

Quantile Panel Data Models with Partially Varying Coefficients: Estimating the Growth Effect of FDI*

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Abstract

In this paper, we propose a new semiparametric quantile panel data model with correlated random effects in which some of the coefficients are allowed to depend on some smooth economic variables while other coefficients remain constant. A three-stage estimation procedure is proposed to estimate both constant and functional coefficients and their asymptotic properties are investigated. We show that the estimator of constant coefficients is root-N consistent and the estimator of varying coefficients converges in a nonparametric rate. A Monte Carlo simulation is conducted to examine the finite sample performance of the proposed estimators. Finally, the proposed semiparametric quantile panel data model is applied to estimating the impact of foreign direct investment (FDI) on economic growth using the cross-country data from 1970 to 1999.

Keywords: Correlated Random Effect; Foreign Direct Investment; Panel Data; Quantile Regression Model; Semiparametric Model; Varying Coefficient Model.

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1 Introduction

Since the seminal paper by Koenker and Bassett (1978), linear quantile models have received heated attentions both in theoretical and empirical studies in economics; see the book by Koenker (2005). Recently, many studies have focused on nonparametric or semiparametric quantile regression models for either independently identically distributed (iid) data or time series data. For example, Chaudhuri (1991) studied nonparametric quantile estimation and derived its local Bahadur representation, He, Ng and Portnoy (1998), He and Ng (1999) and He and Portnoy (2000) considered nonparametric estimation using splines, De Gooijer and Zerom (2003), Yu and Lu (2004) and Horowitz and Lee (2005) focused on additive quantile models, and Honda (2004) and Cai and Xu (2008) studied varying-coefficient quantile models for time series data. In particular, semiparametric quantile models have attracted increasing research interests during the recent years due to their flexibility. For example, He and Liang (2000) investigated the quantile regression of a partially linear errors-in-variable model, Lee (2003) discussed the efficient estimation of a partially linear quantile regression, and Cai and Xiao (2012) proposed a partially varying-coefficient dynamic quantile regression model, among others.

Due to the fact that the approach of taking difference, which is commonly used in conditional mean (linear) panel data models to eliminate individual effects, is invalid in quantile regression settings, even for linear quantile regression model, the literature on quantile panel data models is relatively small. To the best of our knowledge, the paper by Koenker (2004) is the first paper to consider a linear quantile panel data model with fixed effects, where the fixed effects are assumed to have pure location shift effects on the conditional quantiles of the dependent variable but the effects of regressors are allowed to be dependent on quantiles. Koenker (2004) proposed two methods to estimate such a panel data model with fixed effects. The first method is to solve a piecewise linear quantile loss function by using interior point methods and the second one is the so called penalized quantile regression method, in which the quantile loss function is minimized by adding L_1 penalty on fixed effects. Recently, in a penalized quantile panel data regression model as in Koenker (2004), Lamarche (2010) discussed how to select the tuning parameter, which can control the degree of shrinkage for fixed effects, whereas Galvao (2011) extended the quantile regression to a dynamic panel data

model with fixed effects by employing the lagged dependent variables as instrumental variables and by extending Koenker’s first method to Chernozhukov and Hansen (2006)’s quantile instrumental variable framework. Finally, Canay (2011) proposed a simple two-stage method to estimate a quantile panel data model with fixed effects. However, the consistency of the estimator in Canay (2011) relies on the assumption of T going to infinity and the existence of an initial \sqrt{NT} -consistent estimator in the conditional mean model.

An alternative way to deal with individual effects in a panel data model is to treat them as correlated random effects initiated by Chamberlain (1982, 1984). Under the framework of Chamberlain (1982, 1984), to estimate the effect of birth inputs on birth weight, Abrevaya and Dahl (2008) employed a linear quantile panel data model with correlated random effects which are viewed as a linear projection onto some covariates plus an error term. The identification of the effects of covariates only requires two-period information. Furthermore, Gamper-Rabindram, Khan and Timmins (2008) estimated the impact of piped water provision on infant mortality by adopting a linear quantile panel data model with random effects where the random effects were allowed to be correlated with covariates nonparametrically. The model can be estimated through a two-step procedure, in which some conditional quantiles were estimated nonparametrically in the first step and in the second step, the coefficients are estimated by regressing the differenced estimated quantiles on the differenced covariates.

Motivated by examining the impact of foreign direct investment (FDI) on economic growth, we propose a partially varying-coefficient quantile regression model for panel data with correlated random effects, which includes the models in Lee (2003), Cai and Xu (2008) and Cai and Xiao (2012) as a special case. In contrast to Koenker (2004), Galvao (2011) and Canay (2011) by requiring that both N and T go to infinity in their asymptotics, our model requires only N going to infinity with T possibly fixed. Actually, $T \geq 2$ is required. Also, different from Abrevaya and Dahl (2008) and Gamper-Rabindram, Khan and Timmins (2008), we adopt a partially varying-coefficient structure in the conditional quantile model to provide more flexibility in model specification than a linear one. Finally, we apply the aforementioned semiparametric quantile panel data model to study the role of FDI in the economic growth process based on the cross-country data from 1970 to 1999 and the detailed report of this empirical study is given in Section 4.

It is well documented in the growth literature that foreign direct investment plays an im-

portant role in economic growth process in host countries since FDI is often considered as a vehicle to transfer new ideas, advanced capitals, superior technology and know-how from developed countries to developing countries and so on. However, the existing empirical studies provide contradictory results on whether or not FDI promotes an economic development in host countries.¹ The recent studies in the literature concluded that the mixed empirical evidences may be due to nonlinearities in FDI effects on economic growth and the heterogeneity across countries.

Indeed, it is well recognized by many economists in empirical studies that a standard linear growth model may be inappropriate for investigating the nonlinear effect of FDI on economic development. The nonlinearity in FDI effects is mainly due to the so called absorptive capacity in host countries, the fact that host countries need some minimum conditions to absorb the spillovers from FDI.² Most existing literature to deal with the nonlinearity issue used simply some parametric nonlinear models, for example, including an interacted term in the regression model as in Li and Liu (2004) and Kottaridi and Stengos (2010) or running a threshold regression model as in Borensztein, De Gregorio and Lee (1998). A parametric nonlinear model has the risk of misspecifying a model and a misspecified model can lead to biased estimation and misleading empirical results. Recently, to overcome this difficulty, Henderson, Papageorgiou and Parmeter (2011) and Kottaridi and Stengos (2010) adopted nonparametric/semiparametric regression techniques into a growth model. However, due to the so-called curse of dimensionality in a pure nonparametric estimation, such applications are either restricted to the sample size problem or rely heavily on variable selection which is not an easy task.

¹For example, Blomstrom and Persson (1983), Blomstrom, Lipsey and Zejan (1992), De Gregorio (1992), Borensztein, De Gregorio and Lee (1998), De Mello (1999), Ghosh and Wang (2009), Kottaridi and Stengos (2010) among others found positive effects of FDI on promoting economic growth in various environments. On the other hand, many studies including Haddad and Harrison (1992), Aitken and Harrison (1999), Lipsey (2003), and Carkovic and Levine (2005) failed to find beneficial effects of FDI on economic growth in host countries. Grog and Strobl (2001) did a meta analysis of 21 studies using the data from 1974 to 2001 that worked on estimating FDI effects on productivity in host countries, of which 13 studies reported positive results, 4 studies reported negative effects and the remaining reported inconclusive evidence.

²Nunnenkamp (2004) emphasized the importance of the initial condition for host countries to absorb the positive impacts of FDI, Borensztein, De Gregorio and Lee (1998) found that a threshold stock of human capital in host countries is necessary for them to absorb beneficial effects of advanced technologies brought from FDI, and Hermes and Lensink (2003), Alfaro, Chandab, Kalemli-Ozcan and Sayek (2004) and Durham (2004) addressed the local financial market conditions of a country's absorptive capacity.

Heterogeneity among countries is another concern in cross-country studies. Grog and Strobl (2001) found that whether a cross sectional or time series data had been used matters for estimating the effect of FDI on economic growth, because both the cross sectional and time series model cannot control the country-specific heterogeneity. Recent literature focused on using panel data to estimate growth models, which can control the country-specific unobserved heterogeneity using individual effects. However, including individual effects only allows a location shift for each country but it does not have an ability to deal with the heterogeneity effect of FDI on economic growth across countries. For example, some studies found that the empirical results had been changed when the sample included developed countries or not. The existing literature to handle this issue is to split sample into subgroups based on some prior information.³ Generally speaking, splitting sample can lead to potential theoretical and empirical problems. First, splitting sample may lose sample information and degrees of freedom, which may lead to inefficient estimation. Secondly, the selection of thresholds to split sample is often lack of theoretical guideline and justification.

In this paper, to deal with the aforementioned two issues (nonlinearities and heterogeneity) in a simultaneous fashion, we propose a partially varying-coefficient quantile panel data model with correlated random effects to estimate the nonlinear effect of FDI on economic growth with heterogeneity. Different from the existing literature, we resolve the nonlinearity issue by employing a partially varying-coefficient model which allows some of coefficients to be constant but others, reflecting the effects of FDI on economic growth, to depend on the country's initial condition. Compared to a full nonparametric estimation, our model setup can achieve the dimension reduction and accommodate the well recognized economic theory such as the absorptive capacity. In addition to using panel data with individual effects which allow for location shifts for individual countries, we propose a semiparametric conditional quantile regression model instead of the commonly used conditional mean models. A conditional quantile model can provide more flexible structures than conditional mean models to characterize heterogeneity among countries. For example, besides including individual effects allowing country-specific heterogeneity, a conditional quantile model allows different growth equations for different quantiles. Therefore, we can take advantage of all sample information

³For example, Luiz and De Mello (1999) considered OECD and non-OECD samples and Kottaridi and Stengos (2010) split the whole sample into high-income and middle-income groups.

to identify the effect of FDI on growth without splitting sample according to development stages. Moreover, estimating all quantiles can provide a whole picture of the conditional distribution and avoid the possibly misleading conditional mean results.

The rest of the paper is organized as follows. In Section 2, we introduce a partially varying-coefficient quantile panel data model with correlated random effects and propose a three-stage estimation procedure. The asymptotic properties of our estimators are established. In Section 3, a simulation study is conducted to examine the finite sample performance. Section 4 is devoted to reporting the empirical results of the cross-country panel data growth model. Section 5 concludes.

2 Partially varying-coefficient quantile panel data model

2.1 The model and estimation procedures

In this paper, we consider the following partially varying-coefficient conditional quantile panel data model with individual effects,

$$Q_\tau(Y_{it}|\mathbf{U}_{it}, \mathbf{X}_{it}) = \mathbf{X}'_{it,1}\boldsymbol{\gamma}_\tau + \mathbf{X}'_{it,2}\boldsymbol{\beta}_\tau(\mathbf{U}_{it}) + \alpha_i \quad (1)$$

for $i = 1, \dots, N$ and $t = 1, \dots, T$, where $Q_\tau(Y_{it}|\mathbf{U}_{it}, \mathbf{X}_{it})$ is the τ th conditional quantile of Y_{it} given \mathbf{U}_{it} and \mathbf{X}_{it} and both \mathbf{U}_{it} and \mathbf{X}_{it} are covariates. Here, $\boldsymbol{\gamma}_\tau$ and $\boldsymbol{\beta}_\tau(\mathbf{U}_{it})$ denote constant and functional coefficients with a dimension of $K_1 \times 1$ and $K_2 \times 1$, respectively, $\mathbf{X}_{it} = (\mathbf{X}'_{it,1}, \mathbf{X}'_{it,2})'$ is a $K \times 1$ ($K = K_1 + K_2$) vector of covariates with A' denoting the transpose of a matrix or vector A , and α_i is an individual effect which is expected to control the unobserved heterogeneity among individuals. Here, we assume that \mathbf{U}_{it} is a scalar smoothing variable, denoted by U_{it} .⁴ In the quantile panel data literature, Koenker (2004) treated α_i as a fixed effect and proposed a penalized quantile regression method which requires both N and T go to infinity. In this paper, following Abrevaya and Dahl (2008) and Gamper-Rabindran, Khan and Timmins (2008), we view the individual effect as a correlated random effect which is allowed to be correlated with covariates $\mathbf{X}_i = (\mathbf{X}'_{i1}, \dots, \mathbf{X}'_{iT})'$ and $\{U_{it}\}_{t=1}^T$,

⁴For simplicity, we only consider the univariate case for the smoothing variable. The estimation procedure and asymptotic results still hold for the multivariate case with much more complicated notation. Alternatively, one may apply a dimension reduction approach such as a single index method model as $U_{it} = \boldsymbol{\omega}'\mathbf{U}_{it}$ coupled with the iterative backfitting estimation method proposed by Fan, Yao and Cai (2003).

and furthermore, we treat α_i as a nonlinear function such as $\alpha_i = \phi(\mathbf{X}_i, U_{i1}, \dots, U_{iT})$. However, in estimating FDI effect in our empirical study; see the detailed report given in Section 4, the smoothing variable varies only across different individuals but keeps constant over time periods. Therefore, in this paper, we focus on the simple case of $U_{it} = U_i$ for any t and then model (1) becomes to

$$Q_\tau(Y_{it}|U_i, \mathbf{X}_{it}) = \mathbf{X}'_{it,1}\boldsymbol{\gamma}_\tau + \mathbf{X}'_{it,2}\boldsymbol{\beta}_\tau(U_i) + \alpha_i. \quad (2)$$

Note that all estimation procedures and their econometric theory as well as statistical inferences for model (2) developed later (see Sections 2.2 and 2.3) can be easily extended to model (1).

Finally, we approximate the unknown function $\phi(\mathbf{X}_i, U_i)$ by the additive functional-coefficient model⁵ such that

$$\phi(\mathbf{X}_i, U_i) = \sum_{t=1}^T \mathbf{X}'_{it} \boldsymbol{\delta}_t(U_i) = \mathbf{X}'_i \boldsymbol{\delta}(U_i), \quad (3)$$

where $\boldsymbol{\delta}(U_i) = (\boldsymbol{\delta}'_1(U_i), \dots, \boldsymbol{\delta}'_T(U_i))'$ is a $TK \times 1$ vector of unknown functional coefficients. A fully nonparametric model of $\phi(\cdot)$ may lead to the problem of the so-called curse of dimensionality and become infeasible in practice. Compared to a linear projection in Chamberlain (1982) and Abrevaya and Dahl (2008), an additive model with functional coefficients can accommodate more flexibility. Therefore, model (2) can be expressed as

$$Q_\tau(Y_{it}|U_i, \mathbf{X}_i) = \mathbf{X}'_{it,1}\boldsymbol{\gamma}_\tau + \mathbf{X}'_{it,2}\boldsymbol{\beta}_\tau(U_i) + \mathbf{X}'_i \boldsymbol{\delta}(U_i). \quad (4)$$

From the above model, one can see that the conditional quantile effects of \mathbf{X}_{it} on Y_{it} are through two channels: a direct effect $\boldsymbol{\gamma}_\tau$ for constant coefficients or $\boldsymbol{\beta}_\tau(U_i)$ for varying coefficients, and an indirect effect $\boldsymbol{\delta}_t(U_i)$ working through the correlated random effect. Hence, to identify the direct effects $\boldsymbol{\gamma}_\tau$ and $\boldsymbol{\beta}_\tau(U_i)$, one has to estimate at least two conditional quantile models $Q_\tau(\mathbf{Y}_{it} | U_i, \mathbf{X}_i)$ and $Q_\tau(\mathbf{Y}_{is} | U_i, \mathbf{X}_i)$ given by

$$Q_\tau(Y_{it}|U_i, \mathbf{X}_i) = \mathbf{X}'_{it,1}[\boldsymbol{\gamma}_\tau + \boldsymbol{\delta}_{t,1}(U_i)] + \mathbf{X}'_{it,2}[\boldsymbol{\beta}_\tau(U_i) + \boldsymbol{\delta}_{t,2}(U_i)] + \sum_{l \neq t} \mathbf{X}'_{il} \boldsymbol{\delta}_l(U_i)$$

and

$$Q_\tau(Y_{is}|U_i, \mathbf{X}_i) = \mathbf{X}'_{it,1} \boldsymbol{\delta}_{t,1}(U_i) + \mathbf{X}'_{it,2} \boldsymbol{\delta}_{t,2}(U_i) + \mathbf{X}'_{is,1} \boldsymbol{\gamma}_\tau + \mathbf{X}'_{is,2} \boldsymbol{\beta}_\tau(U_i) + \sum_{l \neq t} \mathbf{X}'_{il} \boldsymbol{\delta}_l(U_i),$$

⁵As elaborated in Cai, Das, Xiong and Wu (2006) and Cai (2010), a functional-coefficient model can be actually a good approximation to a general fully nonparametric model, $g(X, Z) \approx \sum_{j=0}^d g_j(Z) X_j = X' g(Z)$.

respectively, where $t \neq s$, $\boldsymbol{\delta}_{t,1}(U_i)$ is the vector which contains the first K_1 components of $\boldsymbol{\delta}_t(U_i)$, and $\boldsymbol{\delta}_{t,2}(U_i)$ is the vector which contains the last K_2 components of $\boldsymbol{\delta}_t(U_i)$. Hence, the estimates of $\boldsymbol{\gamma}_\tau$ and $\boldsymbol{\beta}_\tau(U_i)$ are respectively given by

$$\boldsymbol{\gamma}_\tau = \frac{\partial Q_\tau(\mathbf{Y}_{it} | U_i, \mathbf{X}_i)}{\partial \mathbf{X}_{it,1}} - \frac{\partial Q_\tau(\mathbf{Y}_{is} | U_i, \mathbf{X}_i)}{\partial \mathbf{X}_{it,1}},$$

and

$$\boldsymbol{\beta}_\tau(U_i) = \frac{\partial Q_\tau(\mathbf{Y}_{it} | U_i, \mathbf{X}_i)}{\partial \mathbf{X}_{it,2}} - \frac{\partial Q_\tau(\mathbf{Y}_{is} | U_i, \mathbf{X}_i)}{\partial \mathbf{X}_{it,2}}.$$

However, in order to avoid running two separating conditional quantile models, we adopt Abrevaya and Dahl (2008)'s pooling regression strategy by stacking covariates. From model (4), $Q_\tau(\mathbf{Y}_{it} | U_i, \mathbf{X}_i)$ and $Q_\tau(\mathbf{Y}_{is} | U_i, \mathbf{X}_i)$ can be expressed as

$$Q_\tau(Y_{it}|U_i, \mathbf{X}_i) = \mathbf{X}'_{it,1}\boldsymbol{\gamma}_\tau + \mathbf{X}'_{it,2}\boldsymbol{\beta}_\tau(U_i) + \mathbf{X}'_{i1}\boldsymbol{\delta}_1(U_i) + \cdots + \mathbf{X}'_{iT}\boldsymbol{\delta}_T(U_i)$$

and

$$Q_\tau(Y_{is}|U_i, \mathbf{X}_i) = \mathbf{X}'_{is,1}\boldsymbol{\gamma}_\tau + \mathbf{X}'_{is,2}\boldsymbol{\beta}_\tau(U_i) + \mathbf{X}'_{i1}\boldsymbol{\delta}_1(U_i) + \cdots + \mathbf{X}'_{iT}\boldsymbol{\delta}_T(U_i).$$

Hence, we treat

$$\begin{pmatrix} Y_{11} \\ \vdots \\ Y_{1T} \\ \vdots \\ Y_{i1} \\ \vdots \\ Y_{iT} \\ \vdots \\ Y_{N1} \\ \vdots \\ Y_{NT} \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} \mathbf{X}'_{11,1} & \mathbf{X}'_{11,2} & \mathbf{X}'_{11} & \cdots & \mathbf{X}'_{1T} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ \mathbf{X}'_{1T,1} & \mathbf{X}'_{1T,2} & \mathbf{X}'_{11} & \cdots & \mathbf{X}'_{1T} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ \mathbf{X}'_{i1,1} & \mathbf{X}'_{i1,2} & \mathbf{X}'_{i1} & \cdots & \mathbf{X}'_{iT} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ \mathbf{X}'_{iT,1} & \mathbf{X}'_{iT,2} & \mathbf{X}'_{i1} & \cdots & \mathbf{X}'_{iT} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ \mathbf{X}'_{N1,1} & \mathbf{X}'_{N1,2} & \mathbf{X}'_{N1} & \cdots & \mathbf{X}'_{NT} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ \mathbf{X}'_{NT,1} & \mathbf{X}'_{NT,2} & \mathbf{X}'_{N1} & \cdots & \mathbf{X}'_{NT} \end{pmatrix}$$

as the dependent variable and the right-side explanatory variables, respectively. This pooled regression directly estimates $(\boldsymbol{\gamma}'_\tau, \boldsymbol{\beta}'_\tau(U_i), \boldsymbol{\delta}'_1(U_i), \dots, \boldsymbol{\delta}'_T(U_i))'$. We now consider the following transformed model from (4),

$$Q_\tau(U_i, \{\mathbf{Z}_{it}\}_{t=1}^T) = \mathbf{Z}'_{it,1}\boldsymbol{\gamma}_\tau + \mathbf{Z}'_{it,2}\boldsymbol{\theta}_\tau(U_i), \quad (5)$$

where $\boldsymbol{\theta}_\tau(U_i) = (\boldsymbol{\beta}'_\tau(U_i), \boldsymbol{\delta}'_1(U_i), \dots, \boldsymbol{\delta}'_T(U_i))'$, $\mathbf{Z}_{it,1}$ denotes the corresponding variables in the first column in the above matrix, $\mathbf{Z}_{it,2}$ represents those entries in the remaining columns, and $\mathbf{Z}_{it} = (\mathbf{Z}'_{it,1}, \mathbf{Z}'_{it,2})'$. To estimate the above semiparametric model, similar to Cai and Xiao (2012), we propose a three-stage estimation procedure to the panel data model, described as follows.

At the first stage, we treat all coefficients as functional coefficients depending on U_i , such as $\boldsymbol{\gamma}_\tau = \boldsymbol{\gamma}_\tau(U_i)$. It is assumed throughout that $\boldsymbol{\gamma}_\tau(\cdot)$ and $\boldsymbol{\theta}_\tau(\cdot)$ are both twice continuously differentiable. Then, when U_i is in a neighborhood of u_0 , a given grid point within the domain of U_i , we apply the local constant approximation to $\boldsymbol{\gamma}_\tau(\cdot)$ and the local linear approximation to $\boldsymbol{\theta}_\tau(\cdot)$, respectively. Hence, model (5) is estimated as a fully functional-coefficient model and following Cai and Xu (2008), the localized objective function is given by

$$\min_{\boldsymbol{\gamma}_0, \boldsymbol{\theta}_0, \boldsymbol{\theta}_1} \sum_{i=1}^N \sum_{t=1}^T \rho_\tau(Y_{it} - \mathbf{Z}'_{it,1}\boldsymbol{\gamma}_0 - \mathbf{Z}'_{it,2}\boldsymbol{\theta}_0 - \mathbf{Z}'_{it,2}\boldsymbol{\theta}_1(U_i - u_0))K_h(U_i - u_0), \quad (6)$$

where $\boldsymbol{\gamma}_0 = \boldsymbol{\gamma}_\tau(u_0)$, $\boldsymbol{\theta}_0 = \boldsymbol{\theta}_\tau(u_0)$, $\boldsymbol{\theta}_1 = \dot{\boldsymbol{\theta}}_\tau(u_0)$, $\rho_\tau(y) = y[\tau - I_{y < 0}]$ is called the check function, I_A is the indicator function of any set A , $K_h(u) = K(u/h)/h$, and $K(\cdot)$ is the kernel function. Note that \dot{A} and \ddot{A} denote the first order and second order partial derivatives of A throughout the paper.

Since $\boldsymbol{\gamma}_\tau$ is a global parameter, in order to utilize all sample information to estimate $\boldsymbol{\gamma}_\tau$, at the second stage, we employ the average method to achieve the root-N consistent estimator of $\boldsymbol{\gamma}_\tau$, which is given by

$$\hat{\boldsymbol{\gamma}}_\tau = \frac{1}{N} \sum_{i=1}^N \hat{\boldsymbol{\gamma}}_\tau(U_i). \quad (7)$$

Theorem 1 (see later) shows that indeed, $\hat{\boldsymbol{\gamma}}_\tau$ is a root-N consistent estimator.

Remark 1: First, it is worth to point out that the well known profile least squares type of estimation approach (Robinson (1988) and Speckman (1988)) for classical semiparametric regression models may not be suitable to quantile setting due to lack of explicit normal equations. Secondly, the estimator $\hat{\boldsymbol{\gamma}}_\tau$ given in (7) has the advantage that it is easy to construct and also achieves the \sqrt{N} -rate of convergence (see Theorem 1 later). In addition to this simple estimator, other root-N consistent estimators of $\boldsymbol{\gamma}_\tau$ can be constructed. For example, to estimate the parameter $\boldsymbol{\gamma}_\tau$ without being overly influenced by the tail behavior

of the distribution of U_i , one might use a trimming function $w_i = I_{\{U_i \in \mathcal{D}\}}$ with a compact subset \mathcal{D} of \mathfrak{R} ; see Cai and Xiao (2012) for details. Then, (7) becomes the weighted average estimator as

$$\tilde{\boldsymbol{\gamma}}_{w,\tau} = \left[\frac{1}{N} \sum_{i=1}^N w_i \right]^{-1} \left[\frac{1}{N} \sum_{i=1}^N w_i \hat{\boldsymbol{\gamma}}_{\tau}(U_i) \right].$$

Indeed, this type of estimator was considered by Lee (2003) for a partially linear quantile regression model. To estimate $\boldsymbol{\gamma}_{\tau}$ more efficiently, following Cai and Xiao (2012), a general weighted average approach can be constructed as follows

$$\tilde{\boldsymbol{\gamma}}_{w,\tau} = \left[\frac{1}{N} \sum_{i=1}^N W(U_i) \right]^{-1} \left[\frac{1}{N} \sum_{i=1}^N W(U_i) \hat{\boldsymbol{\gamma}}_{\tau}(U_i) \right],$$

where $W(\cdot)$ is a weighting function (a symmetric matrix) which can be chosen optimally by minimizing the asymptotic variance; see Cai and Xiao (2012) for details. For simplification of presentation, our focus is on $\hat{\boldsymbol{\gamma}}_{\tau}$ given in (7).

At the last step, to estimate the varying coefficients, for a given \sqrt{N} -consistent estimator $\hat{\boldsymbol{\gamma}}_{\tau}$ of $\boldsymbol{\gamma}_{\tau}$, which may be obtained from (7), we plug $\hat{\boldsymbol{\gamma}}_{\tau}$ into model (5) and obtain the partial residual denoted by $Y_{it}^* = Y_{it} - \mathbf{Z}'_{it,1} \hat{\boldsymbol{\gamma}}_{\tau}$. Hence, the functional coefficients can be estimated by using the local linear quantile estimation which is given by

$$\min_{\boldsymbol{\theta}_0, \boldsymbol{\theta}_1} \sum_{i=1}^N \sum_{t=1}^T \rho_{\tau}(Y_{it}^* - \mathbf{Z}'_{it,2} \boldsymbol{\theta}_0 - \mathbf{Z}'_{it,2} \boldsymbol{\theta}_1(U_i - u_0)) K_h(U_i - u_0). \quad (8)$$

By moving u_0 along the domain of U_i , the entire estimated curve of the functional coefficient is obtained. Note that the programming involved in the above local linear quantile estimations given in (6) and (8) can be modified with few efforts from the existing programs for a linear quantile model.

2.2 Asymptotic results

This section provides asymptotic results of $\hat{\boldsymbol{\gamma}}_{\tau}$ and $\hat{\boldsymbol{\theta}}_{\tau}(U_i)$ defined in Section 2.1. All proofs are relegated to the appendices. Firstly, we give the following notations and definitions which will be used in the rest of the paper. Define $\mu_j = \int_{-\infty}^{\infty} u^j K(u) du$ and $\nu_j = \int_{-\infty}^{\infty} u^j K^2(u) du$ with $j > 0$. Let $\Omega(u_0) = E(\mathbf{Z}_{it} \mathbf{Z}'_{it} | U_i = u_0) \equiv E(\mathbf{Z}_{it} \mathbf{Z}'_{it} | u_0)$, in what follows. Define $\Omega^*(u_0) = E(\mathbf{Z}_{it} \mathbf{Z}'_{it} f_{Y|U,\mathbf{Z}}(Q_{\tau}(u_0, \mathbf{Z}_{it})) | u_0)$, and $\Omega_{1t}(u_0) = E(\mathbf{Z}_{i1} \mathbf{Z}'_{it} | u_0)$. Let h_1 be the bandwidth used at the first stage. Let $H = \text{diag}(\mathbf{1}_{K^*}, h_1 \mathbf{1}_{K_2^*})_{(K^*+K_2^*) \times (K^*+K_2^*)}$ and $G =$

$\begin{pmatrix} I_{K^*} \\ I_{K_2^* \times K_1^*}, U_{ih_1} I_{K_2^*} \end{pmatrix}_{(K^* + K_2^*) \times K^*}$, where $U_{ih_1} = (U_i - u_0)/h_1$, $K^* = K_1^* + K_2^*$, $K_1^* = K_1$ and $K_2^* = K_2 + KT$. Finally, denote $f_U(u)$ by the marginal density of U and $\psi_\tau(z) = \tau - I_{\{z < 0\}}$.

The following conditions are necessary to establish the consistency and asymptotic normality of our estimators, although they might not be the weakest ones. Most conditions listed below are similar to Cai and Xu (2008) and Cai and Xiao (2012).

Conditions: A

A1. The kernel function $K(\cdot)$ is a bounded density with a bounded support region.

A2. Assume that the functional coefficients $\boldsymbol{\theta}(u)$ are 2 times continuously differentiable in a small neighborhood of u_0 .

A3. The series $\{U_i\}$ is iid. The series $\{\mathbf{Z}_{it}\}$ is iid across individual i , but can be correlated around t for fixed i .

A4. Assume that bandwidth $h_1 \rightarrow 0$, $h_2 \rightarrow 0$, $Nh_1 \rightarrow \infty$ and $Nh_2 \rightarrow \infty$ as $N \rightarrow \infty$. Furthermore, $Nh_1^4 \rightarrow 0$.

Conditions: B

B1. The error distribution, F , has a continuous and strictly positive density, f .

B2. The marginal density smoothing variable U , $f_U(\cdot)$, is continuous with $f_U(u_0) > 0$.

B3. The conditional density of Y given U and \mathbf{Z} , $f_{Y|U,\mathbf{Z}}(\cdot)$ is bounded and satisfies the Lipschitz continuity condition.

B4. The kernel function $K(\cdot)$ is symmetric.

B5. Assume $\Omega(u_0)$ and $\Omega^*(u_0)$ are positive-definite and continuous in a neighborhood of u_0 .

B6. Assume that $E(\|\mathbf{Z}\|^{2\delta^*}) < \infty$ with $\delta^* > \delta > 2$.

To obtain the asymptotic properties of conditional quantile estimators, we need to firstly derive a local Bahadur representation for both estimators. Following Cai and Xu (2008) and Cai and Xiao (2012), we have

$$\sqrt{Nh_1}H \begin{pmatrix} \hat{\boldsymbol{\gamma}}_\tau(u_0) - \boldsymbol{\gamma}_\tau(u_0) \\ \hat{\boldsymbol{\theta}}_{0,\tau}(u_0) - \boldsymbol{\theta}_\tau(u_0) \\ \hat{\boldsymbol{\theta}}_{1,\tau}(u_0) - \dot{\boldsymbol{\theta}}_\tau(u_0) \end{pmatrix} = \frac{\tilde{\Omega}^{-1}(u_0)}{\sqrt{Nh_1}Tf_U(u_0)} \sum_{i=1}^N \sum_{t=1}^T G\mathbf{Z}_{it}\psi_\tau(\tilde{\varepsilon}_{it})K(U_{ih_1}) + o_p(1),$$

where $\tilde{\varepsilon}_{it} = Y_{it} - \mathbf{Z}'_{it,1}\boldsymbol{\gamma} - \mathbf{Z}'_{it,2}[\boldsymbol{\theta}_0(u_0) + \boldsymbol{\theta}_1(u_0)h_1U_{ih_1}]$, $\tilde{\Omega}(u_0) = \text{diag}(\Omega^*(u_0), \mu_2 e_0' \Omega^*(u_0) e_0)$

and $e'_0 = (\mathbf{0}_{K_2^* \times K_1^*}, \mathbf{I}_{K_2^*})$. In particular, we can obtain

$$\sqrt{Nh_1} \begin{pmatrix} \hat{\boldsymbol{\gamma}}_\tau(u_0) - \boldsymbol{\gamma}_\tau(u_0) \\ \hat{\boldsymbol{\theta}}_{0,\tau}(u_0) - \boldsymbol{\theta}_\tau(u_0) \end{pmatrix} = \frac{(\boldsymbol{\Omega}^*(u_0))^{-1}}{\sqrt{Nh_1 T} f_U(u_0)} \sum_{i=1}^N \sum_{t=1}^T \mathbf{Z}_{it} \psi_\tau(\tilde{\varepsilon}_{it}) K(U_{ih_1}) + o_p(1), \quad (9)$$

which is useful for establishing the asymptotic results for our estimators.

As mentioned above, a root- N consistent estimator of $\boldsymbol{\gamma}_\tau$ at the second stage is constructed by using the average method defined in (7). Theorem 1 states its asymptotic normality result and its detailed proof is presented in Appendices. Indeed, the theoretical proof of Theorem 1 follows by using the U-statistic technique in Powell, Stock and Stoker (1989) instead of the U-statistic technique used in Cai and Xiao (2012).

Theorem 1: *Suppose that Assumptions A and B hold, we have*

$$\sqrt{N} [\hat{\boldsymbol{\gamma}}_\tau - \boldsymbol{\gamma}_\tau - \mathbf{B}_\gamma] \xrightarrow{D} N \left(0, \frac{\tau(1-\tau)}{T} \Sigma_\gamma \right),$$

where $\Sigma_\gamma = E\{e'_1(\boldsymbol{\Omega}^*(U_i))^{-1}[\boldsymbol{\Omega}(U_i) + \sum_{t=2}^T \frac{2(T-t+1)}{T} \boldsymbol{\Omega}_{1t}(U_i)](\boldsymbol{\Omega}^*(U_i))^{-1}e_1\}$ and $\mathbf{B}_\gamma = \mu_2 h_1^2 (2\mathbf{B}_1^* - \mathbf{B}_2^*)$ in which $\mathbf{B}_1^* = e'_1 E[(\boldsymbol{\Omega}^*(U_i))^{-1} \dot{\boldsymbol{\Omega}}^*(U_i)] \begin{pmatrix} 0 \\ \dot{\boldsymbol{\theta}}_\tau(U_i) \end{pmatrix}$, $\mathbf{B}_2^* = e'_1 E[(\boldsymbol{\Omega}^*(U_i))^{-1} \boldsymbol{\Theta}(U_i)]$, $e'_1 = (\mathbf{I}_{K_1^*}, \mathbf{0}_{K_1^* \times K_2^*})$ and $\boldsymbol{\Theta}(U_i) = E\{\dot{f}_{Y|U_i, \mathbf{Z}}(Q_\tau(U_i, \mathbf{Z})) \mathbf{Z} [\mathbf{Z}'_2 \dot{\boldsymbol{\theta}}_\tau(U_i)]^2 | U_i\}$. In particular,

$$\sqrt{N} [\hat{\boldsymbol{\gamma}}_\tau - \boldsymbol{\gamma}_\tau] \xrightarrow{D} N \left(0, \frac{\tau(1-\tau)}{T} \Sigma_\gamma \right)$$

if $\sqrt{N} h_1^2 \rightarrow 0$.

From Theorem 1, the estimator $\hat{\boldsymbol{\gamma}}_\tau$ is root- N consistent when the bandwidth h_1 satisfies $\sqrt{N} h_1^2 \rightarrow 0$, which implies that it requires under-smoothed at the first stage. The bias term \mathbf{B}_γ is exactly the same as the one in Cai and Xiao (2012) but the asymptotic variance is different. The asymptotic variance in Theorem 1 also depends on T and the theorem demonstrates that larger T can make the estimation much better.

At the last stage, the partial residual \mathbf{Y}_{it}^* is used to estimate $\hat{\boldsymbol{\theta}}_{0,\tau}(u_0)$. To this effect, we introduce the following additional notations and definitions: $\boldsymbol{\Omega}_2(u_0) = E(\mathbf{Z}_{it,2} \mathbf{Z}'_{it,2} | u_0)$, $\boldsymbol{\Omega}_2^*(u_0) = E(\mathbf{Z}_{it,2} \mathbf{Z}'_{it,2} \dot{f}_{Y^*|U, \mathbf{Z}_2}(Q_\tau(U_i, \mathbf{Z}_{it,2})) | u_0)$, and $\boldsymbol{\Omega}_{1t,2}(u_0) = E(\mathbf{Z}_{i1,2} \mathbf{Z}'_{it,2} | u_0)$. Denote h_2 by the bandwidth used at the last stage. Finally, we define $G_2 = \begin{pmatrix} \mathbf{I}_{K_2^*} \\ U_{ih_2} \mathbf{I}_{K_2^*} \end{pmatrix}_{2K_2^* \times K_2^*}$ and

$H_2 = \text{diag}(\mathbf{1}_{K_2^*}, h_2 \mathbf{1}_{K_2^*})_{2K_2^* \times 2K_2^*}$. Thus, we can obtain

$$\sqrt{Nh_2} H_2 \begin{pmatrix} \hat{\boldsymbol{\theta}}_{0,\tau}(u_0) - \boldsymbol{\theta}_\tau(u_0) \\ \hat{\boldsymbol{\theta}}_{1,\tau}(u_0) - \dot{\boldsymbol{\theta}}_\tau(u_0) \end{pmatrix} = \frac{\check{\Omega}^{-1}(u_0)}{\sqrt{Nh_2 T f_U(u_0)}} \sum_{i=1}^N \sum_{t=1}^T G_2 \mathbf{Z}_{it,2} \psi_\tau(\check{\varepsilon}_{it}) K(U_{ih_2}) + o_p(1),$$

where $U_{ih_2} = (U_i - u_0)/h_2$, $\check{\varepsilon}_{it} = Y_{it}^* - \mathbf{Z}'_{it,2}[\boldsymbol{\theta}_0(u_0) + \boldsymbol{\theta}_1(u_0)h_2 U_{ih_2}]$ and $\check{\Omega}(u_0) = \text{diag}(\Omega_2^*(u_0), \mu_2 \Omega_2^*(u_0))$.

Similar to (9), we have

$$\sqrt{Nh_2} \begin{pmatrix} \hat{\boldsymbol{\theta}}_{0,\tau}(u_0) - \boldsymbol{\theta}_\tau(u_0) \\ \hat{\boldsymbol{\theta}}_{1,\tau}(u_0) - \dot{\boldsymbol{\theta}}_\tau(u_0) \end{pmatrix} = \frac{(\Omega_2^*(u_0))^{-1}}{\sqrt{Nh_2 T f_U(u_0)}} \sum_{i=1}^N \sum_{t=1}^T \mathbf{Z}_{it,2} \psi_\tau(\check{\varepsilon}_{it}) K(U_{ih_2}) + o_p(1),$$

which is useful to establish the asymptotic result for $\hat{\boldsymbol{\theta}}_{0,\tau}(u_0)$, stated in the following theorem.

Theorem 2: *Suppose that Assumptions A and B hold, given the square root- N consistent estimator of $\boldsymbol{\gamma}_\tau$, we have*

$$\sqrt{Nh_2} \left[\hat{\boldsymbol{\theta}}_{0,\tau}(u_0) - \boldsymbol{\theta}_\tau(u_0) - \frac{h_2^2}{2} \mu_2 \ddot{\boldsymbol{\theta}}_\tau(u_0) \right] \rightarrow N(0, \Sigma_\theta(u_0)),$$

where $\Sigma_\theta(u_0) = \frac{\tau(1-\tau)\nu_0}{T f_U(u_0)} \Sigma(u_0)$ and $\Sigma(u_0) = (\Omega_2^*(u_0))^{-1} [\Omega_2(u_0) + \sum_{t=2}^T \frac{2(T-t+1)}{T} \Omega_{1t,2}(u_0)] (\Omega_2^*(u_0))^{-1}$.

In particular,

$$\sqrt{Nh_2} \left[\hat{\boldsymbol{\beta}}_\tau(u_0) - \boldsymbol{\beta}_\tau(u_0) - \frac{h_2^2}{2} \mu_2 \ddot{\boldsymbol{\beta}}_\tau(u_0) \right] \rightarrow N(0, \Sigma_\beta(u_0)),$$

where $\Sigma_\beta(u_0)$ is the upper corner $K_2 \times K_2$ sub-matrix of $\Sigma_\theta(u_0)$,

Compared to Theorem 1 in Cai and Xu (2008) and Theorem 2 in Cai and Xiao (2012), the asymptotic bias term in the above theorem is the same but the asymptotic variance in our case depends on T . Also, it is easy to see that the asymptotic mean square error of $\hat{\boldsymbol{\beta}}_\tau(u_0)$ is of the order of $O(N^{-4/5})$, when the optimal bandwidth is chosen as $h_2 = cN^{-1/5}$ for some $c > 0$. This means that some conventional bandwidth selection procedures can be applied here to select the optimal bandwidth in a data-driven fashion. This deserves a further investigation.

2.3 Statistical inferences

After deriving the asymptotic results, we now turn to discussing statistical inferences such as constructing confidence intervals and testing hypotheses. To make statistical inferences for $\boldsymbol{\gamma}_\tau$ and $\boldsymbol{\beta}_\tau(\cdot)$ in practice, first one needs some consistent covariance estimators of Σ_γ and $\Sigma(u_0)/f_U(u_0)$. To this end, we need estimate $\Omega^*(u_0)$, $\Omega(u_0)$, $\Omega_{1t}(u_0)$, $\Omega_2^*(u_0)$, $\Omega_2(u_0)$

and $\Omega_{1t,2}(u_0)$ consistently. Since the estimation of $\Omega_2^*(u_0)$, $\Omega_2(u_0)$ and $\Omega_{1t,2}(u_0)$ is similar to $\Omega^*(u_0)$, $\Omega(u_0)$ and $\Omega_{1t}(u_0)$, therefore, the focus is only on the latter to save notations. Following Cai and Xu (2008), we define

$$\begin{aligned}\hat{\Omega}(u_0) &= (NT)^{-1} \sum_{i=1}^N \sum_{t=1}^T \mathbf{Z}_{it} \mathbf{Z}'_{it} K_h(U_i - u_0), \\ \hat{\Omega}_{1t}(u_0) &= (N(T-t))^{-1} \sum_{i=1}^N \sum_{s=1}^{T-t} \mathbf{Z}_{is} \mathbf{Z}'_{i(s+t)} K_h(U_i - u_0),\end{aligned}$$

and

$$\hat{\Omega}^*(u_0) = (NT)^{-1} \sum_{i=1}^N \sum_{t=1}^T \mathbf{Z}_{it} \mathbf{Z}'_{it} \hat{f}_{Y|U,\mathbf{Z}}(Q_\tau(U_i, \mathbf{Z}_{it})) K_h(U_i - u_0),$$

where

$$\hat{f}_{Y|U,\mathbf{Z}}(Q_\tau(u, \mathbf{z})) = \sum_{i=1}^N \sum_{t=1}^T K_h(U_i - u, \mathbf{Z}_{it} - \mathbf{z}) L_h(Y_{it} - Q_\tau(u, \mathbf{z})) \left[\sum_{i=1}^N \sum_{t=1}^T K_h(U_i - u, \mathbf{Z}_{it} - \mathbf{z}) \right]^{-1}$$

is the Nadaraya-Watson type double kernel method as in Fan, Yao and Tong (1996), and $L_h(\cdot)$ is another kernel function. It can be easily shown that $\hat{\Omega}(u_0) = f_U(u_0)\Omega(u_0) + o_p(1)$, $\hat{\Omega}_{1t}(u_0) = f_U(u_0)\Omega_{1t}(u_0) + o_p(1)$, and $\hat{\Omega}^*(u_0) = f_U(u_0)\Omega^*(u_0) + o_p(1)$. Finally, the consistent covariance estimators of Σ_γ and $\Sigma(u_0)/f_U(u_0)$ can be respectively given by

$$e_1' N^{-1} \sum_{i=1}^N (\hat{\Omega}^*(U_i))^{-1} [\hat{\Omega}(U_i) + \sum_{t=2}^T \frac{2(T-t+1)}{T} \hat{\Omega}_{1t}(U_i)] (\hat{\Omega}^*(U_i))^{-1} e_1,$$

and

$$(\hat{\Omega}_2^*(u_0))^{-1} [\hat{\Omega}_2(u_0) + \sum_{t=2}^T \frac{2(T-t+1)}{T} \hat{\Omega}_{1t,2}(u_0)] (\hat{\Omega}_2^*(u_0))^{-1}.$$

Therefore, the consistent estimate of $\Sigma_\theta(u_0)$ can be constructed accordingly in an obvious manner and so is $\Sigma_\beta(u_0)$.

In empirical studies, it is of importance to test the constancy of the varying coefficients. That is to test the null hypothesis defined as $H_0 : \boldsymbol{\beta}_\tau(u) = \boldsymbol{\beta}_\tau$. Under the null hypothesis, following Cai and Xiao (2012), a simple and easily implemented test statistics can be constructed as follows

$$T_N = \max_{1 \leq j \leq q} \|\sqrt{N h_2} \hat{\Sigma}_\beta(v_j)^{-1/2} (\hat{\boldsymbol{\beta}}_\tau(v_j) - \hat{\boldsymbol{\beta}}_\tau)\|^2,$$

where $\{v_j\}_{j=1}^q$ are any distinct points within the domain of U_i and $T_N \rightarrow \max_{1 \leq j \leq q} \chi_j^2(K_2) = T_q$, where $\chi_j^2(K_2)$ is the independent chi-square random variable with K_2 degrees of freedom.

Thus, the null is rejected if T_N is too large. The critical value of T_q can be easily tabulated since the distribution of T_q is a functional of independent chi-square random variables independent of nuisance parameters and quantiles. To improve the finite sample performance, one may use a bootstrap based test to the above hypothesis. Of course, some other type of test statistics may be constructed and it would be warranted as a future research topic to investigate the properties of those test statistics. See Cai and Xiao (2012) for details in particular, on the choice of $\{v_j\}_{j=1}^q$ and q in practice.

3 Monte Carlo Simulations

In this section, Monte Carlo simulations are conducted to demonstrate the finite sample performance of both estimators. To measure the estimation performance of $\hat{\gamma}_{j,\tau}$ for $0 \leq j \leq 2$ and $\hat{\beta}_\tau(\cdot)$, the mean absolute deviation error (MADE) for $\hat{\beta}_\tau(\cdot)$ is defined by

$$\text{MADE}(\hat{\beta}_\tau(\cdot)) = \frac{1}{n_0} \sum_{l=1}^{n_0} |\hat{\beta}_\tau(u_l) - \beta_\tau(u_l)|$$

where $\{u_l\}_{l=1}^{n_0}$ are the grid points within the domain of U_i , and the absolute deviation error (ADE) for $\gamma_{j,\tau}$ is given by

$$\text{ADE}(\hat{\gamma}_{j,\tau}) = |\hat{\gamma}_{j,\tau} - \gamma_{j,\tau}|$$

for $0 \leq j \leq 1$.

We consider the following data generating process

$$Y_{it} = \gamma_0 + X_{it,1}\gamma_1 + X_{it,2}\beta(U_i) + \alpha_i + \epsilon_{it}$$

with $\alpha_i = \sum_{t=1}^T [X_{it,1}\delta_{t,1}(U_i) + X_{it,2}\delta_{t,2}(U_i)]$, and $\epsilon_{it} = (\varphi_0 + X_{it,1}\varphi_1 + X_{it,2}\varphi_2)u_{it}$, where the smoothing variable U_i is generated from *iid* Uniform(-3, 3), $X_{it,1}$ and $X_{it,2}$ are respectively generated from the *iid* Uniform(2, 10) and Uniform(3, 8), and the error term u_{it} is generated from *iid* N(0, 1). Therefore, $\gamma_{0,\tau} = \gamma_0 + \phi_0\Phi^{-1}(\tau)$, $\gamma_{1,\tau} = \gamma_1 + \phi_1\Phi^{-1}(\tau)$, and $\beta_\tau(u) = \beta(u) + \phi_2\Phi^{-1}(\tau)$, where $\Phi(\cdot)$ is the distribution of the standard normal. The constant coefficients are set by $\gamma_0 = 4$, $\gamma_1 = 1.5$, $\varphi_0 = 0.5$, $\varphi_1 = 0.3$, and $\varphi_2 = 0.2$, respectively. The functional coefficients are defined as $\beta(u) = 1.5 \cos(2u) + 0.5u$, $\delta_{1,1}(u) = 1.5e^{-u^2}$, $\delta_{1,2}(u) = \sin(1.5u)$, $\delta_{2,1}(u) = 0.1(u-1)^2 + 0.3u^3$, and $\delta_{2,2}(u) = e^{\sin(u)}$.

We take $T = 2$ and $N = 500, 800$ and 1000 respectively. For a given sample size, we repeat 500 times of simulations to calculate the ADE or MADE values. We compare the

estimation results using different bandwidths, such as $h_1 = 5N^{-2/5}$ and $h_2 = cN^{-1/5}$, where c is chosen from 0.8, 1.0, 1.2, 1.5, 1.7, 2, \dots . In Table 1, the simulation results for the estimator of the constant coefficients are summarized. From Table 1, one can see that

Table 1: The Median and SD of the ADE Values for $\hat{\gamma}_{0,\tau}$ and $\hat{\gamma}_{1,\tau}$

N	$\tau = 0.3$		$\tau = 0.5$		$\tau = 0.7$	
	$\gamma_{0,\tau}$	$\gamma_{1,\tau}$	$\gamma_{0,\tau}$	$\gamma_{1,\tau}$	$\gamma_{0,\tau}$	$\gamma_{1,\tau}$
500	0.109918 (0.045729)	0.074976 (0.025039)	0.026215 (0.022304)	0.015286 (0.013728)	0.116270 (0.042774)	0.074873 (0.024343)
800	0.098042 (0.034524)	0.072410 (0.018952)	0.018198 (0.016489)	0.011117 (0.009983)	0.103390 (0.032699)	0.071485 (0.018808)
1000	0.098407 (0.029876)	0.070720 (0.017287)	0.017121 (0.015570)	0.010360 (0.009283)	0.094826 (0.029437)	0.069539 (0.017233)

Table 2: The Median and SD of the MADE Values for $\hat{\beta}_\tau(\cdot)$

N	$\tau = 0.3$			$\tau = 0.5$			$\tau = 0.7$		
	$c = 0.8$	$c = 1.2$	$c = 1.7$	$c = 0.8$	$c = 1.2$	$c = 1.7$	$c = 0.8$	$c = 1.2$	$c = 1.7$
500	0.0989 (0.0159)	0.0868 (0.0161)	0.1141 (0.0171)	0.0725 (0.0136)	0.0585 (0.0122)	0.0892 (0.0152)	0.0820 (0.0146)	0.0735 (0.0151)	0.0929 (0.0169)
800	0.0747 (0.0134)	0.0767 (0.0139)	0.0982 (0.0134)	0.0446 (0.0087)	0.0471 (0.0098)	0.0734 (0.0123)	0.0682 (0.0127)	0.0638 (0.0123)	0.0773 (0.0128)
1000	0.0832 (0.0120)	0.0730 (0.0126)	0.0930 (0.0119)	0.0558 (0.0097)	0.0406 (0.0082)	0.0678 (0.0101)	0.0648 (0.0109)	0.0583 (0.0112)	0.0733 (0.0116)

the estimation of constant coefficients is not sensitive to the choice of the bandwidth when the first step is under-smoothed. Table 2 reports the simulation results for the estimator of varying coefficients for $c = 0.8$, $c = 1.2$ and $c = 1.7$, respectively. We find from Table 2 that the estimation of $\beta_\tau(\cdot)$ is quite stable when the bandwidth selection is chosen within a reasonable range. In both tables, the standard deviation (SD) of the 500 ADE or MADE values is presented in parentheses.

From both tables, one can observe that both the medians of 500 ADE or MADE values all estimates decrease significantly as N increases at all settings. For example, when the sample size increases from 500 to 1000, the medians of ADE or MADE values for $\hat{\gamma}_{0,0.5}$, $\hat{\gamma}_{1,0.5}$ and $\hat{\beta}_{0.5}(\cdot)$ shrink quickly from 0.0262 to 0.0171, from 0.0153 to 0.0103, and from 0.0585 to 0.0406, respectively. The standard deviations also shrink quickly when the sample size is enlarged.

For example, for $\hat{\gamma}_{0,0.5}$, $\hat{\gamma}_{1,0.5}$ and $\hat{\beta}_{0.5}(U_i)$, the standard deviations shrink from 0.0223 to 0.0156, from 0.0137 to 0.0093, and from 0.0122 to 0.0082, respectively. Similar results can be observed at the lower quantiles, $\tau = 0.3$, and the upper quantile, $\tau = 0.7$. This is in line with our asymptotic theory and implies that our proposed estimators are indeed consistent. Furthermore, the performance at the median quantile, $\tau = 0.5$, is slightly better than those at tails for $\tau = 0.3$ and $\tau = 0.7$, which is due to the fact of the sparsity of data observations in the tailed regions. Compared to Table 2, the shrinkage speed in Table 1 is relatively fast, which is also consistent with the theoretical results in the previous sections. Finally, from Table 2, one may conclude that the performance for $c = 2$ is best among three values of c in the bandwidth at the second stage $h_2 = cN^{-1/5}$, which shows that the optional bandwidth may be around $h_2 = 1.2N^{-1/5}$. Therefore, one may conclude from the above simulation results that the finite sample performance of the proposed estimators is reasonably well.

4 Modeling the effect of FDI on economic growth

4.1 The empirical econometric model

The existing literature presented contradictory empirical evidences on whether or not FDI can promote economic growth in host countries. Recent studies in the literature tried to find the sources of the mixed empirical conclusions and they concluded that the reason may be due to nonlinearities in FDI effects on economic growth and the heterogeneity among countries. In this section, we employ the aforementioned novel quantile panel data mode to deal with the nonlinearities and heterogeneity in a simultaneous fashion. We estimate a semiparametric quantile empirical growth equation which allows the effect of FDI on economic growth to depend on the initial condition in the host country. The following is the typical empirical growth equation

$$y_{it} = \alpha_i + \beta_1(\text{FDI}/Y)_{it} + \beta_2 \log(\text{DI}/Y)_{it} + \beta_3 n_{it} + \beta_4 h_{it} + \epsilon_{it}. \quad (10)$$

In the above model, y_{it} denotes the growth rate of GDP per capita in the country or region i during the period t , n_{it} is the logarithm of population growth rate and h_{it} is the human capital. The FDI and DI in (10) refer to foreign direct investment and domestic investment respectively and Y represents the total output. Hence, $(\text{FDI}/Y)_{it}$ denotes the average ratio between the FDI and the total output during the period t in country i and $(\text{DI}/Y)_{it}$ is defined

in the same fashion for the domestic investment. α_i is the individual effect used to control the unobserved country-specific heterogeneity. To allow the possible joint effect of FDI and human capital, some literatures; see Li and Liu (2004) and Kottaridi and Stengos (2010), among others, considered to add an interacted term between FDI and human capital into the empirical growth model, then (10) becomes

$$y_{it} = \alpha_i + \beta_1(\text{FDI}/Y)_{it} + \beta_2 \log(\text{DI}/Y)_{it} + \beta_3 n_{it} + \beta_4 h_{it} + \beta_5((\text{FDI}/Y)_{it} \times h_{it}) + \epsilon_{it}, \quad (11)$$

which is indeed a nonlinear parametric model.

Since the majority of the literature realized that the effect of FDI on economic growth depends on the absorptive capacity in host countries and the initial GDP per capita is one of the most important indicators to reflect the initial conditions and the absorptive capacity in the host country; see Nunnemkamp (2004), among others, we hereby propose a partially varying-coefficient model which allows the effect of FDI on economic growth to depend on the initial GDP per capita in the host country. Hence, our empirical growth model is given by

$$y_{it} = \alpha_i + \beta_1(U_i)(\text{FDI}/Y)_{it} + \beta_2 \log(\text{DI}/Y)_{it} + \beta_3 n_{it} + \beta_4 h_{it} + \beta_5((\text{FDI}/Y)_{it} \times h_{it}) + \epsilon_{it}, \quad (12)$$

where U_i is the logarithm of initial GDP per capita in country i and $\beta_1(U_i)$ is the varying coefficient over the logarithm of initial GDP per capita U_i . Therefore, model (12) has an ability to characterize how the FDI may have different effects on economic growth under the different initial conditions.

As we discussed in the introduction, the conditional mean model (12) is usually not sufficiently enough to control the heterogeneity among countries. The existing literature dealt with the aforementioned issue by simply looking at sub-samples. Instead, in this paper, we propose to adopt quantile regression approach to investigate the impact of FDI on economic growth. Our method is capable of dealing with heterogeneity among countries by allowing different quantiles to have different empirical growth equations, and at the same time, we can avoid splitting the sample. Different from the mean model, another advance of considering the quantile model is that one can see how the FDI effects differently on the economic growth in the different groups of countries, say the economy fast growing countries (upper quantile) and the economy slowly growing countries (lower quantile).

By assuming that the ϵ_{it} in (12) takes a linearly heteroscedastic form as $\epsilon_{it} = (\mathbf{X}'_{it}\boldsymbol{\varphi})u_{it}$ (Koenker and Bassett (1978)), where \mathbf{X}_{it} includes all regressors in (12) and u_{it} is independent of all covariates but given i , u_{it} is allowed to be correlated around t , then we can obtain the following conditional quantile model:

$$\mathbf{Q}_\tau(y_{it} | U_i, \mathbf{X}_i) = \alpha_i + \beta_{1,\tau}(U_i)(\text{FDI}/Y)_{it} + \beta_{2,\tau} \log(\text{DI}/Y)_{it} + \beta_{3,\tau} n_{it} + \beta_{4,\tau} h_{it} + \beta_{5,\tau} ((\text{FDI}/Y)_{it} \times h_{it}), \quad (13)$$

where $\mathbf{X}_i = \{\mathbf{X}_{it}\}_{t=1}^T$, which can be regarded as a special case of model (2). Imposing the correlated random effect assumption in (3), we can derive the conditional quantile regression model in (4) and then the transformed model in (5) for this empirical example. Therefore, the three-stage estimation procedure described in Section 2 can be applied here to estimate the coefficients.

4.2 The data and empirical results

Our data set includes 95 countries or regions from 1970 to 1999. In order to smooth the yearly fluctuations in aggregate economic variables, we take five-year averages by following the convention of the empirical growth literature as in Maasoumi, Racine and Stengos (2007), Durlauf, Kourtellos and Tan (2008), and Kottaridi and Stengos (2010). The population growth is computed by the average annual growth rate in each period, the human capital is measured as mean years of schooling in each period, and the domestic investment refers to the average of the domestic gross fixed capital formation measured by the US dollars in 2000 constant values. We measure the initial GDP by the GDP per capita of each country in the beginning year of each decade in constant 2000 US dollars.⁶ All the above data are available to be downloaded from World Development Indicators (WDI). The FDI flows, in constant 2000 US dollars, are taken from United Nations Conference on Trade and Development (UNCTAD). The full list of countries and regions can be found in Table 5 in Appendix A.

Firstly, we consider the classical linear regression model in (11). Table 3 presents corresponding estimation results, including coefficient estimates, standard deviations, t -statistic values and p-values from Column 2 to Column 5. The estimate of the FDI effect, denoted by β_1 , is about 0.56, which is positive and significant with a p-value of 0.027. On average, the

⁶We combine three decades, from 1970 to 1979 (69 countries), from 1980 to 1989 (93 countries), and from 1990 to 1999 (95 countries), and then obtain a panel of 514 observations with $N = 257$ and $T = 2$.

linear conditional mean model reports a mild positive effect of FDI on promoting economic growth. Compared to the growth effect of FDI, Table 3 reports a larger effect of domestic investments on economic growth, which is about 2.72 and highly significant with the p-value of 0.009. The effect of population growth (β_2) is also positive and significant, with an estimate of 0.65. However, other estimates (the effect of human capital and the effect of the interacted term between human capital and FDI) are not significant.

Table 3: Empirical Results of a Linear Conditional Mean Model in (11)

Mean Model	Coefficient	Standard Deviation	T-value	P-value
β_1	0.55887	0.25128	2.224	0.02703 *
β_2	2.71795	1.03743	2.620	0.00933 **
β_3	0.64828	0.25749	2.518	0.01243 *
β_4	-0.02904	0.42691	-0.068	0.94582
β_5	-0.03587	0.03752	-0.956	0.34006

Next, we move to the partially varying-coefficient conditional mean model in (12). Compared to the linear model in (11), we now allow the effect of FDI to depend on the initial conditions. Figure 1 and Table 4 present the corresponding estimation results. The solid line in Figure 1 represents the nonparametric estimates of the varying coefficient $\beta_1(\cdot)$ along various values of initial GDP, and the shaded area is the corresponding 90% pointwise confidence intervals with the bias ignored. The solid line in Figure 1 present the estimates of varying coefficients and the dashed lines denote the corresponding 90% confidence intervals. The estimates show a mild but clear pattern that the growth effect of FDI increases as the initial GDP improves, which is in line with the hypothesis of the absorptive capacity. The range of the estimates of the varying coefficient is between 0.9 and 1.5 for different initial GDPs, much larger than 0.56, the estimate of the linear model. Table 4 reports the estimates of constant coefficients in (12), which are quite different from the corresponding estimation results in Table 3. For example, the estimate of β_2 is now 3.81 in stead of 2.72 in Table 3. The impact of population growth rate on economic growth now becomes to be significantly negative with an estimate of -1.18 . Moreover, both the coefficients of human capital and the interacted term between FDI and human capital become significant in Table 4. The estimate of the impact of human capital is positive with a value of 0.17 and the estimate of the interacted term is -0.17 . We attribute the different estimation results to the existence

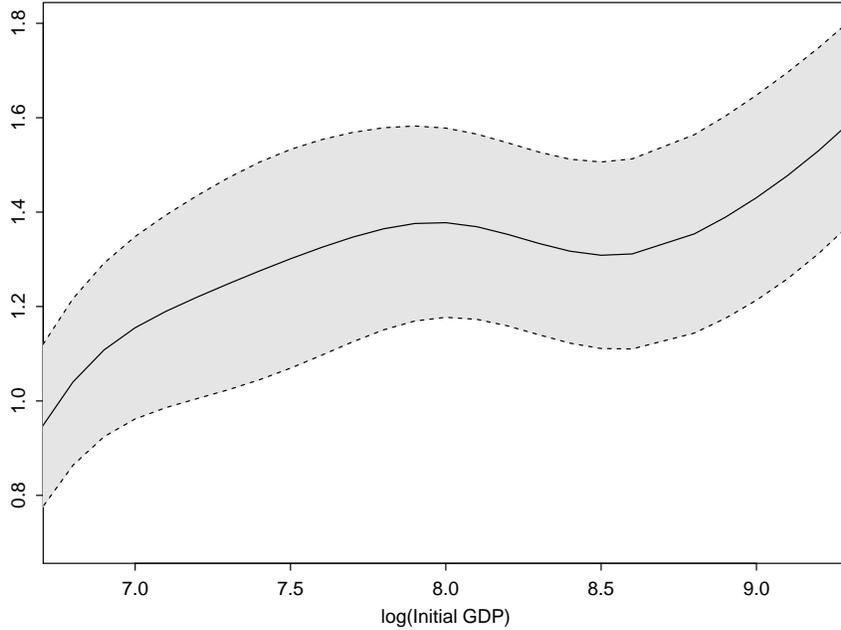


Figure 1: Estimated Curve of Functional Coefficient $\beta_1(\cdot)$ in Model (12).

of nonlinearity in the regression model.

Table 4: Constant Coefficients of a Partial Linear Conditional Mean Model in (12)

Mean Model	Coefficient	Standard Deviation	T-value	P-value
β_2	3.8100361	0.13214332	28.83260463	0.0000 ***
β_3	-1.1837808	0.33564848	-3.526846896	0.0004 ***
β_4	0.1728033	0.02678347	6.45186378	0.0000 ***
β_5	-0.1752624	0.01127884	-15.5390448	0.0000 ***

Finally, we consider the partially varying-coefficient quantile model in (13). Figure 2 presents estimates of all four constant coefficients $\beta_{j,\tau}$ for $2 \leq j \leq 5$ under different quantiles. The horizontal axis represents different quantiles and the vertical axis measures the values of estimators. The curves in solid line denote the estimates under different quantiles and the areas in gray color are corresponding 90% confidence intervals. The horizontal solid lines denote the conditional mean estimates. Except the estimates of $\beta_{3,\tau}$ in the upper left panel in Figure 2, most quantile estimates are outside the 90% confidence intervals of the conditional mean estimates, implying that the conditional mean model is not adequate to catch the heterogeneity effect. Moreover, we observe that the quantile estimates of $\beta_{2,\tau}$ and

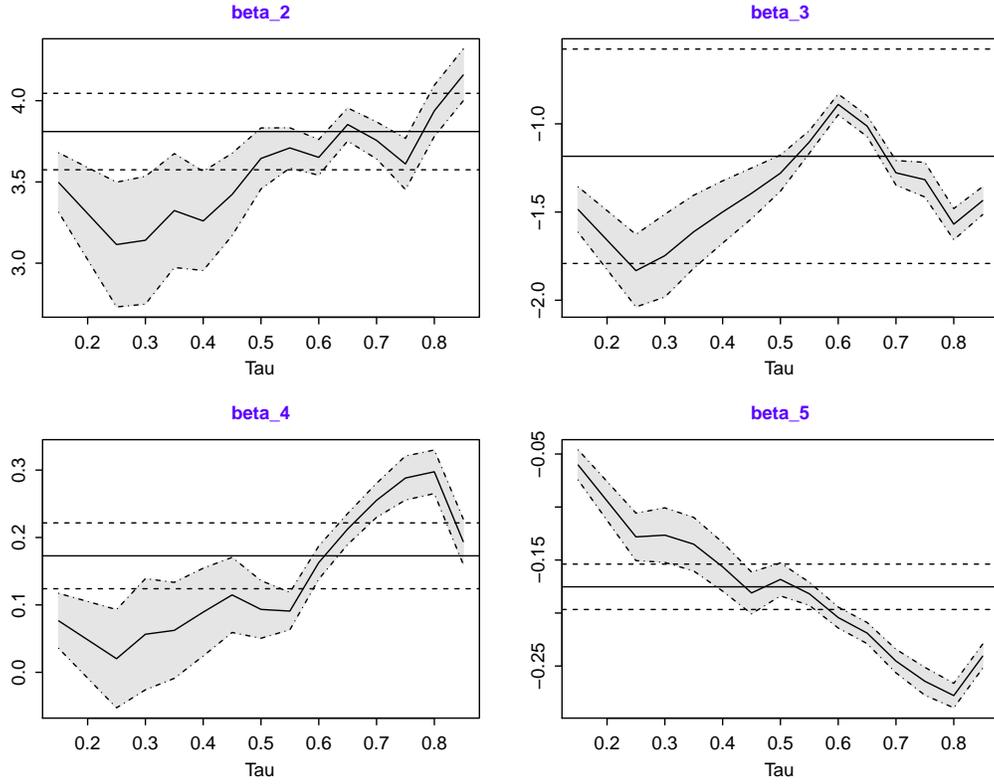


Figure 2: Estimated Results of Constant Coefficients $\beta_{j,\tau}$ in Model (13) for $2 \leq j \leq 5$.

$\beta_{4,\tau}$ increase with τ but the quantile estimates of $\beta_{5,\tau}$ decrease, when τ is in the range from 0.2 to 0.8. Hence, generally speaking, we find evidence that domestic investments and human capitals have positive effects on economic growth, but these effects are larger in countries or regions with better economic growth performance than those with poor growth performance.

The nonparametric estimates of varying coefficient $\beta_{1,\tau}(\cdot)$ with upper ($\tau = 0.85$) and lower ($\tau = 0.15$) quantiles are demonstrated in Figure 3. The horizontal axis measures different values of initial GDP U_i and the vertical axis measures the values of nonparametric estimates. The dark shaded areas represent the 90% pointwise confidence intervals of quantile estimates with the bias ignored. For comparison, we also include the conditional mean varying coefficient, denoted by the solid line in the middle, in Figure 3. For the upper quantile, we observe a strong pattern that the estimated growth effect of FDI increases with the value of initial GDP. However, for the lower quantile, the estimated curve seems to be flat along

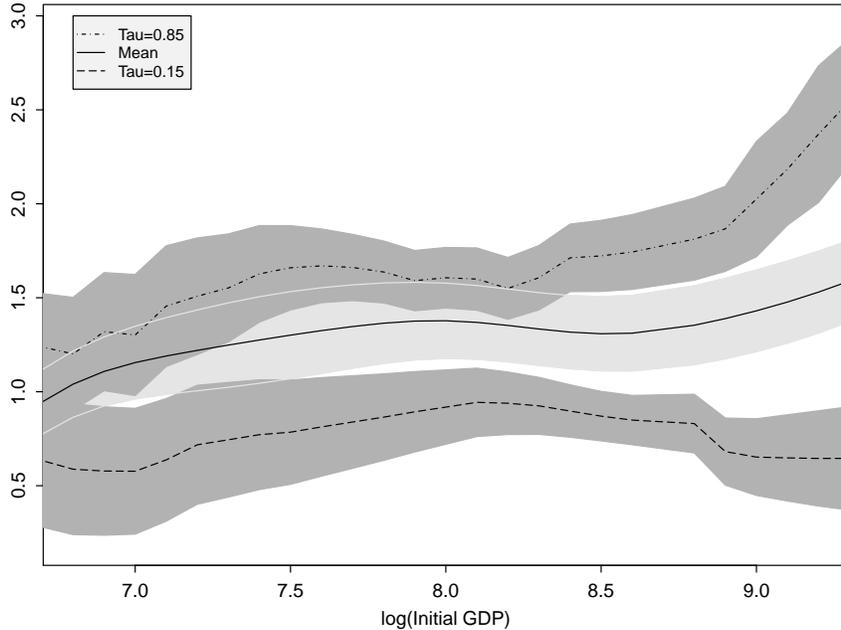


Figure 3: Estimated Curves of Functional Coefficient $\beta_{1,\tau}(\cdot)$ in Model (13).

different levels of initial GDP. We conduct a constancy test as in Section 3.3 to testing whether the coefficient $\beta_{1,\tau}(\cdot)$ does not vary with the initial GDP at different quantiles. It turns out that the p-values are 0.999, 0.000 and 0.000 for 0.15, 0.5 and 0.85 quantiles, respectively. The test strongly rejects the null of constancy in upper and median quantiles but it can not reject the constancy for the lower quantile. All these results verify the existence of the heterogeneity among countries and regions with different development stages.

5 Conclusion

Quantile panel data models have gained a lot of attentions in the literature during the recent years. In this paper, we propose a partially varying-coefficient quantile panel data model with correlated random effects. Compared to quantile panel data models with fixed effect, our estimation only depends on large N but fixed T , while the fixed effect model requires both N and T going to infinity. In our semiparametric model, we allow some of the coefficients to be a function of some smoothing variables while other coefficients are constant. We show that our estimator of varying coefficients is asymptotic normality in a nonparametric rate

and our estimator of constant coefficients is root-N consistent. This novel quantile panel data model is applied to estimate the impact of FDI on economic growth. There are several issues worth of future studies. For example, it is interesting to extending the current model by allowing for cross sectional dependence, fixed individual effect, and endogeneity which is very challenging as addressed in Li and Liu (2004), Durlauf, Kourtellos and Tan (2008), and Henderson, Papageorgiou and Parmeter (2012). Also, from the dynamic growth point of view, our model may be extended to the semiparametric dynamic panel model. Of course, it would be warranted as future research topics to investigate those models.

Appendix A: Table of Countries and Regions

Table 5: Countries

Algeria	Australia	Austria	Bahrain
Bangladesh	Barbados	Belgium	Benin
Bolivia	Botswana	Brazil	Cameroon
Canada	Central African Republic	Chile	China
Colombia	Congo, Rep.	Costa Rica	Cyprus
Denmark	Dominican Republic	Ecuador	Egypt, Arab Rep.
El Salvador	Fiji	Finland	France
Gambia	Germany	Ghana	Greece
Guatemala	Guyana	Honduras	Hong Kong SAR, China
Hungary	Iceland	India	Indonesia
Iran, Islamic Rep.	Ireland	Israel	Italy
Jamaica	Japan	Jordan	Kenya
Korea, Rep.	Lesotho	Malawi	Malaysia
Mali	Malta	Mauritius	Mexico
Mozambique	Nepal	Netherlands	New Zealand
Nicaragua	Niger	Norway	Pakistan
Panama	Papua New Guinea	Paraguay	Peru
Philippines	Poland	Portugal	Rwanda
Senegal	Sierra Leone	Singapore	South Africa
Spain	Sri Lanka	Sudan	Swaziland
Sweden	Switzerland	Syrian Arab Republic	Thailand
Togo	Trinidad and Tobago	Tunisia	Turkey
Uganda	United Kingdom	United States	Uruguay
Venezuela, RB	Zambia	Zimbabwe	

Appendix B: Proof of Theorem 1

It follows from Cai and Xu (2008) and Cai and Xiao (2012) that for any u_0 ,

$$\begin{aligned} & \sqrt{Nh_1} \begin{pmatrix} \hat{\gamma}_\tau(u_0) - \gamma_\tau(u_0) \\ \hat{\boldsymbol{\theta}}_{0,\tau}(u_0) - \boldsymbol{\theta}_\tau(u_0) \end{pmatrix} \simeq \frac{h_1}{\sqrt{Nh_1T}} \sum_{i=1}^N \sum_{t=1}^T B^{-1}(u_0) Z(u_0, \mathbf{Z}_{it}) \\ & + \frac{h_1}{\sqrt{Nh_1T}} \sum_{i=1}^N \sum_{t=1}^T B^{-1}(u_0) \mathbf{Z}_{it} [\psi_\tau(\tilde{\varepsilon}_{it}) - \psi_\tau(u_0, \mathbf{Z}_{it})] K_h(U_i - u_0), \end{aligned}$$

where $B(u_0) = f_U(u_0)\Omega^*(u_0)$, $Z(u_0, \mathbf{Z}_{it}) = \mathbf{Z}_{it}\psi_\tau(u_0, \mathbf{Z}_{it})K_h(U_i - u_0)$ and $\psi_\tau(u_0, \mathbf{Z}_{it}) = \tau - I\{Y_{it} < Q_\tau(u_0, \mathbf{Z}_{it})\} = \tau - I\{Y_{it} < \mathbf{Z}'_{it,1}\boldsymbol{\gamma}_\tau + \mathbf{Z}'_{it,2}\boldsymbol{\theta}_\tau(u_0)\}$ for U_i in a small neighborhood of u_0 . In particular,

$$\hat{\gamma}_\tau(u_0) - \gamma_\tau(u_0) \simeq \frac{1}{NT} \sum_{i=1}^N \sum_{t=1}^T e'_1 B^{-1}(u_0) Z(u_0, \mathbf{Z}_{it}) + B_N(u_0)$$

holds uniformly for all u_0 under Assumption A, where $B_N(u_0) = \frac{1}{NT} \sum_{i=1}^N \sum_{t=1}^T e'_1 B^{-1}(u_0) \mathbf{Z}_{it} [\psi_\tau(\tilde{\varepsilon}_{it}) - \psi_\tau(u_0, \mathbf{Z}_{it})] K_h(U_i - u_0)$. Thus,

$$\begin{aligned} \hat{\gamma}_\tau - \gamma_\tau &= \frac{1}{N} \sum_{i=1}^N [\hat{\gamma}_\tau(U_i) - \gamma_\tau(U_i)] \\ &= \frac{1}{N^2} \sum_{i=1}^N \sum_{j=1}^N \frac{1}{T} \sum_{t=1}^T e'_1 B^{-1}(U_i) Z(U_i, \mathbf{Z}_{jt}) + \frac{1}{N} \sum_{i=1}^N B_N(U_i) \\ &= \frac{2}{N^2} \sum_{1 \leq i < j \leq N} e'_1 B^{-1}(U_i) \frac{1}{T} \sum_{t=1}^T Z(U_i, \mathbf{Z}_{jt}) + \frac{1}{N} \sum_{i=1}^N B_N(U_i) \\ &= \frac{1}{N^2} \sum_{1 \leq i < j \leq N} [e'_1 B^{-1}(U_i) \frac{1}{T} \sum_{t=1}^T Z(U_i, \mathbf{Z}_{jt}) + e'_1 B^{-1}(U_j) \frac{1}{T} \sum_{t=1}^T Z(U_j, \mathbf{Z}_{it})] + \frac{1}{N} \sum_{i=1}^N B_N(U_i) \\ &= \frac{N-1}{2N} \mathbb{U}_N + \mathbb{B}_N, \end{aligned}$$

where $\mathbb{B}_N = \frac{1}{N} \sum_{i=1}^N B_N(U_i)$ and $\mathbb{U}_N = \frac{2}{N(N-1)} \sum_{1 \leq i < j \leq N} p_N(\xi_i, \xi_j)$ with

$$p_N(\xi_i, \xi_j) = e'_1 B^{-1}(U_i) \frac{1}{T} \sum_{t=1}^T Z(U_i, \mathbf{Z}_{jt}) + e'_1 B^{-1}(U_j) \frac{1}{T} \sum_{t=1}^T Z(U_j, \mathbf{Z}_{it}).$$

Define $r_N(\xi_i) = E[p_N(\xi_i, \xi_j) | \xi_i]$, $\theta_N = E[r_N(\xi_i)] = E[p_N(\xi_i, \xi_j)]$, and $\hat{\mathbb{U}}_N = \theta_N + \frac{2}{N} \sum_{i=1}^N [r_N(\xi_i) - \theta_N]$. The following two lemmas are useful to prove Theorem 1 and their detailed proofs are relegated to Appendix D.

Lemma 2.1: Under the assumptions in Theorem 1, we have

$$\begin{aligned}
(i) \quad r_N(\xi_i) &= e'_1(\Omega^*(U_i))^{-1} \frac{1}{T} \sum_{t=1}^T \mathbf{Z}_{it} \psi_\tau(U_i, \mathbf{Z}_{it}) + o(1), \\
(ii) \quad \theta_N &= \mu_2 h_1^2 (2\mathbf{B}_1^* - \mathbf{B}_2^*) + o(h_1^2), \\
(iii) \quad \text{Var}[r_N(\xi_i)] &= \boldsymbol{\Sigma}_\gamma + o(h_1).
\end{aligned}$$

Lemma 2.2: Under the assumptions in Theorem 1, we have

$$\mathbb{B}_N = \mu_2 h_1^2 (-2\mathbf{B}_1^* + \mathbf{B}_2^*) + o(h_1^2).$$

Proof of Theorem 1: First, note that $E[|p_N(\xi_i, \xi_j)|^2] = O(h^{-1}) = O[N(Nh_1)^{-1}] \rightarrow o(N)$ if and only if $Nh_1 \rightarrow \infty$ as $h_1 \rightarrow 0$. Lemma 3.1 in Powell, Stock and Stoker (1989) gives that $\sqrt{N}(\mathbb{U}_N - \hat{\mathbb{U}}_N) = o_p(1)$. Then the result follows from Lemma 2.1, Lemma 2.2 and the Lindeberg-Lévy central limit theorem.

Appendix C: Proof of Theorem 2

For a given root- N consistent estimator $\hat{\gamma}_\tau$ of γ_τ , it follows from (9) that

$$\begin{aligned}
\sqrt{Nh_2}(\hat{\boldsymbol{\theta}}_{0,\tau} - \boldsymbol{\theta}_\tau(u_0)) &\simeq \frac{(\Omega_2^*(u_0))^{-1}}{\sqrt{Nh_2}Tf_U(u_0)} \sum_{i=1}^N \sum_{t=1}^T \mathbf{Z}_{it,2} \psi_\tau(\tilde{\varepsilon}_{it}) K(U_{ih_2}) \\
&= \frac{(\Omega_2^*(u_0))^{-1}}{\sqrt{Nh_2}Tf_U(u_0)} \sum_{i=1}^N \sum_{t=1}^T \mathbf{Z}_{it,2} [\psi_\tau(Y_{it}^* - \check{c}_{1\tau}) - \zeta_{it}] K(U_{ih_2}) \\
&+ \frac{(\Omega_2^*(u_0))^{-1}}{\sqrt{Nh_2}Tf_U(u_0)} \sum_{i=1}^N \sum_{t=1}^T \mathbf{Z}_{it,2} \zeta_{it} K(U_{ih_2}) \\
&\equiv \mathbf{B}_N + \boldsymbol{\zeta}_N,
\end{aligned}$$

where $\check{c}_{1\tau} = Q_\tau(u_0, \mathbf{Z}_{it,2}) + \mathbf{Z}'_{it,2} \dot{\boldsymbol{\theta}}_\tau(u_0) h_2 U_{ih_2}$ and $\zeta_{it} = \psi_\tau[Y_{it}^* - Q_\tau(U_i, \mathbf{Z}_{it,2})]$. We will show that the first term \mathbf{B}_N determines the asymptotic bias and the second term $\boldsymbol{\zeta}_N$ gives the asymptotic normality.

First, note that

$$\begin{aligned}
E(\zeta_{it}|U_i, \mathbf{Z}_{it,2}) &= E(\psi_\tau[Y_{it}^* - Q_\tau(U_i, \mathbf{Z}_{it,2})]|U_i, \mathbf{Z}_{it,2}) \\
&= E(\tau - I\{[Y_{it}^* < Q_\tau(U_i, \mathbf{Z}_{it,2})]\}|U_i, \mathbf{Z}_{it,2}) \\
&= \tau - E(I\{[Y_{it}^* < Q_\tau(U_i, \mathbf{Z}_{it,2})]\}|U_i, \mathbf{Z}_{it,2}) \\
&= \tau - \tau = 0,
\end{aligned} \tag{14}$$

and

$$\begin{aligned}
E(\zeta_{it}^2|U_i, \mathbf{Z}_{it,2}) &= E(\psi_\tau^2[Y_{it}^* - Q_\tau(U_i, \mathbf{Z}_{it,2})]|U_i, \mathbf{Z}_{it,2}) \\
&= E(\tau^2 - (2\tau - 1)I\{[Y_{it}^* < Q_\tau(U_i, \mathbf{Z}_{it,2})]\}|U_i, \mathbf{Z}_{it,2}) \\
&= \tau^2 - (2\tau - 1)E(I\{[Y_{it}^* < Q_\tau(U_i, \mathbf{Z}_{it,2})]\}|U_i, \mathbf{Z}_{it,2}) \\
&= \tau^2 - (2\tau - 1)\tau = \tau(1 - \tau).
\end{aligned}$$

Thus,

$$E(\boldsymbol{\zeta}_N) = \frac{(\Omega_2^*(u_0))^{-1}}{\sqrt{Nh_2}f_U(u_0)}NE[\mathbf{Z}_{it,2}E(\zeta_{it}|U_i, \mathbf{Z}_{it,2})K(U_{ih_2})] = 0,$$

and

$$\text{Var}(\boldsymbol{\zeta}_N) = \Sigma_\theta(u_0) = \frac{\tau(\tau - 1)\nu_0}{Tf_U(u_0)}\Sigma(u_0).$$

Let $Q_N = \frac{1}{NT} \sum_{i=1}^N \sum_{t=1}^T \mathbf{Z}_{it,2} \zeta_{it} K(U_{ih_2})$. Using the Cramer-Wold device, for any $\mathbf{d} \in R^{K_2^*}$, define $Z_{N,it} = \sqrt{\frac{h_2}{T}} \mathbf{d}' \mathbf{Z}_{it,2} \zeta_{it} K(U_{ih_2})$, then we have

$$\sqrt{Nh_2} \mathbf{d}' Q_N = \frac{1}{\sqrt{NT}} \sum_{i=1}^N \sum_{t=1}^T Z_{N,it} = \frac{1}{\sqrt{NT}} \sum_{i=1}^N Z_{N,i}^*,$$

where $Z_{N,i}^* = \sum_{t=1}^T Z_{N,it}$, which is iid across i . Hence, it follows by the Lindeberg-Lévy central limit theorem that the asymptotic normality holds.

Next, we move to work on the first term \mathbf{B}_N . Note that

$$\begin{aligned}
&E\{\mathbf{Z}_{it,2}[\psi_\tau(Y_{it}^* - \check{c}_{1\tau}) - \zeta_{it}]K(U_{ih_2})\} \\
&= E[\mathbf{Z}_{it,2}\psi_\tau(Y_{it}^* - \check{c}_{1\tau})K(U_{ih_2})] - E[\mathbf{Z}_{it,2}(\zeta_{it}|U_i, \mathbf{Z}_{it,2})K(U_{ih_2})] \\
&= E[\mathbf{Z}_{it,2}\psi_\tau(Y_{it}^* - \check{c}_{1\tau})K(U_{ih_2})] \equiv E(\check{\mathbf{Z}}_{it}),
\end{aligned}$$

and furthermore, we have

$$\begin{aligned}
& E\{\psi_\tau[Y_{it}^* - \check{c}_{1\tau}]|U_i, \mathbf{Z}_{it,2}\} \\
&= E\{\tau - I\{Y_{it}^* < \check{c}_{1\tau}\}|U_i, \mathbf{Z}_{it,2}\} \\
&= F_{Y^*|U, \mathbf{Z}}(Q_\tau(U_i, \mathbf{Z}_{it,2})) - F_{Y^*|U, \mathbf{Z}_2}(\check{c}_{1\tau}|U_i, \mathbf{Z}_{it,2}) \\
&= f_{Y^*|U, \mathbf{Z}}(\check{c}_{1\tau} + \xi(Q_\tau(U_i, \mathbf{Z}_{it,2}) - \check{c}_{1\tau})|U_i, \mathbf{Z}_{it,2})(Q_\tau(U_i, \mathbf{Z}_{it,2}) - \check{c}_{1\tau}) \\
&= \frac{h_2^2}{2} f_{Y^*|U, \mathbf{Z}_2}(\check{c}_{1\tau} + \xi \mathbf{Z}'_{it,2} \ddot{\theta}_\tau(\tilde{u}) U_{ih_2}^2 |U_i, \mathbf{Z}_{it,2}) \mathbf{Z}'_{it,2} \ddot{\theta}_\tau(\tilde{u}) U_{ih_2}^2 \\
&= \frac{h_2^2}{2} [f_{Y^*|U, \mathbf{Z}_2}(Q_\tau(U_i, \mathbf{Z}_{it,2})) + o(h_2)] U_{ih_2}^2 \mathbf{Z}'_{it,2} [\ddot{\theta}_\tau(u_0) + o(1)] \\
&= \frac{h_2^2}{2} f_{Y^*|U, \mathbf{Z}_2}(Q_\tau(U_i, \mathbf{Z}_{it,2})) U_{ih_2}^2 \mathbf{Z}'_{it,2} \ddot{\theta}_\tau(u_0) + o(h_2^2).
\end{aligned}$$

Since

$$Q_\tau(U_i, \mathbf{Z}_{it,2}) - \check{c}_{1\tau} = \frac{h_2^2}{2} \mathbf{Z}'_{it,2} \ddot{\theta}_\tau(\tilde{u}) U_{ih_2}^2,$$

we obtain

$$\begin{aligned}
E(\check{\mathbf{Z}}_{it,2}) &= E\{\mathbf{Z}_{it,2} E(\psi_\tau[Y_{it}^* - \check{c}_{1\tau}]|U_i, \mathbf{Z}_{it,2}) K(U_{ih_2})\} \\
&= \frac{h_2^2}{2} E\{\mathbf{Z}_{it,2} [f_{Y^*|U, \mathbf{Z}}(Q_\tau(U_i, \mathbf{Z}_{it,2})) + o(h_2)] U_{ih_2}^2 K(U_{ih_2})\} [\ddot{\theta}_\tau(u_0) + o(1)] \\
&= \frac{h_2^3}{2} f_U(u_0) \mu_2 \Omega_2^*(u_0) (\ddot{\theta}_\tau(u_0) + o(1)).
\end{aligned}$$

Thus,

$$\begin{aligned}
E(\mathbf{B}_N) &= \frac{(\Omega_2^*(u_0))^{-1}}{\sqrt{N h_2} f_U(u_0)} N E(\check{\mathbf{Z}}_{it,2}) \\
&= \frac{(\Omega_2^*(u_0))^{-1}}{\sqrt{N h_2} f_U(u_0)} N \frac{h_2^3}{2} f_U(u_0) \mu_2 \Omega_2^*(u_0) \ddot{\theta}_\tau(u_0) [1 + o(1)] \\
&= \frac{h_2^2 \sqrt{N h_2}}{2} \mu_2 \ddot{\theta}_\tau(u_0) [1 + o(1)].
\end{aligned}$$

Let $B_{it} = \mathbf{Z}_{it,2} [\psi_\tau(Y_{it}^* - \check{c}_{1\tau}) - \zeta_{it}] K(U_{ih_2})$ and then

$$\begin{aligned}
[\psi_\tau(Y_{it}^* - \check{c}_{1\tau}) - \zeta_{it}]^2 &= [I\{Y_{it}^* < Q_\tau(U_i, \mathbf{Z}_{it,2})\} - I\{Y_{it}^* < \check{c}_{1\tau}\}]^2 \\
&= I_{\{q_{\tau, \min} < Y_{it}^* < q_{\tau, \max}\}}
\end{aligned}$$

where $q_{\tau, \min} = \min(Q_\tau(U_i, \mathbf{Z}_{it,2}), \check{c}_{1\tau})$ and $q_{\tau, \max} = \max(Q_\tau(U_i, \mathbf{Z}_{it,2}), \check{c}_{1\tau})$.

Finally, we show that

$$\begin{aligned}
E[B_{it}^2] &= E[\mathbf{Z}_{it,2}\mathbf{Z}'_{it,2}I_{\{q_{\tau,min} < Y_{it} < q_{\tau,max}\}}K^2(U_{ih_2})] \\
&= E[\mathbf{Z}_{it,2}\mathbf{Z}'_{it,2}[F_{Y^*|U,\mathbf{Z}_2}(q_{\tau,max}) - F_{Y^*|U,\mathbf{Z}_2}(q_{\tau,min})]^2K^2(U_{ih_2})] \\
&= O(h_2^3)
\end{aligned}$$

and similarly, we have $E[B_{it_1}B_{it_2}] = O(h_2^3)$. Hence, $Var(\mathbf{B}_N) = o(1)$. This completes the proof of Theorem 2.

Appendix D: Proofs of Lemmas

Proof of Lemma 2.1: First, note that

$$\begin{aligned}
E[\psi_\tau(U_i, \mathbf{Z}_{jt})|\xi_j] &= \tau - F_{Y|U,\mathbf{Z}}(Q_\tau(U_j, \mathbf{Z}_{jt}) - \mathbf{Z}'_{it,2}[\boldsymbol{\theta}_\tau(U_j) - \boldsymbol{\theta}_\tau(U_i)]) \\
&\simeq \tau - \{F_{Y|U,\mathbf{Z}}(Q_\tau(U_j, \mathbf{Z}_{jt})) - f_{Y|U,\mathbf{Z}}(Q_\tau(U_j, \mathbf{Z}_{jt}))\mathbf{Z}'_{it,2}[\boldsymbol{\theta}_\tau(U_j) - \boldsymbol{\theta}_\tau(U_i)] \\
&\quad + \frac{1}{2}\dot{f}_{Y|U,\mathbf{Z}}(Q_\tau(U_j, \mathbf{Z}_{jt}))\{\mathbf{Z}'_{it,2}[\boldsymbol{\theta}_\tau(U_j) - \boldsymbol{\theta}_\tau(U_i)]\}^2\} \\
&= f_{Y|U,\mathbf{Z}}(Q_\tau(U_j, \mathbf{Z}_{jt}))\mathbf{Z}'_{jt} \begin{pmatrix} 0 \\ \boldsymbol{\theta}_\tau(U_j) - \boldsymbol{\theta}_\tau(U_i) \end{pmatrix} \\
&\quad - \frac{1}{2}\dot{f}_{Y|U,\mathbf{Z}}(Q_\tau(U_j, \mathbf{Z}_{jt}))\{\mathbf{Z}'_{jt,2}[\boldsymbol{\theta}_\tau(U_j) - \boldsymbol{\theta}_\tau(U_i)]\}^2.
\end{aligned}$$

Hence, we have

$$\begin{aligned}
E[Z(U_i, \mathbf{Z}_{jt})] &= E\{\mathbf{Z}_{jt}E[\psi_\tau(U_i, \mathbf{Z}_{jt})|\xi_j]K_h(U_j - U_i)\} \\
&= E[f_{Y|U,\mathbf{Z}}(Q_\tau(U_j, \mathbf{Z}_{jt}))\mathbf{Z}_{jt}\mathbf{Z}'_{jt} \begin{pmatrix} 0 \\ \boldsymbol{\theta}_\tau(U_j) - \boldsymbol{\theta}_\tau(U_i) \end{pmatrix} K_h(U_j - U_i)] \\
&\quad - \frac{1}{2}E\{\dot{f}_{Y|U,\mathbf{Z}}(Q_\tau(U_j, \mathbf{Z}_{jt}))\mathbf{Z}_{jt}\{\mathbf{Z}'_{it,2}[\boldsymbol{\theta}_\tau(U_j) - \boldsymbol{\theta}_\tau(U_i)]\}^2K_h(U_j - U_i)\} \\
&\equiv I_1 - \frac{I_2}{2}.
\end{aligned}$$

Then, we obtain that

$$\begin{aligned}
I_1 &= E[\Omega^*(U_j) \begin{pmatrix} 0 \\ \boldsymbol{\theta}_\tau(U_j) - \boldsymbol{\theta}_\tau(U_i) \end{pmatrix} K_h(U_j - U_i)] \\
&= \int [\Omega^*(U_i) + uh_1\dot{\Omega}^*(U_i)] \begin{pmatrix} 0 \\ \boldsymbol{\theta}_\tau(U_j) - \boldsymbol{\theta}_\tau(U_i) \end{pmatrix} K(u)[f_U(U_i) + uh_1\dot{f}_U(U_i)]du + o(h_1^2) \\
&= \mu_2h_1^2B(U_i) \begin{pmatrix} 0 \\ \frac{\dot{\boldsymbol{\theta}}_\tau(U_i)}{2} + \frac{\dot{f}_U(U_i)}{f_U(U_i)}\boldsymbol{\theta}_\tau(U_i) \end{pmatrix} + \mu_2h_1^2f_U(U_i)\dot{\Omega}^*(U_i) \begin{pmatrix} 0 \\ \dot{\boldsymbol{\theta}}_\tau(U_i) \end{pmatrix} + o(h_1^2),
\end{aligned}$$

and

$$\begin{aligned}
I_2 &= E[\Theta(U_j)(U_j - U_i)^2 K_h(U_j - U_i)] \\
&= \int \Theta(U_i)(uh_1)^2 K(u) f_U(U_i) du + o(h_1^2) \\
&= \mu_2 h_1^2 f_U(U_i) \Theta(U_i) + o(h_1^2).
\end{aligned}$$

It follows that

$$\begin{aligned}
r_N(\xi_i) &= E[e_1' B^{-1}(U_i) \frac{1}{T} \sum_{t=1}^T Z(U_i, \mathbf{Z}_{jt}) + e_1' B^{-1}(U_j) \frac{1}{T} \sum_{t=1}^T Z(U_j, \mathbf{Z}_{it}) | \xi_i] \\
&= E[e_1' B^{-1}(U_i) \frac{1}{T} \sum_{t=1}^T Z(U_i, \mathbf{Z}_{jt}) | \xi_i] + E[e_1' B^{-1}(U_j) \frac{1}{T} \sum_{t=1}^T Z(U_j, \mathbf{Z}_{it}) | \xi_i] \\
&= o_p(h_1) + \frac{1}{T} \sum_{t=1}^T e_1' \int B^{-1}(U_j) \mathbf{Z}_{it} \psi_\tau(U_j, \mathbf{Z}_{it}) K_h(U_i - U_j) f_U(U_j) dU_j \\
&= o_p(h_1) + \frac{1}{T} \sum_{t=1}^T e_1' B^{-1}(U_i) \mathbf{Z}_{it} \psi_\tau(U_i, \mathbf{Z}_{it}) f_U(U_i) (1 + o(1)) \\
&= e_1' (\Omega^*(U_i))^{-1} \frac{1}{T} \sum_{t=1}^T \mathbf{Z}_{it} \psi_\tau(U_i, \mathbf{Z}_{it}) + o(1),
\end{aligned}$$

and furthermore, we obtain that

$$\begin{aligned}
\theta_N &= E[p_N(\xi_i, \xi_j)] = 2E[e_1' B^{-1}(U_i) Z(U_i, \mathbf{Z}_{jt})] \\
&= \mu_2 h_1^2 \left\{ E \left[e_1' \begin{pmatrix} 0 \\ \ddot{\theta}_\tau(U_i) + 2 \frac{\dot{f}_U(U_i)}{f_U(U_i)} \dot{\theta}_\tau(U_i) \end{pmatrix} \right] + 2E[e_1' (\Omega^*(U_i))^{-1} \dot{\Omega}^*(U_i) \begin{pmatrix} 0 \\ \dot{\theta}_\tau(U_i) \end{pmatrix}] \right. \\
&\quad \left. - E[e_1' (\Omega^*(U_i))^{-1} \Theta(U_i)] \right\} + o(h_1^2) \\
&\equiv \mu_2 h_1^2 (2\mathbf{B}_1^* - \mathbf{B}_2^*) + o(h_1^2),
\end{aligned}$$

Since $E[r_N(\xi_i)] = o_p(h_1)$ holds, it follows that

$$\begin{aligned}
\text{Var}[r_N(\xi_i)] &= E[e_1' (\Omega^*(U_i))^{-1} \frac{1}{T} \sum_{t=1}^T \mathbf{Z}_{it} \psi_\tau(U_i, \mathbf{Z}_{it})]^2 + o(h_1) \\
&\equiv \boldsymbol{\Sigma}_\gamma + o(h_1).
\end{aligned}$$

Therefore, Lemma 2.1 is established.

Proof of Lemma 2.2: Similar to Lemma 2.1 (ii), we show that $E(\mathbb{B}_N) = \mu_2 h_1^2 (-2\mathbf{B}_1^* + \mathbf{B}_2^*) + o(h_1^2)$. The lemma is established due the fact that $\text{Var}(\mathbb{B}_N) = o(h_1^4)$.

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