab) & (b, ab) & (a, ac) & (c, ac) & (b, bc) & (c, bc) & (a, abc) & (b, abc) & (c, abc) \\
 e^* = e & 8 & 0 & 9 & 0 & 4 & 0 & 13 & 4 & 0.
 \end{array}$$

Accordingly, \succeq satisfies Axioms 1–5 here.

3 Discussion

The above axioms and identifications are formulated in terms of a complete preference relation \succeq over menus. However, it appears more common in applications to focus on ex post choices in menus, such as budget sets or investment portfolios. Moreover, empirical observations (such as household choices) are usually available only for some menus in $A \in \mathcal{M}$, but missing for many others. Our model can be restated in terms of such incomplete choice data as follows.

Let $\Phi \subset \mathcal{D}$ be a *dataset* that consists of all pairs $(x, A) \in \mathcal{D}$ such that x is an observed choice in a menu A . All menus where choices are not observed are omitted from the dataset Φ . For simplicity, let Φ be *single-valued* so that $(x, A), (y, A) \in \Phi$ should imply $x = y$. Multi-valued datasets allow extensions for our delegation model, but make them more complicated with limited additional insights.

Suppose that there is a utility index $u \in \mathbb{R}^X$ and a selective cost function $h \in \mathcal{H}$ such that for all $(z, A) \in \Phi$,

$$z = \arg \max_{x \in A} [u(x) - h(x, A)]. \quad (12)$$

Then the dataset Φ is compatible with the existence of some planner who delegates a choice x in A to maximize the difference between her commitment utility $u(x)$ and hidden delegation costs $h(x, A)$.

Adapt the classic notion of revealed preferences. Write $x \succ_A y$ if $(x, A) \in \Phi$ and $y \in A \setminus x$ so that x is chosen in a menu where $y \neq x$ is feasible, but not chosen. Say that x is *revealed preferred* to y in the menu $A \in \mathcal{M}$.

The classic Weak Axiom of Revealed Preference (WARP) prohibits reversals

$$x \succ_A y \succ_B x \tag{13}$$

for any alternatives $x, y \in X$ and menus $A, B \in \mathcal{M}$. By contrast, representation (12) allows such reversal under suitable conditions on the commitment utility u . More precisely, the combination of (12) and (13) implies that either xPz for some $B \setminus A$ or yPz for some $z \in A \setminus B$. This claim is Lemma A.4 in the appendix.

Accordingly, a total order $R \in \mathcal{T}$ is called *principal* for the dataset Φ if for any reversal $x \succ_A y \succ_B x$, either xPz for some $z \in B \setminus A$ or yPz for some $z \in A \setminus B$.

Theorem 4. *A dataset Φ has a principal order $R \in \mathcal{T}$ if and only if Φ is represented by (12) for some $u \in \mathbb{R}^X$ and selective cost function $h \in \mathcal{H}$. Without loss in generality, u represents R , and h is subadditive.*

This result fits representation (12) into a dataset Φ in two steps. First, one needs to find a principal order $R \in \mathcal{T}$ that is consistent with all preference reversals (13). If this task is doable, then it is guaranteed that there is a commitment utility index $u \in \mathbb{R}^X$ and a selective cost function $h \in \mathcal{H}$ that satisfies (12). Moreover, h is subadditive without loss in generality here. Roughly speaking, this freedom is possible because it cannot be observed ex post which delegations have zero costs to the planner. Therefore, subadditivity does not imply any counterpart of CM for ex post choice patterns.

For example, let $X = \{a, b, c, d\}$ and consider a dataset

$$\Phi = \{(a, abcd), (b, abc), (a, ab)\}$$

that violates the Weak WARP of Manzini and Mariotti [15]. This dataset exhibits two preference reversals:

$$a \succ_{abcd} b \succ_{abc} a \quad \text{and} \quad b \succ_{abc} a \succ_{ab} b.$$

Accordingly, any $R \in \mathcal{T}$ that satisfies aPd and bPc is a principal order and allows representation (12). In particular, (12) holds if for $u(a) = 40 > u(b) = 30 > u(c) = 10 > u(d) = 0$ and

$$h(x, A) = \begin{cases} 15 & \text{if } x = a \text{ and } c \in A \\ 15 & \text{if } x = b \text{ and } d \in A \\ 0 & \text{otherwise.} \end{cases}$$

This function h is both selective and subadditive.

Given a principal order $R \in \mathcal{T}$, the fitting of u and h in representation (12) in our proofs requires finding a Richter-Peleg utility function that is consistent with a suitable acyclic pair of binary relations. This task can be achieved by standard algorithms, such as Tarjan's. By contrast, the definition of a principal order can appear problematic because it does not suggest any clear algorithm that will find R or reject its existence. Indeed, the definition produced a conjunctive formula, and the consistency of such formulas is a well-known NP problem.

To alleviate this concern, we provide a more direct criterion for the existence of principal orders when the dataset Φ is observed in all menus $A \in \mathcal{M}$. Say that Φ is *complete* if for any $A \in \mathcal{M}$, there is $(x, A) \in \Phi$ for some $x \in X$. If Φ is complete, then any weakly principal order R is principal.

Consider a simple preference reversal where menus $A, B \in \mathcal{M}$ and distinct elements $x, y, z \in X$ are such that

$$(x, A) \in \Phi \quad \text{and} \quad (y, A \cup z) \in \Phi. \quad (14)$$

Then (12) has a deterministic implication $u(x) > u(z)$. In this case, write $x \succ_p z$ and call the incomplete relation \succ_p the *incremental revealed preference*.

Theorem 5. *For any complete dataset Φ , $R \in \mathcal{T}$ is a principal order if and only if R extends the incremental revealed preference \succ_p so that for all $x, y \in X$,*

$$x \succ_p y \quad \Rightarrow \quad xPy. \quad (15)$$

Moreover, a complete dataset Φ can be represented by (12) if and only if \succ_p is acyclic.

The second part of this theorem follows from the first in combination with the standard extension results. In the complete case, the construction of a principal order is also reduced to the standard problem of extending an acyclic order to a total one. We provide another example in Section 3.1 below.

The incremental revealed preference \succ_p can be related to several other definitions that are based on choice reversals (14). Such definitions appear in models of rationalization (Cherepanov, Feddersen, and Sandroni [6]), inattention (Masatlioglu, Nakajima, and Ozbay [16]), and their refinements (e.g Lleras, Masatlioglu, Nakajima, and Ozbay [14]). In all of these models, each menu $A \in \mathcal{M}$ has some unobserved consideration subset $\Psi(A) \subset A$ that can be constrained by rationalizations, inattention, and other cognitive issues. Depending on the interpretation of Ψ , such models require that any reversal (14) implies that

- $x \succ_r z$ because the presence of y can make the choice of x violate some rationality principles, as in Cherepanov et al.,
- $z \succ_q y$ because x is not even noticed in the menu $y \cup A$, but both y and z are revealed to be in the attention filter $\Psi(y \cup A)$, as in Masatlioglu et al.,

- $x \succ_s z \succ_s y$ when rationalization principles are assumed to be transitive (Au and Kawai [3]) or the attention filter Ψ is taken to be path-independent, as in Lleras et al.

Clearly, the principal preference \succ_p is a subrelation of \succ_s , but the other rankings \succ_r and \succ_q need not include or belong to \succ_p .

Our definition of \succ_p relaxes the assumption that each choice must belong to some unobserved filter $\Psi(A) \subset A$. Instead, we postulate the existence of a selective cost function $h(\cdot)$ that does not directly prohibit any elements in A from being chosen. Another distinct feature of our model is the novel extensions for incomplete datasets.

3.1 More Identifications

Any selective cost function $h \in \mathcal{H}$ can be translated into several equivalent forms. These translations can be applied to the corresponding representations for preferences over menus and for datasets.

Consider three cost functions such that for all $(x, A) \in \mathcal{D}$,

$$h_1(x, A) = \min_{R \in \Theta, x \in R(A)} f(R) \quad (16a)$$

$$h_2(x, A) = \min_{t \in \Gamma} [g(t) + t(A) - t(x)] \quad (16b)$$

$$h_3(x, A) = \min_{C \in \Pi, x \in R^d(A \cap C)} v(C) \quad (16c)$$

where the functions $f : \Theta \rightarrow \mathbb{R}_+$, $g : \Gamma \rightarrow \mathbb{R}_+$, and $v : \Pi \rightarrow \mathbb{R}_+$ are grounded on their domains $\Theta \subset \mathcal{R}$, $\Gamma \subset \mathbb{R}^X$, and $\Pi \subset \mathcal{M}$ respectively. Moreover, the sets Θ , Γ , and Π are all finite, $R^d \in \mathcal{R}$ is a weak order, and $v(X) = 0$. It is routine to check that all functions h_1, h_2, h_3 are selective.⁵

The selective functions $h_1, h_2, h_3 \in \mathcal{H}$ reflect several distinct delegation strategies. First, h_1 is the minimal expense $f(R)$ of selecting a doer's order R such that x maximizes R in A . Second, h_2 is the minimal combination of the expense $g(t)$ of selecting a doer's utility index t with the performance-based incentive $t(A) - t(x)$ that stimulates the doer to choose x in A . Third, h_3 is the minimal cost of verification that the doer's choice belongs to a set $C \in \Pi$ such that x is maximal in $A \cap C$ for the doer's order R^d .

Note that h_3 must be also subadditive if for all $C, C' \in \Pi$ such that $C \cap C' \neq \emptyset$,

$$C \cap C' \in \Pi \quad \text{and} \quad v(C \cap C') \leq v(C) + v(C'). \quad (17)$$

⁵H1 and H2 are immediate. H3 holds because for all $A \in \mathcal{M}$, $x \in A$, and $y \in X$,

- (a) if $R \in \mathcal{R}$ and $x \in R(A)$, then $x \in R(y \cup A)$ or $y \in R(x \cup A)$,
- (b) if $t \in \mathbb{R}^X$, then $t(y \cup A) = t(A)$ or $t(y \cup A) = t(y)$
- (c) if $C \in \mathcal{M}$ and $x \in R^d(A \cap C)$, then either x or y belong to $R^d(C \cap (y \cup A))$.

This property reflects the intuition that any two verification tests $C, C' \in \Pi$ can be used together.

Say that $h \in \mathcal{H}$ is *bounded* if there is $\alpha > 0$ such that $h(x, A) < \alpha$ for all $(x, A) \in \mathcal{D}$. If h is bounded, then for each $R \in \mathcal{T}$, fix a representation $t_R : X \rightarrow \mathbb{R}$ such that $t_R(x) - t_R(y) > \alpha$ for all $x, y \in X$ such that xPy .

Say that $R \in \mathcal{R}$ is a *free order* for a cost function $h \in \mathcal{H}$ if $h(x, A) = 0$ for all $A \in \mathcal{M}$ and $x \in R(A)$.

By convention, let $+\infty$ be the minimal value of a function on an empty set.

Theorem 6. *Given any selective function $h \in \mathcal{H}$,*

(i) *h satisfies (16a) where $\Theta = \mathcal{T}$ and*

$$f(R) = \max_{A \in \mathcal{M}} h(R(A), A) \quad \text{for all } R \in \mathcal{T}, \quad (18a)$$

(ii) *if h is bounded, then h satisfies (16b) where $\Gamma = \{t_R : R \in \mathcal{T}\}$ and*

$$g(t_R) = f(R) \quad \text{for all } R \in \mathcal{T}, \quad (18b)$$

(iii) *h satisfies (16c) where $\Pi = \mathcal{M}$, R^d is any free order for h , and*

$$v(C) = \max_{A \in \mathcal{M}, x \in R^d(A \cap C)} h(x, A) \quad \text{for all } C \in \mathcal{M}. \quad (18c)$$

If h is subadditive, then v satisfies (17) as well.

This result identifies components in (16a)–(16c) via (18a)–(18c) respectively. Subadditivity guarantees the dual condition (17) for v . None of these identifications achieve uniqueness.⁶

Our identification results—Theorem 3 and Theorem 6—can be combined to obtain utility representations (6)–(9) where U is regular, $\Theta = \mathcal{T}$, $\Gamma = \{t_R : R \in \mathcal{T}\}$, $\Pi = \mathcal{M}$, R^d is represented by $-u$,

$$f(R) = g(t_R) = \max_{B \in \mathcal{M}} [u(R(B)) - U(B)] \quad \text{for all } R \in \mathcal{T} \quad (19)$$

$$v(C) = \max_{B \in \mathcal{M}} [u(R^d(B \cap C)) - U(B)] \quad \text{for all } C \in \mathcal{M}. \quad (20)$$

Here $-u$ provides an easy representation for a free order R^d , but (20) can be applied to any other order $R \in \mathcal{R}$ such that $A \succeq x$ for all $A \in \mathcal{M}$ and $x \in R(A)$. This condition implies that R is free for the cost function e in Theorem 3.

⁶In particular, the domain $\Theta = \mathcal{T}$ consists only of total orders, but in some applications it can be convenient to use (16a) where weak orders $R \in \Theta$ are allowed. For example, if X consists of lotteries, then risk neutral preferences need not be total on X .

3.2 Informal Commitments

The model of *informal commitments* proposed by Chandrasekher [5] is a special case of the utility representation (9) where the cost function $v = 0$ is zero on the entire Π . Accordingly, the preference \succeq is represented for all $A \in \mathcal{M}$ by

$$U(A) = \max_{C \in \Pi, x \in R^d(A \cap C)} u(x) \quad (21)$$

for some domain $\Pi \subset \mathcal{M}$ and weak order $R^d \in \mathcal{R}$.

In Chandrasekher's interpretation, the planner can freely impose any set $C \in \Pi$ such that $A \cap C \neq \emptyset$ as an informal (hidden) commitment on the doer's choice in a menu A .

Axiom 6 (Costless Verifications (CV)). *For all $A, B \in \mathcal{M}$ and $x \in A$,*

$$x \succ A \quad \text{and} \quad A \subset B \quad \Rightarrow \quad B \sim B \setminus x.$$

Take any $A, B \in \mathcal{M}$ and $x \in X$ such that $x \succ A$ and $A \subset B$. Then x cannot maximize the doer's order R^d in $A \cap C$ for any $C \in \Pi$. Otherwise, x could be delegated (verified) for free in A and $A \succeq x$ should hold. Thus the maximizers of R^d should be the same in $B \cap C$ and in $(B \setminus x) \cap C$. If the planner can delegate only such maximizers, then the indifference $B \sim B \setminus x$ should hold.

Theorem 7. *\succeq satisfies Order, PSB, CV if and only if there is $u \in \mathbb{R}^X$, $\Pi \subset \mathcal{M}$, and $R \in \mathcal{T}$ such that \succeq is represented by (21).*

Moreover, \succeq satisfies Order, PSB, CV, CM if and only if \succeq is represented by (21) where Π is closed under intersections.

The first part corresponds to Chandrasekher's main Theorem 1 and follows as a quick corollary to our results. PSB and CV are arguably more transparent than Chandrasekher's counterparts. The refinement where Π is closed under intersections is novel. It reflects the case when informal commitments can be freely combined by the planner.

The ex post choices in this model can be characterized via the result in Lleras, Masatalioglu, Nakajima, and Ozbay [14].

A APPENDIX: PROOFS

We start with some preliminary findings.

Take any function $U : \mathcal{M} \rightarrow \mathbb{R}$. Let $u : X \rightarrow \mathbb{R}$ be the restriction of U to X .

For any $(x, A) \in \mathcal{D}$, let $\mathcal{M}(x, A) = \{B \in \mathcal{M} : x \in B \subset A\}$. Accordingly,

$$e(x, A) = \max_{B \in \mathcal{M}(x, A)} [u(x) - U(B)]. \quad (22)$$

By definition, e satisfies H1 and H2, but in general, it can violate H3.

Let $E(x, A) \in \mathcal{M}(x, A)$ be a menu that has the smallest size among all maximizers in (22). Then

$$e(x, A) = u(x) - U(E(x, A)) \quad \text{and} \quad |E(x, A)| \leq |B| \quad (23)$$

for all $B \in \mathcal{M}(x, A)$ such that $e(x, A) = u(x) - U(B)$.

Let \succeq be the preference that is represented by U on \mathcal{M} . Order is implied.

Lemma A.1. *If \succeq satisfies PSB, then U aggregates e . If U aggregates some $h \in \mathcal{H}$ that satisfies H1–H2, then \succeq satisfies PSB, and $e(x, A) \leq h(x, A)$ for all $(x, A) \in \mathcal{D}$.*

Proof. Assume PSB. Take any $A \in \mathcal{M}$. Then

$$U(A) \geq \max_{x \in A} [u(x) - e(x, A)] \quad (24)$$

because for all $x \in A$, $e(x, A) \geq u(x) - U(A)$ by (22) with $B = A$. Suppose that (24) holds strictly. Then for all $x \in A$,

$$U(A) > u(x) - e(x, A) = u(x) - (u(x) - U(E(x, A))) = U(E(x, A)).$$

By PSB, $A \succ \bigcup_{x \in A} E(x, A) = A$. By contradiction, (24) holds as equality. Thus U aggregates e .

Conversely, suppose that U aggregates some $h \in \mathcal{H}$ that satisfies H1–H2. Take any $A, B \in \mathcal{M}$. As U aggregates h , then $U(A \cup B) = u(z) - h(z, A \cup B)$ for some $z \in A \cup B$. If $z \in A$, then by H2,

$$U(A) \geq u(z) - h(z, A) \geq u(z) - h(z, A \cup B) = U(A \cup B).$$

Similarly, if $z \in B$ then $U(B) \geq U(A \cup B)$. Thus PSB holds. Take any $x \in A$. Then for all $C \in \mathcal{M}(x, A)$,

$$U(C) \geq u(x) - h(x, C) \geq u(x) - h(x, A)$$

and hence, $e(x, A) \leq h(x, A)$. □

Lemma A.1 characterizes all preferences \succeq that can be represented by an aggregation of some monotonic cost function $h \in \mathcal{H}$. Moreover, the endogenous e is the minimal cost function that allows such aggregation for the given U .

Lemma A.2. *If \succeq satisfies Axioms 1–3, then for any $(x, A) \in \mathcal{D}$ and $B \in \mathcal{M}$,*

$$A_x \supset E(x, A) \setminus x \quad (25)$$

$$x \in B \text{ and } B_x = A_x \quad \Rightarrow \quad e(x, A) = e(x, B) \quad (26)$$

$$x \text{ is costly in } A \quad \Leftrightarrow \quad e(x, A) > 0. \quad (27)$$

Proof. Take any $(x, A) \in \mathcal{D}$. Recall that $A_x = \{y \in A : x \succ y\}$.

Take any $y \in E(x, A)$. Let y maximize u in $E(x, A)$. Suppose that $y \neq x$. Let $C = E(x, A) \setminus y$. As $C \in \mathcal{M}(x, A)$ is smaller than $E(x, A)$, then by (23),

$$u(x) - U(C) < e(x, A) = u(x) - U(E(x, A)).$$

Thus $C \succ E(x, A)$, which violates Dominance. Therefore, $y = x$ is the only maximizer of u in $E(x, A)$, that is, $A_x \supset E(x, A) \setminus x$.

Take any $B \in \mathcal{M}$ such that $x \in B$ and $B_x = A_x$. By (25), $E(x, B) \subset A$ and $E(x, A) \subset B$. Thus $e(x, A) = e(x, B)$.

Let x be costly in A , that is, $x \succ x \cup A_x$. Then

$$e(x, A) \geq u(x) - U(x \cup A_x) > 0.$$

Conversely, let $e(x, A) > 0$. Then $x \succ E(x, A)$. By PSB,

$$x \succ E(x, A) \cup \bigcup_{y \in A_x} y = E(x, A) \cup A_x.$$

By (25), $E(x, A) \cup A_x = x \cup A_x$. By definition, x is costly in A . □

A.1 Proof of Theorem 3

Suppose that \succeq satisfies Axioms 1–4, and is represented by a regular function $U : \mathcal{M} \rightarrow \mathbb{R}_{++}$. By definition, for all $x \in X$ and $A \in \mathcal{M}$,

$$x \succ A \quad \Rightarrow \quad U(x) \geq 2U(A). \quad (28)$$

By Lemma A.1, U aggregates e . Show that e is selective. H1 and H2 follow from definition (22). Prove H3. Take any $x, y \in X$ and $A \in \mathcal{M}$ such that $x \in A$. If $e(x, y \cup A) = 0$ or $e(y, y \cup A) = 0$, then H3 is trivial. Let $e(x, y \cup A) > 0$ and $e(y, y \cup A) > 0$. By Lemma A.2, both x and y are costly in $y \cup A$. By Reduction, x is costly in A . If $y \succeq x$, then by (25), $E(x, y \cup A) \subset A$. Thus

$$e(x, A) \geq u(x) - U(E(x, y \cup A)) = e(x, y \cup A).$$

Suppose that $x \succ y$. As $e(x, A) > 0$, then $x \succ E(x, A)$ and by (28),

$$e(x, A) = u(x) - U(E(x, A)) \geq \frac{1}{2}u(x). \quad (29)$$

As $x \succ y$, then by (28), $u(y) \leq \frac{1}{2}u(x)$. Therefore,

$$e(y, y \cup A) = u(y) - U(E(y, y \cup A)) < u(y) \leq \frac{1}{2}u(x) \leq e(x, A).$$

Thus the function e satisfies H3, and hence, e is selective.

Next, suppose that \succeq satisfies Axioms 1–5. The function $e \in \mathcal{H}$ is selective, but it need not be subadditive even if \succeq satisfies CM. To obtain subadditivity, replace e with another function e^* such that for all $(x, A) \in \mathcal{M}$,

$$e^*(x, A) = e(x, A \cup E^*(x)) \quad (30)$$

where $E^*(x) = \{y \in X : x \sim \{x, y\} \succ y\}$. The case of zero costs is preserved.

Lemma A.3. $e(x, A) = 0$ if and only if $e^*(x, A) = 0$.

Proof. Take any $(x, A) \in \mathcal{D}$. Note first that for all $B \in \mathcal{M}$,

$$B \subset E^*(x) \quad \Rightarrow \quad x \sim x \cup B \succ B. \quad (31)$$

If $|B| = 1$, then (31) is true by definition of $E^*(x)$. Suppose that (31) is true for all menus B of size n . Take any $B' \subset E^*(x)$ of size $n + 1$. Then $B' = B \cup y$ for some $y \in E^*(x)$ and $B \subset E^*(x)$ of size n . By CM,

$$(x \cup B) \cup \{x, y\} \succeq \{x, y\} \sim x$$

because $x \sim \{x, y\} \succ y$ and $x \sim x \cup B \succ B$. By PSB, $(x \cup B) \cup \{x, y\} \preceq x$. Thus (31) holds by induction with respect to the size of B .

If $e^*(x, A) = 0$, then $e(x, A) = 0$ because $e(x, A) \leq e^*(x, A)$.

Suppose that $e(x, A) = 0$. By (30) and Lemma A.2,

$$e^*(x, A) = u(x) - U(x \cup B)$$

for some $B \subset A_x \cup E^*(x)$. If $B \subset A_x$, then $e^*(x, A) \leq e(x, A) = 0$. If $B \subset E^*(x)$, then $U(x \cup B) = u(x)$ by (31), and $e^*(x, A) = 0$. Suppose that $B \cap A_x$ and $B \cap E^*(x)$ are not empty. As $e(x, A) = 0$, then

$$x \sim x \cup (B \cap A_x) \succ B \cap A_x.$$

By (31), $x \sim x \cup (B \cap E^*(x)) \succ B \cap E^*(x)$. By CM and PSB,

$$x \sim (x \cup (B \cap A_x)) \cup (x \cup (B \cap E^*(x))) = x \cup B.$$

Thus $e^*(x, A) = 0$. □

As $e(x, x) = 0$, then $e^*(x, x) = 0$ for all $x \in X$. Thus e^* satisfies H1. By definition, e^* satisfies H2 as well.

Show H3 for e^* . Take any $x, y \in X$ and $A \in \mathcal{M}$ such that $x \in A$. If $e^*(x, y \cup A) = 0$ or $e^*(y, y \cup A) = 0$, then H3 is trivial. Suppose that $e^*(x, y \cup A) > 0$ and $e^*(y, y \cup A) > 0$. By Lemma A.3, both x and y are costly in $y \cup A$. By Reduction, x is costly in A and hence, $e^*(x, A) > 0$. If $u(y) \geq u(x)$, then $A_x = (y \cup A)_x$ and hence, $e^*(x, y \cup A) = e^*(x, A)$. Assume that $u(x) > u(y)$. Then $e^*(y, y \cup A) \leq u(y) \leq \frac{1}{2}u(x) \leq e^*(x, A)$.

Show subadditivity. Take any $A, B \in \mathcal{M}$ and $x \in A \cap B$. Suppose that $e^*(x, A) > 0$ and $e^*(x, B) > 0$. By (28),

$$e^*(x, A) = u(x) - U(E(x, A \cup E^*(x))) \geq \frac{u(x)}{2}.$$

Similarly, $e^*(x, B) \geq \frac{u(x)}{2}$ and hence,

$$e^*(x, A \cup B) \leq u(x) \leq e^*(x, A) + e^*(x, B).$$

Suppose next that $e^*(x, B) = 0$. Let $C = E(x, A \cup B \cup E^*(x))$. Take any $y \in C \setminus x$. By Lemma A.2, $x \succ y$. If $y \in B$, then $x \sim \{x, y\} \succ y$ because $x \succ \{x, y\}$ would imply $e^*(x, B) > 0$. Thus $C \subset A \cup E^*(x)$, and hence,

$$e^*(x, A \cup B) = u(x) - U(C) \leq e^*(x, A).$$

Similarly, if $e^*(x, A) = 0$, then $e^*(x, A \cup B) \leq e^*(x, B)$.

For all $A \in \mathcal{M}$, let

$$U^*(A) = \max_{x \in A} [u(x) - e^*(x, A)].$$

Then $U^*(x) = u(x)$ for all $x \in X$ and hence, U^* is an aggregation of e^* . Show that $U = U^*$. Take any $A \in \mathcal{M}$. By (30), $e(x, A) \leq e^*(x, A)$ for all $x \in A$. Thus $U(A) \geq U^*(A)$. Show that $U^*(A) \geq U(A)$.

By Lemma A.1, U aggregates e and hence, the set

$$B = \{y \in A : U(A) = u(y) - e(y, A)\}$$

is not empty. Let x minimize u in B . Assume that $A_x = \emptyset$. By Lemma A.2, $e(x, A) = 0$. By Lemma A.3, $e^*(x, A) = 0$, and

$$U^*(A) \geq u(x) - e^*(x, A) = u(x) - e(x, A) = U(A).$$

Assume that $A_x \neq \emptyset$. Then $u(x) - e(x, A) > u(y) - e(y, A)$ for all $y \in A_x$. By (26), $e(x, A) = e(x, x \cup A_x)$ and for all $y \in A_x$, $e(y, A) = e(y, x \cup A_x) = e(y, A_x)$. As U aggregates e , then

$$U(A) = u(x) - e(x, A) = U(x \cup A_x) > U(A_x) = \max_{y \in A_x} [u(y) - e(y, A)].$$

Thus $A \sim x \cup A_x \succ A_x$. Let $C = E(x, A \cup E^*(x))$. If $C = x$, then $e^*(x, A) = u(x) - U(C) = 0$ and hence, $U^*(A) \geq u(x) \geq U(A)$. Suppose that $C \neq x$. By (25), $C \subset x \cup A_x \cup E^*(x)$. Show that $C \succeq x \cup A_x$. Suppose $x \cup A_x \succ C$. By PSB,

$$x \cup A_x \succ C \cup A_x = x \cup A_x \cup (C \cap E^*(x)).$$

By (31), $x \sim x \cup (C \cap E^*(x)) \succ C \cap E^*(x)$. By CM,

$$x \cup A_x \cup (C \cap E^*(x)) \succeq x \cup A_x.$$

This contradiction implies $C \succeq x \cup A_x$. Thus

$$e^*(x, A) = u(x) - U(C) \leq u(x) - U(x \cup A_x) = u(x) - U(A)$$

and hence, $U(A) \leq u(x) - e^*(x, A) = U^*(A)$. Thus Axioms 1–5 imply that \succeq is represented by $U^* = u \circ e^*$ where e^* is both selective and subadditive.

A.2 Proof of Theorem 1

Suppose that \succeq is represented for all $A \in \mathcal{M}$ by

$$U(A) = \max_{x \in A} [u(x) - h(x, A)] \quad (32)$$

where $u \in \mathbb{R}^X$ and $h \in \mathcal{H}$ is a selective cost function. Show Axioms 1–4. Order is obvious. PSB follows from Lemma A.1. Show Dominance. Take any $A \in \mathcal{M}$ and $y \in X$. By (32), $U(A) = u(x) - h(x, A)$ for some $x \in A$. If $y \succeq x$, then $u(y) \geq u(x)$ and by H3,

$$U(y \cup A) \geq \max\{u(y) - h(y, y \cup A), u(x) - h(x, x \cup A)\} \geq u(x) - h(x, A) = u(A).$$

Thus Dominance holds. Show Reduction. Take any $A, B \in \mathcal{M}$, $x \in A$, $y \in X$ such that both x and y are costly in $y \cup A$. If $y \succeq x$, then $A_x = (y \cup A)_x$ and Reduction is trivial. Let $x \succ y$. Then $(y \cup A)_x = y \cup A_x$. Assume that $h(x, x \cup A_x) = 0$. By H3, either $h(x, x \cup y \cup A_x) = 0$ or $h(y, x \cup y \cup A_x) = 0$. By H2 and (32),

$$\begin{aligned} h(x, x \cup y \cup A_x) = 0 &\Rightarrow x \cup y \cup A_x \succeq x, \\ h(y, x \cup y \cup A_x) = 0 &\Rightarrow h(y, y \cup A_y) = 0 \Rightarrow y \cup A_y \succeq y. \end{aligned}$$

However, $x \succ y \cup A_y$ and $y \succ y \cup A_y$ because both x and y are costly in $y \cup A$. By contradiction, $h(x, x \cup A_x) > 0$. Thus $u(x) > U(x \cup A_x)$, and x is costly in A .

Conversely, assume Axioms 1–4 for \succeq . As \mathcal{M} is finite, then Order implies that \succeq has a utility representation $V : \mathcal{M} \rightarrow \mathbb{N}$ with natural values. Let $U = 2^V$. Then U is regular. By Lemma A.1, U aggregates e . By Theorem 3, e is selective.

A.3 Proof of Theorem 2

Suppose that \succeq is represented by (32) where $h \in \mathcal{H}$ is both selective and subadditive. Axioms 1–4 hold by Theorem 1. Show CM. Take any $A, B \in \mathcal{M}$ and $x \in A \cap B$ such that $A \succ A \setminus x$ and $B \succeq x \succ B \setminus x$. By (5), $U(A) = u(x) - h(x, A)$. Otherwise, if $U(A) > u(x) - h(x, A)$, then $A \succeq A \setminus x$ should hold. Similarly, $U(B) = u(x) - h(x, B)$. Moreover, $h(x, B) = 0$ because $B \succeq x$. As h is subadditive, then $h(x, A \cup B) \leq h(x, A)$ and hence,

$$U(A \cup B) \geq u(x) - h(x, A \cup B) \geq u(x) - h(x, A) = U(A),$$

that is, $A \cup B \succeq A$.

Conversely, assume that \succeq satisfies Axioms 1–5. Take a regular utility representation $U : \mathcal{M} \rightarrow \mathbb{R}_{++}$. By Theorem 3, U aggregates e^* that is both selective and additive.

A.4 Proof of Theorem 4

Take a dataset $\Phi \subset \mathcal{D}$.

Suppose first that there is a utility index $u \in \mathbb{R}^X$ and a selective cost function $h \in \mathcal{H}$ such that for all $(x, A) \in \Phi$,

$$x = \arg \max_{z \in A} [u(z) - h(z, A)]. \quad (33)$$

This representation has the following implications for the commitment utility u .

Lemma A.4. *If $x \succ_A y \succ_B x$, then either $u(x) > u(z)$ for some $z \in B \setminus A$ or $u(y) > u(z)$ for some $z \in A \setminus B$.*

Proof. Take any preference reversal $x \succ_A y \succ_B x$. Then $(x, A), (y, B) \in \Phi$ and $x, y \in A \cap B$. Suppose that $u(x) - h(x, A) \geq u(y) - h(y, B)$. As $x \in B$, then $u(x) - h(x, B) < u(y) - h(y, B)$ and hence, $h(x, A) < h(x, B)$. By H2, $h(x, A \cap B) \leq h(x, A)$. By H3, there is $z \in x \cup (B \setminus A)$ such that $h(z, B) \leq h(x, A \cap B)$. Note that $z \neq x$ because $h(x, A) < h(x, B)$. Thus $z \in B \setminus A$. As $z \in B$, but z is not chosen in B , then $u(z) - h(z, B) < u(y) - h(y, B) \leq u(x) - h(x, A)$. Thus $u(z) < u(x)$. Similarly, if $u(x) - h(x, A) \leq u(y) - h(y, B)$, then $y P z$ for some $z \in A \setminus B$. \square

Next, take any total order $R \in \mathcal{T}$ such that for all $x, y \in X$,

$$u(x) > u(y) \quad \Rightarrow \quad x P y. \quad (34)$$

For example, such R can be found as follows. Let

$$\alpha = \min_{x, y \in X: u(x) > u(y)} [u(x) - u(y)]$$

be the minimal possible variation of u between two distinct utility levels. Take some injective function $v : X \rightarrow \mathbb{R}_{++}$ such that $v(x) < \alpha$ for all $x \in X$. Then $u + v$ represents a total order R that satisfies (34).

By Lemma A.4, R is a principal order.

Conversely, suppose that Φ has a principal order R . Construct a representation (33). For any $x \in X$ and $A \in \mathcal{M}$, let

$$P(x, A) = \{z \in X : x P z\}$$

be the set of all elements in A that are inferior to x according to R .

Define an incomplete binary relation

$$\succ^* = \succ_1 \cup \succ_2 \cup \succ_3$$

on \mathcal{D} as a combination of three relations such that for all $(x, A), (y, B) \in \mathcal{D}$,

- (i) $(x, A) \succ_1 (y, B)$ if $A = B$, $x \neq y$, and $(x, A) \in \Phi$,

- (ii) $(x, A) \succeq_2 (y, B)$ if $x = y$ and $P(x, A) \subset B$,
- (iii) $(x, x) \succ_3 (y, y)$ if xPy .

Let \succ^* and \sim^* be the asymmetric and symmetric parts of \succeq^* . Note that $\succ^* = \succ_1 \cup \succ_2 \succ_3$ and $\sim^* = \sim_2$ because the relations \succ_1 and \succ_3 are asymmetric.

Let \succ^* be the strict part of \succeq^* .

Lemma A.5. *The pair (\succeq^*, \succ^*) is acyclic.*

Proof. Suppose that there is a cycle

$$(x_1, A_1) \succeq^* (x_2, A_2) \succeq \cdots \succeq^* (x_n, A_n) \succeq^* (x_{n+1}, A_{n+1}) = (x_1, A_1) \quad (35)$$

where $(x_i, A_i) \in \mathcal{D}$ and some of the comparisons is strict. Suppose that n be the shortest possible length for such a cycle. Consider two cases.

Case 1. $x_1 = x_2 = \cdots = x_n$. Then for all $i = 1, \dots, n$, the second preference $(x_i, A_i) \succeq_2 (x_{i+1}, A_{i+1})$ must hold because otherwise $x_i \neq x_{i+1}$ would be implied. By definition of \succeq_2 ,

$$P(x_1, A_1) \subset P(x_1, A_2) \subset \cdots \subset P(x_1, A_n) \subset P(x_1, A_1).$$

Thus all of these set inclusions must hold as equality, which implies that $(x_i, A_i) \sim^* (x_{i+1}, A_{i+1})$ for all $i = 1, \dots, n$.

Case 2. There is $i = 1, \dots, n$ such that $x_{i+1}Px_i$. Such i can be found so that x_jRx_i for all $j \in 1, \dots, n$. Moreover, it is without loss in generality, that such $i = 1$. Otherwise, just relabel cycle (35) starting from (x_i, A_i) . As $(x_1, A_1) \succeq^* (x_2, A_2)$ and x_2Px_1 , then

$$(x_1, A_1) \succ_1 (x_2, A_2) \quad (36)$$

must hold, that is, $(x_1, A_1) \in \Phi$ and $x_2 \in A_1 = A_2$. Then $(x_2, A_2) \notin \Phi$ because $A_1 = A_2$ and $x_1 \neq x_2$. Moreover, A_2 is not a singleton. Thus

$$(x_2, A_2) \succeq_2 (x_3, A_3)$$

must hold, that is, $x_2 = x_3$ and $P(x_2, A_2) \subset A_3$. In particular, $x_1 \in P(x_2, A_2)$ and hence $x_1 \in A_3$. Next, we claim that

$$(x_3, A_3) \succ_1 (x_4, A_4).$$

Suppose that $(x_3, A_3) \succeq_2 (x_4, A_4)$. Then $(x_2, A_2) \succeq_2 (x_4, A_4)$ because \succeq_2 is transitive. As (x_3, A_3) can be omitted from cycle (35), then it is not the shortest cycle. This contradiction shows that $(x_3, A_3) \succ_1 (x_4, A_4)$ should hold. Thus $(x_3, A_3) \in \Phi$ and $x_1 \in A_3$. The preference reversal

$$x_1 \succ_{A_1} x_3 \succ_{A_3} x_1$$

implies that either $x_1 P z$ for some $z \in A_3 \setminus A_1$ or $x_3 P z$ for some $z \in A_1 \setminus A_3$. The latter case is excluded by the inclusion $P(x_3, A_1) = P(x_3, A_2) \subset A_3$. Thus there is $z \in A_3 \setminus A_1$ such that $x_1 P z$. As $x_j R x_1$ for all j , then $x_j P z$ as well. The definition of \succeq^* implies that if $z \in A_j$ for any $j \geq 3$, then $z \in A_{j+1}$ as well because $z \in A_{j+1} = A_j$ or $z \in P(x_j, A_j) \subset A_{j+1}$ must hold. This is a contradiction with $z \notin A_1 = A_{n+1}$. \square

By Lemma A.5, there is a utility function $U : \mathcal{D} \rightarrow \mathbb{R}$ such that for all $(x, A), (y, B)$

$$\text{Let } u(x) = U(x, x) \text{ and } h(x, A) = U(x, x) - U(x, A).$$

Suppose instead that \succ_p is acyclic, and show that ϕ is represented by (33). Take any total order $R \in \mathcal{T}$ such that $\succ_p \subset P$. Such R exists because \succ_p is acyclic. Therefore, the transitive closure of \succ_p is a partial order and hence, can be extended to a total order by Szpilrajn's Theorem.

Let $u \in \mathbb{R}_+^X$ represent R .

A.5 Proof of Theorem 5

Suppose that Φ is complete. Take any principal order R . Take any $A, B \in \mathcal{M}$ and distinct elements $x, y, z \in X$ such that $(x, A) \in \Phi$ and $y, A \cup z$. Then $x \succ_A y \succ_{A \cup z} x$ and hence, $x P z$ by the definition of the principal order.

Conversely, take any $R \in \mathcal{T}$ that extends the incremental revealed preference \succ_p . Say that $z \in X$ is *special* for a preference reversal $x \succ_A y \succ_A B$ if $x P z$ and $z \in B \setminus A$ or $y P z$ and $z \in A \setminus B$. We claim that for any preference reversal $x \succ_A y \succ_B x$, there is a special element $z \in X$. The proof of this claim is by induction with respect to the number $k = |A \setminus B| + |B \setminus A|$ of elements that belong exactly to one of the menus A or B . If $k = 0$, then $A = B$ and $x \succ_A y \succ_B y$ is impossible because Φ is single-valued. Suppose that our claim is true if $k = n$. Assume that $x \succ_A y \succ_B y$ and $k = n + 1$. As $k > 0$, then $A \setminus B \neq \emptyset$ or $B \setminus A \neq \emptyset$. For concreteness, let $A \setminus B \neq \emptyset$. Let $z \in A \setminus B$ be a minimal element for the order R in $A \setminus B$. As Φ is complete, then there is a choice $y' \in B \cup z$ such that $(y', B \cup z) \in \Phi$. If $y' \notin \{y, z\}$, then the incremental revealed preference $y \succ_p z$ holds by definition. As R extends \succ_p , then $y P z$ and hence, z is special for $x \succ_A y \succ_B x$. Consider two other cases where $y' = y$ or $y' = z$. If $y' = y$, then $x \succ_A y \succ_{B \cup z} x$ is a preference reversal where $k = n$. By the inductive assumption, this reversal has a special element $z' \in X$. By definition, z' is also special for $x \succ_A y \succ_B x$. Finally, suppose that $y' = z$. Then $x \succ_A z \succ_{B \cup z} x$ is a preference reversal where $k = n$. Thus it has a special element z' . If $z' \in A \setminus (B \cup z)$ and $z P z'$, then z is not the minimal element in $A \setminus B$ for R . Thus $z' \in (B \cup z) \setminus A$ and $x P z'$. By definition, z' is special for the reversal $x \succ_A y \succ_B x$.

We have shown that R is a principal order.

A.6 Proofs of Theorem 6

Take any triple (\succeq, U, u) where $U : \mathcal{M} \rightarrow \mathbb{R}$ represents the preference \succeq , and u is the restriction of U to X .

Let $R^d \in \mathcal{R}$ be the antagonistic order represented by $-u$. For all $A \in \mathcal{M}$, let

$$U_1(A) = \max_{R \in \mathcal{T}} [u(R(A)) - f(R)] \quad (37a)$$

$$U_2(A) = \max_{C \in \mathcal{M}} [u(R^d(A \cap C)) - v(C)] \quad (37b)$$

where for all $R \in \mathcal{T}$ and $C \in \mathcal{M}$,

$$f(R) = \max_{B \in \mathcal{M}} [u(R(B)) - U(B)] \quad (38a)$$

$$v(C) = \max_{B \in \mathcal{M}} [u(R^d(B \cap C)) - U(B)]. \quad (38b)$$

Here the functions $f : \mathcal{T} \rightarrow \mathbb{R}$ and $v : \mathcal{M} \rightarrow \mathbb{R}$ are defined in terms of the given U . The functions U_1 and U_2 reconstruct U from f and v respectively.

Lemma A.6. *If $U = u \circ h$ for some $h \in \mathcal{H}$, then*

$$U = u \circ e = U_1 = U_2.$$

Moreover, the function e is selective, f and v are grounded, and $v(X) = 0$.

Proof. Let $U = u \circ h$ for some $h \in \mathcal{H}$.

By Theorem 1, \succeq satisfies Axioms 1–4. By Lemma A.1, $U = u \circ e$.

Take any $A \in \mathcal{M}$ and show that

$$U(A) \geq U_2(A) \geq U_1(A) \geq U(A). \quad (39)$$

By (37b), $U_2(A) = u(x) - v(C)$ for some $C \in \mathcal{M}$ and $x \in R^d(A \cap C)$. By (38b) with $B = A$, $v(C) \geq u(x) - U(A)$. Thus $U(A) \geq U_2(A)$.

By (37a), $U_1(A) = u(x) - f(R)$ for some $R \in \mathcal{T}$ and $x = R(A)$. Let

$$C' = \{x' \in X : x'Rx\}.$$

Then x maximizes R^d in C' and hence, $x \in R^d(A \cap C')$. By (37b),

$$U_2(A) \geq u(x) - v(C').$$

By (38b), $v(C') = u(z) - U(B)$ for some $B \in \mathcal{M}$ and $z \in R^d(B \cap C')$. Take any $y \in R(B)$. As yRz and $z \in C'$, then $y \in C'$. As z maximizes $-u$ in $B \cap C'$, then $u(z) \leq u(y)$, and

$$v(C') = u(z) - U(B) \leq u(y) - U(B) \leq f(R).$$

Thus $U_2(A) \geq u(x) - v(C') \geq u(x) - f(R) = U_1(A)$.

As $U = u \circ h$, then $U(A) = u(z) - h(z, A)$ for some $z \in A$. For each $B \in \mathcal{M}$, let

$$\mu(B) = \{x \in B : h(x, B) \leq h(z, A)\}.$$

The set $\mu(B) \subset B$ is not empty. Indeed, by H1, $\mu(x) = x$ for all $x \in X$. By H3, if $x \in \mu(B)$, then for all $y \in X$, either $x \in \mu(y \cup B)$ or $y \in \mu(y \cup B)$. By induction with respect to the size of B , $\mu(B)$ is not empty for all $B \in \mathcal{M}$. By H2 and H3, $\mu : \mathcal{M} \rightarrow \mathcal{M}$ satisfies Sen's α and Aizerman conditions. By the multi-utility representation theorem of Aizerman and Malishevski [1] (Theorem 5 in Moulin), there is a set $\Theta \subset \mathcal{T}$ such that

$$\mu(B) = \bigcup_{R \in \Theta} R(B) \quad \text{for all } B \in \mathcal{M}.$$

As $z \in \mu(A)$, then there is $R_z \in \Theta$ such that $z = R_z(A)$. By (38a), there is $B \in \mathcal{M}$ and $y = R_z(B)$ such that $f(R_z) = u(y) - U(B)$. By (32), $U(B) \geq u(y) - h(y, B)$ and hence,

$$f(R_z) \leq u(y) - (u(y) - h(y, B)) = h(y, B) \leq h(z, A)$$

because $R_z \in \Theta$ and $y \in \mu(B)$. Thus

$$U(A) = u(z) - h(z, A) \leq u(z) - f(R_z) \leq \max_{R \in \mathcal{T}} [u(R(A)) - f(R)] = U_1(A).$$

We have shown all inequalities (39), which must hold as equalities.

Show that e is selective. Take any $x, y \in X$ and $A \in \mathcal{M}$ such that $x \in A$. Use (23) to define $B = E(x, y \cup A)$ and $C = E(y, y \cup A)$ so that

$$e(x, y \cup A) = u(x) - U(B) \quad \text{and} \quad e(y, y \cup A) = u(y) - U(C).$$

Suppose that $y \succeq x$. Then by (25), $y \notin B$ and $B \subset A$. Thus H3 holds because

$$e(x, y \cup A) = u(x) - U(B) \leq e(x, A).$$

Suppose that $x \succ y$, $e(x, y \cup A) > e(x, A)$ and $e(y, y \cup A) > e(x, A)$. Then

$$\begin{aligned} U(B) &= u(x) - e(x, y \cup A) < u(x) - e(x, A) \\ U(C) &= u(y) - e(y, y \cup A) < u(x) - e(x, A) \quad \text{and by PSB} \\ U(B \cup C) &< u(x) - e(x, A). \end{aligned}$$

Let $D = (B \cup C) \setminus y$. Take any $z \in D$. If $h(z, B \cup C) < e(y, B \cup C)$, then

$$h(z, D) = h(z, B \cup C)$$

because h satisfies H3. Thus

$$u(z) - h(z, D) \leq U(B \cup C) < u(x) - e(x, A).$$

If $h(z, B \cup C) \geq e(y, B \cup C)$, then $h(z, D) \geq h(y, B \cup C)$ because h satisfies H3. Thus

$$u(z) - h(z, D) \leq u(z) - h(y, B \cup C) < u(x) - h(x, A) \leq u(x) - e(x, A).$$

Thus $U(D) < u(x) - e(x, A)$ which contradicts the definition of $e(x, A)$ because $D \in \mathcal{M}(x, A)$. By contradiction, e satisfies H3 and hence, is selective.

Show that f is grounded. Take any total order $R \in \mathcal{T}$ such that for all $x, y \in X$, $u(x) > u(y)$ implies yPx . Then

$$u(R(A)) = \min_{x \in A} u(x) \leq U(B)$$

for all $B \in \mathcal{M}(x, A)$. Thus $f(R) = 0$. Similarly, $v(X) = 0$. \square

Turn to Theorem ???. Suppose that $U = u \circ h$ for some $h \in \mathcal{H}$. By Lemma A.6, U can be rewritten in the forms (6) and (9). To derive (7), let

$$\alpha = \max_{A \in \mathcal{M}} u(A) - \min_{A \in \mathcal{M}} u(A).$$

For any $R \in \mathcal{T}$, represent it by a function $t_R : X \rightarrow \mathbb{R}$ such that for all $x, y \in X$,

$$xPy \Leftrightarrow t_R(x) - t_R(y) > \alpha. \quad (40)$$

Then for all $A \in \mathcal{M}$

$$u(R(A)) = \max_{x \in A} (u(x) + t_R(x) - t_R(A))$$

because this maximization implies $t_R(x) = t_R(A)$ by (40). Thus (7) holds for $\Gamma = \{t_R : R \in \mathcal{T}\}$ and $g(t_R) = f(R)$ for all $R \in \mathcal{T}$.

Suppose that U satisfies (6) where $u \in \mathbb{R}^X$, $\Theta \subset \mathcal{R}$, and $f : \Theta \rightarrow \mathbb{R}_+$ is grounded. For all $(x, A) \in \mathcal{D}$, let

$$h(x, A) = \min\{\alpha, \min_{R \in \Theta: x \in R(A)} f(R)\}.$$

Here $h(x, A) = \alpha$ if $x \notin R(A)$ for all $R \in \Theta$. Then $U = u \circ h$ and $h \in \mathcal{H}$.

Suppose that U satisfies (7) where $u \in \mathbb{R}^X$, $\Gamma \subset \mathbb{R}^X$ is finite, and $g : \Gamma \rightarrow \mathbb{R}_+$ is grounded. For all $(x, A) \in \mathcal{D}$, let

$$h(x, A) = \min_{t \in \Gamma} [g(t) + t(A) - t(x)].$$

Then $U = u \circ h$ and $h \in \mathcal{H}$.

Suppose that U satisfies (9) where $u \in \mathbb{R}^X$, $R^d \in \mathcal{R}$, $X \in \Pi \subset \mathcal{M}$, and $v : \Pi \rightarrow \mathbb{R}_+$ is such that $v(X) = 0$. For all $(x, A) \in \mathcal{D}$, let

$$h(x, A) = \min\{\alpha, \min_{C \in \Pi, x \in R^d(A \cap C)} v(C)\}.$$

Here $h(x, A) = \alpha$ if $x \notin R^d(A \cap C)$ for all $C \in \Pi$. Then $U = u \circ h$ and $h \in \mathcal{H}$.

A.7 Proof of Theorem 7

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