The Identifying Information in Vector Autoregressions with Time-Varying Volatilities: An Application to Endogenous Uncertainty*

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Abstract

We develop a structural VAR with stochastic volatility in which one of the variables can impact both the mean and the variance of the other variables. We provide conditional posterior distributions for this model, develop an MCMC algorithm for estimation, and show how stochastic volatility can be used to provide useful restrictions for the identification of structural shocks. We then use the model to show that macroeconomic uncertainty can be considered as exogenous when assessing its effects on the U.S. economy. Instead, financial uncertainty can at least in part arise as an endogenous response to some macroeconomic developments, and overlooking this channel leads to distortions in the estimated effects of financial uncertainty shocks on the economy.

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1 Introduction

Starting from the seminal work of Bloom (2009), the business cycle relationship between uncertainty and output growth and the transmission mechanism from one to the other have received substantial attention in the literature; see Bloom (2014) for an exhaustive survey. Various measures of uncertainty have been put forward, and several efforts have been made to study the macroeconomic effects and broader importance of uncertainty shocks. A non-exhaustive list of studies in this area includes Bachmann, Elstner, and Sims (2013), Baker, Bloom, and Davis (2016), Basu and Bundick (2017), Bloom (2009), Bloom, et al. (2018), Caldara, et al. (2016), Caggiano, Castelnuovo, and Groshenny (2014), Carriero, Clark, and Marcellino (2017, CCM), Cesa-Bianchi, Pesaran, and Rebucci (2018), Jurado, Ludvigson, and Ng (2015, JLN), Rossi and Sekhposyan (2015), and Shin and Zhong (2018).

While the definitions and measurements of uncertainty differ in all these contributions, the common denominator in this line of research is the way in which the effects of uncertainty shocks are identified and assessed. Specifically, most econometric studies typically estimate the effects of uncertainty on economic variables by using structural VARs with some recursive identification scheme, which all inevitably assume some type of causal direction between uncertainty and economic variables. The assumption typically made is that uncertainty (be it macroeconomic or financial) is exogenous; i.e., it does not react contemporaneously to economic variables, while economic variables react contemporaneously to uncertainty.[1]

However, as is well known, recursive schemes have the advantage of simplicity of implementation and interpretation, but in some cases, they can be hard to defend as a credible identification strategy. This is particularly true when economists have very little a priori, generally accepted, and theoretically grounded reasons to believe that a specific recursive ordering is valid. The study of uncertainty shocks is such a case, since the existing evidence and economic wisdom make us unable to take a stand on the direction of the causality between uncertainty and economic variables such as GDP growth.

The existing literature has shown that both directions of causality are plausible. For example, the case has been made that uncertainty has effects on the economy through firms' behavior. Firms' behavior can be influenced by uncertainty for several reasons, e.g., because of the real option value of waiting before taking investment decisions (e.g., Bernanke (1983), McDonald and Siegel (1986)); because of the postponement of hiring and capital investment decisions (e.g., Bloom (2009), Bloom, et al. (2018), and Leduc and Liu (2012)); and because of the interaction with financial frictions constraining firms' decisions (e.g., Arellano, Bai, and Kehoe (2018), Gilchrist, Sim, and Zakrajsek (2014)). From the consumers' side, the

[1] Exogenous as used here and in the rest of the paper is not meant to mean strict exogeneity. Rather, we use it as shorthand for uncertainty being predetermined within the period.

effects of uncertainty on the macroeconomy are possible via precautionary savings (e.g., Basu and Bundick (2017) and Fernandez-Villaverde, et al. (2011)). Equivalently, it is reasonable to conjecture that lower growth, typically associated with higher unemployment, tighter credit conditions, and larger volatility in financial markets, in turn can increase uncertainty. One of the first papers to stress the possible endogeneity of uncertainty is Bachmann, Elstner, and Sims (2013). Using an identification strategy in which uncertainty shocks have no long-run effects on aggregate economic activity, they find that the uncertainty shocks then also have no effects in the short run. Instead, various measures of uncertainty substantially increase after a negative shock to aggregate economic activity (see, e.g., Bachmann and Moscarini (2011) and Fajgelbaum, Schaal, and Taschereau-Dumouchel (2017)).

In this paper, to allow business cycle endogeneity and assess the macroeconomic effects of uncertainty, we develop a general, novel approach to identification based on time variation in the volatilities of macroeconomic variables, in the form of stochastic volatility. Identification is obtained via a heteroskedasticity structure in which the time-varying conditional variances of the variables are driven by an uncertainty measure plus a stochastic idiosyncratic component, or just a stochastic idiosyncratic component. [While we focus on uncertainty shocks, the identification procedure developed in this paper can be applied in any vector autoregression (VAR) featuring time variation in the volatilities.]. Differently from most existing approaches in the uncertainty literature based on recursive schemes, our identification strategy permits both a causal channel going from uncertainty to the macroeconomy and the opposite causal channel going from the macroeconomy to uncertainty (which we will refer to as the "feedback channel") to be potentially relevant and quantifiable. Our identification strategy rests on an empirical feature of macroeconomic data — time-varying volatility — that has been well established, starting with the seminal work of Cogley and Sargent (2005) and Primiceri (2005) on VARs with stochastic volatility and Justiniano and Primiceri (2008) on DSGE models with stochastic volatility. More recent studies providing corroborating evidence include, among others, Carriero, Clark, and Marcellino (2016), Chan and Eisenstat (2018), Clark (2011), Clark and Ravazzolo (2015), D'Agostino, Gambetti, and Giannone (2013), and Diebold, Schorfheide, and Shin (2017).

Methodologically, the model developed in this paper is a structural VAR with time-varying volatility in which one of the variables (the uncertainty measure) can impact both the mean and the variance of the other variables. We provide conditional posterior distributions for this model, which is a substantial extension of the leverage model of Jacquier, Polson, and Rossi (2004), a widely used model in the finance literature. These distributions are nontrivial because, with respect to the model of Jacquier, Polson, and Rossi (2004), our model entails an additional layer of complication insofar as the stochastic volatility factor also enters the conditional mean of the process. The correctness, efficiency, and reliability of the algorithm are established in Monte Carlo experiments with simulated data and with Geweke's (2004) test, prior to use with monthly and quarterly U.S. datasets.

With the model, we show that macroeconomic uncertainty can be considered as mostly exogenous when assessing its effects on the U.S. economy. Instead, financial uncertainty can at least in part arise as an endogenous response to some macroeconomic developments, and overlooking this channel leads to a distorted estimate of the effects of financial uncertainty shocks on the economy. We obtain these empirical findings with an econometric model in which current and past values of uncertainty affect the current levels of economic variables, and uncertainty is in turn also affected by them contemporaneously.

More specifically, our empirical application, based on both monthly and quarterly U.S. data over the period 1960-2017, leads to four main findings. First, when allowing for the simultaneous feedback effect, shocks to macro and financial uncertainty have a depressive effect on output growth, investment, and consumption, in line with previous empirical studies such as JLN and CCM and the theoretical contributions mentioned above. Second, when looking at macro uncertainty, we find strong evidence that coefficients related to the feedback channel are close to zero, which means that treating macroeconomic uncertainty as exogenous is likely harmless. Third, we find that some of the coefficients measuring the feedback effect of macroeconomic variables on financial uncertainty are significantly different from zero, thereby indicating that financial uncertainty can be endogenous to some extent. This pattern is particularly evident in the monthly dataset, with variables such as consumer spending, inflation, industrial production, and the federal funds rate all featuring negative feedback coefficients, implying that an increase in these indicators leads to a reduction in financial uncertainty. Broadly, our results on financial uncertainty are in line with the common treatment of financial indicators as "fast" variables that can react contemporaneously to macroeconomic shocks; see, e.g., Bernanke, Boivin, and Eliasz (2005).

To put our approach and results in the broader context of the literature, our identification method belongs to the heteroskedasticity-based identification tradition (see, e.g., Rigobon (2003), Sentana and Fiorentini (2001), and the review in Kilian and Lütkepohl (2017, chapter 14)). However, differently from this tradition, our methodology is based on modeling the conditional variances via stochastic volatility. This difference is nontrivial, because it allows much more flexibility in the evolution of the conditional variances than regime switching or GARCH specifications, since the time-varying volatilities have their own shocks that are independent from the shocks hitting the level of the variables. Bertsche and Braun (2018) recently developed a related approach to identification in VARs with stochastic volatility, with frequentist estimation and inference. They focus on allowing the A matrix of the standard VAR with stochastic volatility to take a form more general than the usual recursive ordering-based format, with the aim of testing the over-identifying restrictions involved in traditional triangular schemes. However, as emphasized by Lanne, Meitz, and Saikkonen (2017), with such an approach, additional information is required to be able to attach economic meaning to the structural shocks. We instead condition on the triangularity of the macroeconomic block of variables and focus on modeling the simultaneity between the macro block and the measure of uncertainty.

It is also worth mentioning that the presence of stochastic volatility makes the errors of our model non-Gaussian, so that another way to interpret our identification procedure is that it exploits the information in higher-order moments, rather than only in the second moments as in traditional Gaussian VARs. In this sense, our approach also belongs to the strand of the literature on identification in non-Gaussian models; see, for example, Lanne, Meitz, and Saikkonen (2017).

Our emphasis on potential simultaneity between uncertainty and economic conditions echoes Ludvigson, Ma, and Ng (2019, LMN), which points out that most of the existing results in the uncertainty literature could be biased by an endogeneity problem. LMN develop an alternative identification based on external information, using event constraints and correlation constraints. Their estimates indicate that macro uncertainty is mostly endogenous; i.e., it mainly reacts to growth conditions rather than being an exogenous source of business cycle fluctuations, whereas financial uncertainty is mostly exogenous. As described above, our approach yields somewhat different results, in which macroeconomic uncertainty appears to be exogenous and financial uncertainty does not. Supplementary analysis we have conducted indicates the difference in results stems primarily from a few of the particular identifying assumptions in LMN, mostly relating to shock size and simultaneity. With adjustments around these identifying assumptions, the LMN method yields results qualitatively similar to ours. In addition, Angelini, et al. (2018), using discrete variance breaks to achieve identification, reach a conclusion similar to ours, in which shocks to macroeconomic uncertainty can be treated as exogenous.

The paper is structured as follows. Section 2 presents the model and the identification approach. Section 3 develops the estimation algorithm. Section 4 provides an illustrative application and Monte Carlo experiments based on a small-scale version of the model. Section 5 presents the main empirical results. Section 6 summarizes our main findings and concludes. The paper's appendix contains derivations. A supplementary appendix available online provides additional results and robustness checks.

2 A model of endogenous uncertainty

2.1 Model specification

Our interest is in modeling the relationship between a set of economic variables, which we collect in the n-dimensional vector process y_t, and an observable scalar process, which we label m_t. We specify the following model:

$$y_t = \Pi_y(L)y_{t-1} + \Pi_m(L)\ln m_{t-1} + \phi \ln m_t + A^{-1}\Lambda_t^{0.5}\epsilon_t^* \quad (1)$$

$$\ln m_t = \delta_y(L)y_{t-1} + \delta_m(L)\ln m_{t-1} + \psi\epsilon_t^* + \tilde{u}_t, \quad (2)$$

where $\Pi_y(L)$ is an $n \times n$ matrix polynomial, $\Pi_m(L)$ is an $n \times 1$ vector polynomial, ϕ is a $n \times 1$ vector, $\delta_y(L)$ is a $1 \times n$ vector polynomial, $\delta_m(L)$ is a scalar polynomial, and ψ is a $1 \times n$ vector. The matrix A^{-1} is a lower triangular $n \times n$ matrix with ones on the main diagonal, which describes the contemporaneous relationships across the economic variables, and Λ_t is an $n \times n$ diagonal matrix of state variables. The shocks are $\epsilon_t^* \sim iid\ N(0, I_n)$, $\tilde{u}_t \sim iid\ N(0, \sigma_{\tilde{u}}^2)$, and are mutually independent. In the model above we have omitted intercepts for notational simplicity. Intercepts and any other exogenous variables can be added to equations (1) and (2) without any substantial change in the identification and estimation analysis presented below.

The model (1)-(2) is a structural VAR for the $(n+1)$-dimensional vector $(y_t' \ \ln m_t)'$. In our application $\ln m_t$ will be an observable scalar measure of uncertainty.[2] Clearly in a more general context $\ln m_t$ could be any other variable which the researcher is interested in modeling as potentially endogenous.

There are two major features that differentiate this model from the VARs typically used in the uncertainty literature (e.g., in studies such as Bloom (2009) and JLN).

First, the model allows for bilateral simultaneity between economic variables and uncertainty. Specifically, the model allows for both i) the contemporaneous effects of a shock to uncertainty on the economic variables, as measured by $\partial y_t/\partial\tilde{u}_t = \phi$, and ii) the contemporaneous effect of a shock to economic variables on uncertainty, as measured by $\partial \ln m_t/\partial\epsilon_t^* = \psi$ (we will refer to this as the "feedback effect"). This bilateral simultaneity is typically not present in the traditional implementations of uncertainty VARs, and is in general not achievable within the class of Gaussian models, since the number of reduced-form coefficients available in such a class of models is insufficient to pin down all of the contemporaneous relations across variables.

[2] The uncertainty measure could also be treated as unobservable. In this case, an additional step in the MCMC sampler would be needed in order to draw from its conditional posterior distribution. For an example of this approach in a model that does not allow for endogenous uncertainty, see Carriero, Clark, and Marcellino (2017).

The second major feature of the model proposed here is that the disturbance term to the first block of equations (i.e., $A^{-1}\Lambda_t^{0.5}\epsilon_t^*$) is heteroskedastic, as opposed to the large majority of uncertainty VARs which are specified as homoskedastic. The assumption of heteroskedasticity in a VAR of macroeconomic variables has overwhelming support in the recent literature (see, e.g., Chan and Eisenstat 2018), and many uncertainty measures are constructed on the basis of some variant of a time-varying volatility model (e.g., the measures put forward by JLN and LMN). Therefore, having a model featuring time-varying volatility is key in assessing the effects of uncertainty. In this paper we show that — besides providing a better description of the data — the assumption of heteroskedasticity allows us to simultaneously identify the coefficient vectors ϕ and ψ. Indeed this assumption implies that the VAR is unconditionally not Gaussian, which provides additional identifying information in the form of additional reduced-form moments.

The model in (1)-(2) nests some other models that previously appeared in the literature. Setting $n = 1$, $\Lambda_t = m_t$, $\phi = 0$ and dropping VAR dynamics provides the model of Jacquier, Polson, and Rossi (2004). Setting $\psi = 0$ provides the model of Carriero, Clark, and Marcellino (2017). Finally, setting $\psi = 0$ and shutting down time variation in volatilities ($\Lambda_t = \Lambda$) provides the homoskedastic VAR specification of Jurado, Ludvigson, and Ng (2015). All these contributions set either ϕ or ψ to a vector of zeros. As we shall see, this is equivalent to achieving identification by means of a triangular recursive structure in which uncertainty is ordered first ($\psi = 0$) or last ($\phi = 0$) in a VAR.

At a first inspection, the model in (1)-(2) might look somewhat asymmetric, in the sense that while the first equation contains the contemporaneous value of uncertainty ($\phi \ln m_t$) as a regressor, the second equation contains the shock to the macroeconomic data ($\psi\epsilon_t^*$) as a regressor. It is important to stress that this choice does not impact the key feature of the model, namely the simultaneity in the conditional means of y_t and $\ln m_t$. Indeed the model in (1)-(2) is obviously a re-parameterization of a model in which both shocks ϵ_t^* and $\tilde{u}_t$ appear in both the equations for y_t and the equation for $\ln m_t$. This can be easily seen by inserting the expression for $\ln m_t$ shown in (2) into the right-hand side of (1), which provides a representation in which both equations contain both shocks ϵ_t^* and $\tilde{u}_t$.[3]

[3] A slightly different specification would be one having the level of y_t as a regressor in (2), and this would introduce a second channel of simultaneity, this time between the conditional variance of y_t and the conditional mean of $\ln m_t$. Such a model would be highly nonlinear, increasing substantially the computational complexity without providing any major insights into the simultaneity in the conditional means with respect to the specification in (1)-(2).

2.2 Law of motion of the volatilities

The structural VAR in (1)-(2) admits a wide array of possibilities for modeling the latent state variables in the matrix Λ_t. Importantly, the choice of a law of motion for these state variables is purely a specification choice and does not impact the identification results which we will present below. Let λ_{jt} denote the j-th element on the diagonal of Λ_t. A general specification for the unobserved volatility process λ_{jt} is given by:

$$\lambda_{jt} = \Pi_{i=1}^{k} z_{it}^{\beta_{ji}} h_{jt}(\omega_j, \tilde{\eta}_{jt}), \tag{3}$$

where $z_{it},\ i = 1, ...k$, are k log-normally distributed observable variables and β_{ji} are loadings measuring the effect of the i-th variable z_{it} on the j-th volatility λ_{jt}. The h_{jt} are log-normal unobservable states featuring the Markov property and ω_j are coefficients modeling their evolution. The shocks $\tilde{\eta}_{jt}$ are i.i.d. across time (but they can be mutually contemporaneously correlated) and are assumed to be independent from the shocks to equations (1)-(2).

Clearly, equation (3) (for $j = 1, ...n$) represents the transition equation of a state-space system in which the observation equations are given by (1)-(2). Importantly, equation (3) forms a well defined log-normal linear regression model for λ_{jt}, in which the states h_{jt} and the coefficients β_{ji} and ω_j are fully identified. This in turns implies (as we shall see) identification of the entire state-space system (1)-(2)-(3). In our application we focus on two alternative specifications of the volatility process (3).

Specification 1 The first specification we consider is given by:

$$\lambda_{jt} = h_{jt},\ j = 1, ..., n, \tag{4}$$

where h_{jt} are idiosyncratic volatility states with transition equation:

$$\ln h_{jt} = \alpha_j + \delta_j \ln h_{jt-1} + \tilde{\eta}_{jt},\ j = 1, ..., n, \tag{5}$$

with $\tilde{\eta}_{jt} \sim iid\ N(0, \sigma^2_{\tilde{\eta}_j})$ and independent from $\tilde{\eta}_{it},\ i \neq j,\ \epsilon^*_t,\ \tilde{u}_t$. This specification is the one originally introduced by Cogley and Sargent (2005) and Primiceri (2005), eventually becoming a standard specification for VARs with time variation in volatility in the empirical macroeconomics literature.[4] Equation (5) is specified in logarithms, as is common in stochastic volatility models to implicitly implement non-negativity constraints. Note that in this specification λ_{jt} is an entirely unobservable state variable, with a log-normal distribution.

[4] To be precise, the Primiceri (2005) formulation is slightly different as it allows for correlation across the shocks $\tilde{\eta}_{it}$. This could be also allowed for in our approach with no consequences on identification. Note also that Koop and Potter (2007) show that stochastic volatility can be interpreted as a more continuous version of change-points.

Specification 2 The second specification is more general, and allows the states λ_{jt} to have a factor structure. Specifically the volatilities on the diagonal of Λ_t are defined as:

$$\lambda_{jt} = m_t^{\beta_j} h_{jt}, \ j = 1, ..., n, \tag{6}$$

where m_t is the measure of uncertainty (and is common to all the volatilities for which $\beta_j \neq 0$), and h_{jt} are the idiosyncratic volatilities, again following (5). Taking the logarithms of both sides of (6) shows that the (log) time-varying conditional variance of each variable in y_t is decomposed into a component common to all variables and given by the observable uncertainty measure m_t, plus a variable-specific, unobservable stochastic component, given by h_{jt}:

$$\ln \lambda_{jt} = \beta_j \ln m_t + \ln h_{jt}, \ j = 1, ..., n. \tag{7}$$

In the expression above, the loading β_j measures the elasticity of the volatility of variable j to the common volatility factor m_t. Moreover, note that specification (6) implies that the measure of uncertainty is allowed to impact not only the conditional mean of the economic variables (which happens through the term $\phi \ln m_t$ in (1)) but also the conditional variance. Even though λ_{jt} depends on an observable variable m_t, the term h_{jt} implies that the overall λ_{jt} is still an unobservable variable. Finally, note that specification 1 can be obtained as a special case simply by setting $\beta_j = 0, \ j = 1, ..., n$.

All of the results of this paper (including the Monte Carlo experiments) have been computed using both Specification 1 and Specification 2. The qualitative results in terms of coefficient estimates and impulse responses under the two specifications are very similar. To save space, in the remainder of the paper we will not always present the results for both specifications; we will rather focus mainly on Specification 2, only occasionally providing some results and insights based on Specification 1. This is because, as we discuss in Section 5.2, the overall evidence points towards Specification 2 being more appealing. Results based on Specification 1 can be found in the supplementary appendix (section 2).

Of course, more general specifications for λ_{jt} are possible, as shown in (3). Also, consistent with (3), $\ln h_{jt}$ does not need to be an autoregression, but it could be modelled as a discrete state variable describing a limited number of regimes, as in Markov switching or threshold models. The choice of a good specification is key to obtain efficiency, but does not impact identification.

In this respect it is worth mentioning that Lewis (2018) shows that heteroskedasticity per se can be sufficient to provide identification under fairly general conditions, regardless of whether the heteroskedasticity is well specified in the estimated model.

2.3 Identification

In this section we illustrate our identification strategy. To fix the ideas, we start from an illustrative example based on a simple bivariate version of the model, which we describe in Section 2.3.1. Then, we generalize the discussion to the general model in Section 2.3.2.

2.3.1 An illustrative example

Consider the special case of the model defined by (1)-(2), without the lags, and with $n = 1$:

$$\begin{aligned} y_t &= \phi \ln m_t + \sqrt{\lambda_t}\epsilon_t^* &(8)\\ \ln m_t &= \psi\epsilon_t^* + \tilde{u}_t, &(9)\end{aligned}$$

with $\epsilon_t^* \sim iid\ N(0,1)$, $\tilde{u}_t \sim iid\ N(0,\sigma_{\tilde{u}}^2)$ mutually independent (structural) shocks.

Here, y_t is a scalar economic variable of interest, for example, GDP growth, while m_t denotes an observable uncertainty measure. The conditional mean of y_t depends on contemporaneous (log) uncertainty through the term $\phi \ln m_t$. Uncertainty is endogenous, as it depends on the contemporaneous value of y_t through the term $\psi\epsilon_t^*$. We now relate the simultaneous equation model (8)-(9) with a standard VAR and show how identification is achieved. To do so, first consider the simpler homoskedastic model obtained by replacing (8) with:

$$y_t = \phi \ln m_t + \sqrt{\sigma_\epsilon^2}\epsilon_t^*. \quad (10)$$

The heteroskedasticity from the shock to the first equation has been removed and this shock now has a fixed variance σ_ϵ^2.

We will now derive the reduced form of this simpler model. Using equation (10) we can derive an expression for $\psi\epsilon_t^* = \psi\frac{1}{\sqrt{\sigma_\epsilon^2}}(y_t - \phi \ln m_t)$ which can be fed into equation (9), so that we can write the following system:

$$\begin{bmatrix} 1 & -\phi \\ -\frac{\psi}{\sqrt{\sigma_\epsilon^2}} & 1 + \frac{\phi\psi}{\sqrt{\sigma_\epsilon^2}} \end{bmatrix} \begin{bmatrix} y_t \\ \ln m_t \end{bmatrix} = \begin{bmatrix} \sqrt{\sigma_\epsilon^2}\epsilon_t^* \\ \tilde{u}_t \end{bmatrix}.$$

The system above is a simultaneous equation model of output and uncertainty. By inverting the matrix on the left-hand side, we obtain the reduced form model:

$$\begin{bmatrix} y_t \\ \ln m_t \end{bmatrix} = \begin{bmatrix} 1 + \frac{\phi\psi}{\sqrt{\sigma_\epsilon^2}} & \phi \\ \frac{\psi}{\sqrt{\sigma_\epsilon^2}} & 1 \end{bmatrix} \begin{bmatrix} \sqrt{\sigma_\epsilon^2}\epsilon_t^* \\ \tilde{u}_t \end{bmatrix} = \begin{bmatrix} \epsilon_t \\ u_t \end{bmatrix},$$

where the reduced form errors $(\epsilon_t'\ u_t')'$ have variance:

$$\begin{bmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{12} & \Sigma_{22} \end{bmatrix} = \begin{bmatrix} 1 + \frac{\phi\psi}{\sqrt{\sigma_\epsilon^2}} & \phi \\ \frac{\psi}{\sqrt{\sigma_\epsilon^2}} & 1 \end{bmatrix} \begin{bmatrix} \sigma_\epsilon^2 & 0 \\ 0 & \sigma_{\tilde{u}}^2 \end{bmatrix} \begin{bmatrix} 1 + \frac{\phi\psi}{\sqrt{\sigma_\epsilon^2}} & \frac{\psi}{\sqrt{\sigma_\epsilon^2}} \\ \phi & 1 \end{bmatrix}.$$

The reduced-form model contains 3 coefficients $(\Sigma_{11}, \Sigma_{12}, \Sigma_{22})$, but there are 4 parameters in the structural form $(\phi, \psi, \sigma^2_\epsilon, \sigma^2_{\tilde{u}})$; therefore, the model is not identified. In the standard approach, identification is usually achieved by setting $\psi = 0$, which gives exactly 3 coefficients in both the reduced form and the structural form.

Instead, in model (8)-(9) identification is achieved because λ_t appears in place of σ^2_ϵ. In other words, using (8) rather than (10) allows us to estimate both ϕ and ψ. Indeed — at any given time t — we have:

$$\begin{bmatrix} \Sigma_{11t} & \Sigma_{12t} \\ \Sigma_{12t} & \Sigma_{22t} \end{bmatrix} = \begin{bmatrix} 1 + \frac{\phi\psi}{\sqrt{\lambda_t}} & \phi \\ \frac{\psi}{\sqrt{\lambda_t}} & 1 \end{bmatrix} \begin{bmatrix} \lambda_t & 0 \\ 0 & \sigma^2_{\tilde{u}} \end{bmatrix} \begin{bmatrix} 1 + \frac{\phi\psi}{\sqrt{\lambda_t}} & \frac{\psi}{\sqrt{\lambda_t}} \\ \phi & 1 \end{bmatrix},$$

which — for any fixed t — has 3 coefficients in both the reduced form and the structural form. Computing the products in the right-hand side and vectorizing leads to the following system of equations:

$$\begin{cases} \Sigma_{11t} = \phi^2(\psi^2 + \sigma^2_{\tilde{u}}) + \lambda_t + 2\phi\psi\sqrt{\lambda_t} \\ \Sigma_{12t} = \phi(\psi^2 + \sigma^2_{\tilde{u}}) + \psi\sqrt{\lambda_t} \\ \Sigma_{22} = \psi^2 + \sigma^2_{\tilde{u}} \end{cases}. \tag{11}$$

Note that the reduced-form equation for y_t has time-varying conditional variance (and covariance), but such time variation is entirely driven by λ_t. It follows that the solution for $(\phi, \psi, \sigma^2_{\tilde{u}})$ of the system in (11) is the same for any t. Note that under knowledge of ψ one can obtain $\sigma^2_{\tilde{u}}$ by using the last equation. The last equation can also be substituted in the equations for Σ_{11t} and Σ_{12t}, and the problem reduces to solving

$$\begin{cases} \Sigma_{11t} = \phi^2\Sigma_{22} + \lambda_t + 2\phi\psi\sqrt{\lambda_t} \\ \Sigma_{12t} = \phi\Sigma_{22} + \psi\sqrt{\lambda_t} \end{cases} \tag{12}$$

for ϕ and ψ. To verify the existence of a unique local solution, we follow Hamilton (1994, p.334) and compute the Jacobian:

$$J = \begin{bmatrix} 2\phi\Sigma_{22} + 2\psi\sqrt{\lambda_t} & 2\phi\sqrt{\lambda_t} \\ \Sigma_{22} & \sqrt{\lambda_t} \end{bmatrix}.$$

Since $|J| = 2\lambda_t\psi$, it is $rank(J) = 2$, unless $\psi = 0$, but in this case we would have over-identification, as we would have $\Sigma_{11t} = \phi^2\sigma^2_{\tilde{u}} + \lambda_t$, $\Sigma_{12} = \phi\sigma^2_{\tilde{u}}$, $\Sigma_{22} = \sigma^2_{\tilde{u}}$, which would give $\phi = \Sigma_{12}/\Sigma_{22}$ and $\phi^2 = (\Sigma_{11t} - \lambda_t)/\sigma^2_{\tilde{u}}$, which over-identifies ϕ.

To complete the argument above we have to show that the unobservable state variable λ_t is separately identified. This is immediate to verify, since λ_t is entirely characterized by the transition equations (3). For example, under the more general Specification 2 we have:

$$\ln\lambda_t = \alpha + \beta\ln m_t + \delta\ln h_{t-1} + \tilde{\eta}_t;\ \tilde{\eta}_t \sim iidN(0, \sigma^2_{\tilde{\eta}}),$$

which is a well defined log-normal linear regression model. Such a model features a log-normal likelihood, which automatically implies that both the state variable λ_t and the coefficients $\alpha,\ \beta,\ \delta,\ \sigma^2_{\tilde{\eta}}$ regulating its law of motion are identified.

The issue of $\ln \lambda_t$ being unobservable and consistently estimable should not be confused with the issue of $\ln \lambda_t$ being identified. However it is worth noting how $\ln \lambda_t$ can be filtered out. Consider the heteroskedastic structural error $y^*_t = y_t - \phi \ln m_t = \sqrt{\lambda_t}\epsilon^*_t$, which can be obtained using (8), and take the logs of the squares:

$$\ln y^{*2}_t = \ln \lambda_t + 2 \ln \epsilon^*_t. \tag{13}$$

Since $\ln y^{*2}_t$ is observable conditioning on the parameters, expression (13) can be used as an observation equation to filter out the states λ_t.[5]

Information comes from the time-varying volatility term λ_t. To further clarify this point, suppose that, in Specification 1, $\sigma^2_{\tilde{\eta}} \longrightarrow 0$. In this case λ_t will no longer be a random state variable, but rather will converge to a fixed $\bar{\lambda}$, and this will make this case similar to the standard one (that is, there is one more parameter to be estimated, i.e. $\bar{\lambda}$, which is equivalent to the parameter σ^2_ϵ in the homoskedastic case described in (10)), and the model would again be unidentified.[6] Hence, the question arises: why is the situation so different when $\sigma^2_{\tilde{\eta}} > 0$?

The intuition behind this result can be explained in two ways: by looking at either the conditional or the unconditional moments of the shocks. Starting with the intuition based on conditional moments, when $\sigma^2_{\tilde{\eta}} > 0$, more moments become available from the reduced form, because time variation in h_t means that we have more reduced-form error-variance matrices (each corresponding to a different point in time t). The simplest way to think about this is within the simple textbook example $u_t = B^{-1}e_t$, where e_t is the structural shock (with an identity variance matrix) and u_t the reduced-form shock, with variance Σ. The matrix B^{-1} is a full matrix describing the contemporaneous relationships among the variables. Since Σ has only $n(n+1)/2$ free parameters, while B has n^2 free parameters, there is incomplete identification. Now, let us assume we have two regimes: one in which $\Sigma = \Sigma(h_1)$ and the second in which $\Sigma = \Sigma(h_2)$. This doubles the number of parameters we have from the reduced form, which become $n(n+1)$, and therefore, the order condition for identification is satisfied.[7] This approach to identification makes the implicit assumption

[5] Specifically, the equation would be used in conjunction with either specification (4) or (6) and the transition equations (5). An Expectation-Maximization algorithm could be employed to perform the estimation of the transition equation coefficients $\alpha,\ \beta,\ \delta,\ \sigma^2_{\tilde{\eta}}$ and filtering of the states $\ln \lambda_t$. An example of this approach can be found in Bertsche and Braun (2018). We instead use a MCMC algorithm and simulate the entire posterior distribution of λ_t.

[6] In the Monte Carlo section we will provide an example showing precisely this effect.

[7] This argument is similar to that in Rigobon (2003), even though it is important to stress that in our

that the contemporaneous relationships among the variables (those described by the matrix B^{-1}) are constant over time.

The intuition based on unconditional moments is related to Gaussianity. A Gaussian random variable is entirely defined by its first two moments (which are sufficient statistics). In the case of shocks, the first moment is 0 so we are left with the second moments (variances) only. The problem of identification precisely arises because the variance-covariance matrix of the Gaussian shocks has only $n(n+1)/2$ free coefficients, which often are not enough to identify all the contemporaneous relations we would like to (which are typically n^2). However, in the case of a non-Gaussian random variable, higher-order moments can provide additional information for identification. In the case at hand, if $\sigma^2_{\tilde{\eta}} = 0$, then the shocks are Gaussian, and therefore, we have the identification problem. Instead, if $\sigma^2_{\tilde{\eta}} > 0$, the shocks are not Gaussian (they are a mixture of Gaussians with mixture weights $\sqrt{\lambda_t}$), and therefore, we have identification.

The previous discussion clarifies that our method belongs to the family of methods for heteroskedasticity-based identification, considered in papers such as Rigobon (2003), Sentana and Fiorentini (2001), and Lanne and Lütkepohl (2008).[8] The use of a continuously changing volatility is studied in a more general context by Lewis (2018), who also considers the case of heteroskedasticity of arbitrary and unknown form. Another related strand of research considers identification in non-Gaussian models; see, for example, Lanne, Meitz, and Saikkonen (2017).[9] Kilian and Lütkepohl (2017) provide an excellent survey. There is, however, a key difference between our approach and those in these studies: our approach is based on stochastic volatility. This difference is not trivial, because the stochastic volatilities are state variables, driven by their own shocks rather than being either deterministic or driven by (functions of) the same shocks driving the variables in levels.

Finally, our identification strategy differs from the procedure introduced by LMN. The latter achieves identification by imposing "event constraints," in the terminology of LMN, requiring the identified shocks to be coherent with economic reasoning when some extraordinary events happen, and "correlation constraints," which restrict the identified uncertainty shocks to minimize their correlation with the stock market. There is also an important conceptual difference in reporting of impulse responses, reflecting the fact that the model

approach λ_t is a state variable and not a vector of parameters. As discussed below, this implies that we have a sequence of conditional variance matrices that are restricted by the fact that they must obey the laws of motion specified in (3).

[8] Angelini, et al. (2018) extend this approach to identify the effects of uncertainty shocks allowing for endogeneity. In line with our results, they find that the uncertainty shocks can be treated as exogenous.

[9] The Lanne, Meitz, and Saikkonen (2017) approach nests a number of other identification procedures based on conditional heteroskedasticity, including Normandin and Phaneuf (2004), Lanne, Lütkepohl, and Maciejowska (2010), and Lütkepohl and Netšunajev (2017).

we present is point-identified, while LMN's model is set-identified. The impulse response bounds reported in LMN represent the boundaries of a set of alternative point estimates of the impulse responses, each corresponding to a different identification scheme. As such, the bounds do not take into account parameter uncertainty around the point estimates. In contrast, in our approach there is only one identification scheme, and we estimate the entire posterior distribution of the impulse responses (complete posteriors are provided in section 4 of the supplementary appendix), which fully takes into account parameter uncertainty.

2.3.2 Identification in the general model

The logic illustrated in Section 2.3.1 can be extended to the general case of the multivariate model of equations (1)-(2)-(3).

Define $\mathbf{y} = (y_t, m_t)_{t=1}^T$, $\mathbf{\Lambda} = (h_{jt})_{t=1}^T$, $\vartheta = \{\Pi_y(L), \Pi_m(L), \phi, A, \delta_y(L), \delta_m(L), \psi\}$, and $\zeta = \{\beta_j, \omega_j, j = 1, ..., n\}$. Note that ζ contains the coefficients of the transition equations (3), while ϑ contains the coefficients of the observation equations (1)-(2). The model (1)-(2)-(3) forms a state-space system with joint data density given by:

$$\mathcal{L}(\mathbf{y}, \mathbf{\Lambda}|\vartheta, \zeta). \tag{14}$$

We defer the details of this function until later, when we get to estimation (see equation (25)). The fact that $\Lambda_{\mathbf{t}}$ is unobservable will of course need to be dealt with, but is entirely unrelated to the issue of identification.

The assumptions underlying our identification results, which we use as indicated in the discussion above and below, consist of the following, some implied by the VAR structure of equations (1) and (2):

- for each variable contained in y_t, a time-varying volatility process of the general form of (3), with innovation variance $\sigma^2_{\tilde{\eta}_j} > 0$;
- Gaussianity of innovations to y_t, $\ln m_t$, and log volatility (as noted below, some generalizations to allow fat tails are possible);
- in the error term $A^{-1}\Lambda_t^{0.5}\epsilon_t^*$ of the VAR equation for y_t, A is lower triangular with ones on the main diagonal (as discussed below, generalizations are possible), and Λ_t is diagonal.

Following Rothenberg (1971), the point $\{\mathbf{\Lambda}_0, \vartheta_0, \zeta_0\}$ is said to be locally identifiable if there exists an open neighborhood containing no other point $\{\mathbf{\Lambda}, \vartheta, \zeta\}$ which is observationally equivalent, i.e.

$$\mathcal{L}(\mathbf{y}, \mathbf{\Lambda}_0|\vartheta_0, \zeta_0) \neq \mathcal{L}(\mathbf{y}, \mathbf{\Lambda}|\vartheta, \zeta), \ \forall \vartheta \in \Theta_0, \ \forall \zeta \in Z_0, \ \forall \mathbf{\Lambda} \in L_0. \tag{15}$$

Verifying analytically condition (15) is difficult because $\mathcal{L}(\mathbf{y}, \mathbf{\Lambda}|\vartheta, \zeta)$ is non-Gaussian and is highly nonlinear. However, note that the density of the state-space model can be factorized as follows:

$$\mathcal{L}(\mathbf{y}, \mathbf{\Lambda}|\vartheta, \zeta) = \mathcal{L}(\mathbf{y}|\mathbf{\Lambda}, \vartheta, \zeta)\mathcal{L}(\mathbf{\Lambda}|\vartheta, \zeta) = \mathcal{L}(\mathbf{y}|\mathbf{\Lambda}, \vartheta)\mathcal{L}(\mathbf{\Lambda}|\zeta), \tag{16}$$

where $\mathcal{L}(\mathbf{y}|\mathbf{\Lambda}, \vartheta)$ is the (likelihood) density of the observation equations, and $\mathcal{L}(\mathbf{\Lambda}|\zeta)$ is the density of the transition equations.[10] Using (16), condition (15) can be written as:

$$\mathcal{L}(\mathbf{y}|\mathbf{\Lambda}_0, \vartheta_0)\mathcal{L}(\mathbf{\Lambda}_0|\zeta_0) \neq \mathcal{L}(\mathbf{y}|\mathbf{\Lambda}, \vartheta)\mathcal{L}(\mathbf{\Lambda}|\zeta),\ \forall\vartheta \in \Theta_0,\ \forall\zeta \in Z_0,\ \forall\mathbf{\Lambda} \in L_0. \tag{17}$$

Expression (17) offers a simpler way to check condition (15). Indeed one can check separately identification in the observation equations, i.e., $\mathcal{L}(\mathbf{y}|\mathbf{\Lambda}_0, \vartheta_0) \neq \mathcal{L}(\mathbf{y}|\mathbf{\Lambda}, \vartheta)$, and identification in the transition equations, i.e., $\mathcal{L}(\mathbf{\Lambda}_0|\zeta_0) \neq \mathcal{L}(\mathbf{\Lambda}|\zeta)$. If both these conditions are verified, then condition (15) is satisfied and there is identification in the joint density.[11] Since (3) is a well defined log-normal regression model, the identification condition on the transition equations is immediately satisfied. Hence all is left to verify is the condition for identification in the observation equations.

Identifying the observation equations The goal of this subsection is to verify that $\mathcal{L}(\mathbf{y}|\mathbf{\Lambda}_0, \vartheta_0) \neq \mathcal{L}(\mathbf{y}|\mathbf{\Lambda}, \vartheta),\ \forall\vartheta \in \Theta_0,\ \forall\mathbf{\Lambda} \in L_0$**.** Conditioning on $\mathbf{\Lambda}$ the model reduces to (1)-(2). Using steps similar to those illustrated in the previous subsection, the system can be cast in its reduced form:

$$\begin{bmatrix} y_t \\ \ln m_t \end{bmatrix} = \begin{bmatrix} C_{11}(L) & C_{21}(L) \\ C_{12}(L) & C_{22}(L) \end{bmatrix} \begin{bmatrix} y_{t-1} \\ \ln m_{t-1} \end{bmatrix} + \begin{bmatrix} \epsilon_t \\ u_t \end{bmatrix}, \tag{18}$$

where

$$Var\left(\begin{bmatrix} \epsilon_t \\ u_t \end{bmatrix}\right) = \begin{bmatrix} \Sigma_{11t} & \Sigma_{12t} \\ \Sigma_{21t} & \Sigma_{22} \end{bmatrix}. \tag{19}$$

Since the likelihood $\mathcal{L}(\mathbf{y}|\mathbf{\Lambda}, \vartheta)$ is Gaussian (as a function of $\mathbf{y}$), the reduced form coefficients appearing in (18) and (19) are uniquely identified. The paper's appendix shows that they

[10] The second equality in (16) follows from the fact that the coefficients ζ are redundant in the observation equations (conditionally on Λ), and the coefficients ϑ are redundant in the transition equations.

[11] For completeness, to ensure that (15) is satisfied there is the need to rule out from (17) the degenerate case in which, notwithstanding the fact that $\mathcal{L}(\mathbf{y}|\mathbf{\Lambda}_0, \vartheta_0) \neq \mathcal{L}(\mathbf{y}|\mathbf{\Lambda}, \vartheta)$ and $\mathcal{L}(\mathbf{\Lambda}_0|\zeta_0) \neq \mathcal{L}(\mathbf{\Lambda}|\zeta)$, their ratios are such that $\frac{\mathcal{L}(\mathbf{y}|\mathbf{\Lambda}, \vartheta)}{\mathcal{L}(\mathbf{y}|\mathbf{\Lambda}_0, \vartheta_0)} = \frac{\mathcal{L}(\mathbf{\Lambda}_0|\zeta_0)}{\mathcal{L}(\mathbf{\Lambda}|\zeta)}$. Ruling this case out is straightforward by simply noting that the ratio $\mathcal{L}(\mathbf{\Lambda}_0|\zeta_0)/\mathcal{L}(\mathbf{\Lambda}|\zeta)$ is independent from $\mathbf{y}$. Hence, the degenerate case would require the ratio $\mathcal{L}(\mathbf{y}|\mathbf{\Lambda}, \vartheta)/\mathcal{L}(\mathbf{y}|\mathbf{\Lambda}_0, \vartheta_0)$ to also be constant for all $\mathbf{y}$, i.e. for any possible realization of the data, which obviously cannot be the case.

are related to the structural coefficients appearing in (1)-(2) through the following equations:

$$C_{11}(L) = \Pi_y(L) + \phi\delta_y(L) \tag{20a}$$
$$C_{21}(L) = \Pi_m(L) + \phi\delta_m(L) \tag{20b}$$
$$C_{12}(L) = \delta_y(L) \tag{20c}$$
$$C_{22}(L) = \delta_m(L) \tag{20d}$$
$$\Sigma_{11t} = (\phi\psi + A^{-1}\Lambda_t^{0.5})(\phi\psi + A^{-1}\Lambda_t^{0.5})' + \phi\phi'\sigma_{\tilde{u}}^2 \tag{20e}$$
$$\Sigma_{12t} = \Sigma_{21t}' = \left(\phi\psi + A^{-1}\Lambda_t^{0.5}\right)\psi' + \phi\sigma_{\tilde{u}}^2 \tag{20f}$$
$$\Sigma_{22} = \psi\psi' + \sigma_{\tilde{u}}^2. \tag{20g}$$

Under knowledge of ϕ, the conditional mean coefficients $\Pi_y(L)$, $\Pi_m(L)$, $\delta_y(L)$, and $\delta_m(L)$ are immediately identified through equations (20a)-(20d). Similarly, under knowledge of ψ the coefficient $\sigma_{\tilde{u}}^2$ is identified through equation (20g). Completing the multiplication of the parenthetical components of (20e) and (20f) yields:

$$\Sigma_{11t} = \Sigma_{22}\phi\phi' + \phi\psi\Lambda_t^{0.5}A^{-1'} + (\phi\psi\Lambda_t^{0.5}A^{-1'})' + A^{-1}\Lambda_t A_t^{-1\prime} \tag{21a}$$
$$\Sigma_{12t} = \phi\Sigma_{22} + A^{-1}\Lambda_t^{0.5}\psi', \tag{21b}$$

where we also used equation (20g) to substitute any instance of $\psi\psi' + \sigma_{\tilde{u}}^2$ with Σ_{22}. The system above is a generalization of the system (12) laid out in the simple example. Note that (21a) is a system of equations arranged in matrices and let the pair (i, j) identify the equation in row i and column j. Also, note that both sides of (21a) are composed of symmetric matrices, which implies that the equations above the diagonal $(i < j)$ are redundant. In order to remove these redundant equations we define the operators:

$$tril(\cdot);\ diag(\cdot);$$

which respectively select the elements on the lower-triangular portion of a matrix (below and not including the main diagonal) and on the main diagonal of the matrix, and arrange them in a column vector. Applying these operators we can re-write the system (21) as:

$$tril(\Sigma_{11t}) = tril(\Sigma_{22}\phi\phi') + tril(\phi\psi\Lambda_t^{0.5}A^{-1'} + (\phi\psi\Lambda_t^{0.5}A^{-1'})') + tril(A^{-1}\Lambda_t A^{-1\prime}) \tag{22a}$$
$$diag(\Sigma_{11t}) = diag(\Sigma_{22}\phi\phi') + diag(\phi\psi\Lambda_t^{0.5}A^{-1'} + (\phi\psi\Lambda_t^{0.5}A^{-1'})') + diag(A^{-1}\Lambda_t A^{-1\prime}) \tag{22b}$$
$$\Sigma_{12t} = \phi\Sigma_{22} + A^{-1}\Lambda_t^{0.5}\psi'. \tag{22c}$$

The system above consists of $n(n-1)/2 + n + n$ equations. The coefficients to estimate are $2n$ in the vectors ϕ and ψ, and $n(n-1)/2$ in the matrix A. Hence there are sufficient equations to solve for (A, ϕ, ψ). The Jacobian of the entire system is given by:

$$J_t = \begin{bmatrix} \underset{\frac{n}{2}(n-1)\times\frac{n}{2}(n-1)}{\left[\frac{\partial tril(\Sigma_{11t})}{\partial tril(A)}\right]} & \underset{\frac{n}{2}(n-1)\times n}{\left[\frac{\partial tril(\Sigma_{11t})}{\partial \phi}\right]} & \underset{\frac{n}{2}(n-1)\times n}{\left[\frac{\partial tril(\Sigma_{11t})}{\partial \psi}\right]} \\ \underset{n\times\frac{n}{2}(n-1)}{\left[\frac{\partial diag(\Sigma_{11t})}{\partial tril(A)}\right]} & \underset{n\times n}{\left[\frac{\partial diag(\Sigma_{11t})}{\partial \phi}\right]} & \underset{n\times n}{\left[\frac{\partial diag(\Sigma_{11t})}{\partial \psi}\right]} \\ \underset{n\times\frac{n}{2}(n-1)}{\left[\frac{\partial \Sigma_{12t}}{\partial tril(A)}\right]} & \underset{n\times n}{\left[\frac{\partial \Sigma_{12t}}{\partial \phi}\right]} & \underset{n\times n}{\left[\frac{\partial \Sigma_{12t}}{\partial \psi}\right]} \end{bmatrix}, \tag{23}$$

and has determinant:

$$2^n \prod_{i=1}^{n} \lambda_{it}^{n-i+1} \psi_i, \tag{24}$$

which implies that it has full $rank(J) = n(n-1)/2 + 2n$ when $\psi \neq 0$. Since the Jacobian has full column rank, the system is identified. In the case of k elements of ψ being 0, the Jacobian would have column rank $n(n-1)/2 + 2n - k$. Since the number of rows in the system would not change, this means the system would have k less coefficients to solve for and the same number of equations, which leads to over-identification.

Structure of the A matrix The identification result has been derived under a specification in which the matrix A^{-1} is lower triangular. Recall the equations in (1), which we will refer to as the "macro block," and equation (2), the uncertainty equation. The triangularity assumption on A^{-1} allows the identification of the ε_t^* shocks of the macro block. Of course, this restriction does not impact the existence of simultaneity of the macro block and the uncertainty equation, which still can affect each other contemporaneously through the coefficients ϕ and ψ. Moreover, the shocks of interest are those to uncertainty, so the identification scheme used within the macro block might seem irrelevant. There is however an effect to keep in mind: since in equation (2) the identified effect of these shocks on uncertainty is given by $\psi\varepsilon_t^*$, rotations of ε_t^* will induce a rotation (in the opposite direction) in ψ so that the product $\psi\varepsilon_t^*$ stays constant. This means that different permutations within the macro block deliver different shocks ε_t^*, and this can translate into different coefficients ψ. In practice, we have experimented with several alternative orderings in both the monthly and the quarterly model, in both specifications 1 and 2, and the effect of having different orderings within the macro block has a rather limited impact on the estimated ψ coefficients. Section 3 in the supplementary appendix provides these results.

It is worth noting that a lower triangular structure is not strictly necessary to obtain the identification result. A sufficient, milder condition would be the one reported in Rubio-Ramirez, Waggoner, and Zha (2010), i.e. that every row j in A^{-1} has at least $n - j$

restrictions. This condition ensures that there exists a permutation of the rows of the matrix A^{-1} which is lower triangular, and therefore there exists a permutation of the equations in (21) that can be solved recursively. More details are given in the paper's appendix, in Section 7.1.2.

Even more general specifications for the matrix A^{-1} are possible. Since (21) holds for $t = 1, ..., T$, the model is over-identified when we consider the whole data-sample T. This means that it would be possible to allow for specifications in which the matrix A^{-1} is full, as in, e.g., Lanne, Meitz, and Saikkonen (2017). We do not take this route for two reasons. The first reason is simplicity in estimation. While specifying a full A^{-1} matrix is possible, the resulting model would entail a very high level of nonlinearity because in such a model the simultaneity would be possible not only between the macro-block and the uncertainty measure, but also within the macro-block. Instead, the model with lower triangular A^{-1} is reasonably easy to estimate, and indeed it has become a workhorse of macro-econometrics since the seminal papers of Cogley and Sargent (2005) and Primiceri (2005).[12] The second reason is that even though a full A^{-1} would provide full statistical identification of the shocks to the macro block of the model, economic identification of such shocks would still be necessary, as discussed in Lanne, Meitz, and Saikkonen (2017). Since our focus is on shocks to uncertainty rather than on shocks happening within the macro block, we maintain the assumption of a triangular A^{-1}, using the ordering conventional in many macroeconomic applications (i.e. ordering the so-called "slow" variables before the "fast" variables). As mentioned, we have experimented with alternative orderings, finding our results robust to such changes (see section 3 of the supplementary appendix).

3 Model estimation

In this section we describe the Markov Chain Monte Carlo (MCMC) algorithm for the estimation of the model. The model is a structural VAR with time-varying volatility in which one of the regressors (the uncertainty measure) possibly impacts both the mean and the variance of the others. This model nests as a special case the leverage model of Jacquier, Polson, and Rossi (2004). The conditional posterior distributions of this model are nontrivial because, with respect to the model of Jacquier, Polson, and Rossi (2004), our model entails an additional layer of complication insofar as the stochastic volatility factor also enters the conditional mean of the process (and our model includes VAR dynamics). Section 3.2 discusses the efficiency and convergence of the algorithm. Section 3.3 and Section

[12] For a VAR with time-varying parameters and stochastic volatility, Bognanni (2018) develops an alternative model formulation and estimation algorithm that addresses the ordering dependency which exists with the formulation that has become common.

3.4 discuss, respectively, the priors used in the empirical application and the computation of impulse response functions.

3.1 MCMC algorithm

We only discuss the more general Specification 2 since Specification 1 can be obtained as a special case by simply setting $\beta_j = 0$ for $j = 1, ..., n$. We collect the model coefficients in three sets. First, θ_1 groups all the coefficients of the y_t equation, plus the loadings: ϕ, A, $\Pi_y(L)$, $\Pi_m(L)$, β_j, $j = 1, ..., n$. Second, θ_2 groups all the coefficients of the uncertainty equation: $\delta_y(L)$, $\delta_m(L)$, ψ, $\sigma^2_{\tilde{u}}$. Third, θ_3 groups all the coefficients of the latent volatility processes: α_j, δ_j, $\sigma^2_{\tilde{\eta}_j}$, $j = 1, ..., n$. Both y_t and m_t are observable, while h_t is a vector of state variables. The vector θ contains θ_1, θ_2, and θ_3. We collect the h_{jt}'s in the $n \times 1$ vector process h_t, and we define the following matrices:

$$H_t = \begin{bmatrix} h_{1t} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & h_{nt} \end{bmatrix}; \; M_t^{(\beta)} = \begin{bmatrix} m_t^{\beta_1} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & m_t^{\beta_n} \end{bmatrix},$$

which implies (6) can be written as follows:

$$\Lambda_t = M_t^{(\beta)} H_t,$$

where the β appearing as a superscript indicates dependence on the vector composed of β_j for $j = 1, ..., n$.

The density of data and states is given by:

$$p(y_t, m_t, h_t|\theta) = m_t^{-1} \Pi_{j=1}^{n} m_t^{-0.5\beta_j} h_{jt}^{-1.5} p_G(\epsilon_t^*) \times p_G(\tilde{u}_t) \times p_G(\tilde{\eta}_t), \quad (25)$$

where the shocks ϵ_t^*, $\tilde{u}_t$, and $\tilde{\eta}_t$ are those in equations (1), (2), and (5). A derivation of expression (25) can be found in the paper's appendix.

The data density (25) will be the basis of a Gibbs sampler that will draw from:

1. $\mathbf{h}_{1:T}|\theta, \mathbf{y}_{1:T}, \mathbf{m}_{1:T}$
2. $\theta|\mathbf{h}_{1:T}, \mathbf{y}_{1:T}, \mathbf{m}_{1:T}$,

where $\mathbf{h}_{1:T}$, $\mathbf{y}_{1:T}$, and $\mathbf{m}_{1:T}$ are the matrices containing the states and variables for $t = 1, ..., T$. Step (2) of the algorithm will be further partitioned into three blocks:

2a. $\theta_1|\theta_2, \theta_3, \mathbf{h}_{1:T}, \mathbf{y}_{1:T}, \mathbf{m}_{1:T}$

2b. $\theta_2|\theta_1, \theta_3, \mathbf{h}_{1:T}, \mathbf{y}_{1:T}, \mathbf{m}_{1:T}$

2c. $\theta_3|\theta_1,\theta_2,\mathbf{h}_{1:T},\mathbf{y}_{1:T},\mathbf{m}_{1:T}$.

While steps (2b) and (2c) are straightforward, step (1) and step (2a) are nontrivial due to the simultaneity inherent in our model. We now proceed to analyze these steps in more detail.

3.1.1 Drawing $\mathbf{h}_{1:T}|\theta,\mathbf{y}_{1:T},\mathbf{m}_{1:T}$

Under the assumption that the processes h_{jt} are idiosyncratic volatilities with independent shocks $\tilde{\eta}_{it}$, the density $p(h_t|h_{t-1},h_{t+1},\theta,\mathbf{y}_{1:T},\mathbf{m}_{1:T})$ can be decomposed into the product of $\Pi_{j=1}^{n}p(\mathbf{h}_{j1:T}|\theta,\mathbf{y}_{1:T},\mathbf{m}_{1:T},\mathbf{h}_{\neq j1:T})$ and drawn in blocks $j=1,...,n$. The generic block $\mathbf{h}_{j1:T}|\theta,\mathbf{y}_{1:T},\mathbf{m}_{1:T},\mathbf{h}_{\neq j1:T}$ is Markov and can be simulated via a sequence of:

$$\begin{aligned}&p(h_{jt}|h_{jt-1},h_{jt+1},\theta,\mathbf{y}_{1:T},\mathbf{m}_{1:T},\mathbf{h}_{\neq j1:T})\\ \propto\quad &h_{jt}^{-0.5}\exp\left\{-\frac{e_{jt}^2}{2h_{jt}}\left[1+\frac{\psi_j^2}{\sigma_{\tilde{u}}^2}\right]+\frac{e_{jt}}{\sqrt{h_{jt}}}\frac{\psi_j}{\sigma_{\tilde{u}}^2}[u_t-\psi_{\neq j}\epsilon^*_{\neq jt}]\right\}\\ &\times h_{jt}^{-1}\exp\left\{\frac{-(\ln h_{jt}^2-\mu_{jt})^2}{2s_{\tilde{\eta}_j}^2}\right\},\end{aligned}\tag{26}$$

where $\psi_{\neq j}$ and $\epsilon^*_{\neq jt}$ are the vectors obtained by removing the j-th elements from ψ and ϵ_t^*, e_{jt} is the j-th element of the vector $e_t=(M_t^{(\beta)})^{-0.5}A(y_t-\Pi_y(L)y_{t-1}-\Pi_m(L)\ln m_{t-1}-\phi\ln m_t)$, $\mu_{jt}=(\delta_j(\ln h_{jt-1}+\ln h_{jt+1})+\alpha_j(1-\delta_j))/(\delta_j^2+1)$, and $s_{\tilde{\eta}_j}^2=\sigma_{\tilde{\eta}_j}^2/(\delta_j^2+1)$.[13] Note that $\epsilon^*_{\neq jt}$ and e_{jt} are observable under the conditioning set of (26). The derivation of the conditional posterior in (26) is detailed in the paper's appendix. Using an independence chain Metropolis step with the transition equation as proposal, i.e., $q\propto h_{jt}^{-1}\exp\{-(\ln h_{jt}^2-\mu_{jt})^2/2s_{\tilde{\eta}_j}^2\}$, we can accept/reject with acceptance probability $a=\min(1,\iota)$, where

$$\iota=\frac{\frac{1}{\sqrt{h_{jt}^{cand}}}\exp\left\{-\frac{{e_{jt}}^2}{2h_{jt}^{cand}}\left[1+\psi_j^2/\sigma_{\tilde{u}}^2\right]+\frac{e_{jt}}{\sqrt{h_{jt}^{cand}}}\frac{\psi_j}{\sigma_{\tilde{u}}^2}[u_t-\psi_{\neq j}\epsilon^*_{\neq jt}]\right\}}{\frac{1}{\sqrt{h_{jt}^{pres}}}\exp\left\{-\frac{{e_{jt}}^2}{2h_{jt}^{pres}}\left[1+\psi_j^2/\sigma_{\tilde{u}}^2\right]+\frac{e_{jt}}{\sqrt{h_{jt}^{pres}}}\frac{\psi_j}{\sigma_{\tilde{u}}^2}[u_t-\psi_{\neq j}\epsilon^*_{\neq jt}]\right\}}.\tag{27}$$

Finally, we initialize the states by drawing a set of initial conditions h_{j0}, $j=1,...n$. We assume a log-normal prior $\ln h_{j0}\sim N(\mu_{h_{j0}},\sigma_{h_{j0}}^2)$, yielding a log-normal posterior $(\ln h_{j0}|\ln h_{jt+1},...)\sim N(\bar{\mu}_{h_j},\bar{\sigma}_{h_j}^2)$ where $\bar{\mu}_{h_j}=\bar{\sigma}_{h_j}^2\left(\frac{\mu_{h_{j0}}}{\sigma_{h_{j0}}^2}+\frac{(\ln h_{j1}-\alpha_j)/\delta_j}{\sigma_{\tilde{\eta}_j}^2/\delta_j^2}\right)$ and $\bar{\sigma}_{h_j}^2=\frac{\sigma_{h_{j0}}^2\sigma_{\tilde{\eta}_j}^2/\delta_j^2}{\sigma_{h_{j0}}^2+\sigma_{\tilde{\eta}_j}^2/\delta_j^2}$.

[13] The conditional moments μ_{jt} and $s_{\tilde{\eta}_j}^2$ are slightly different in the first and last period of the sample.

3.1.2 Drawing $\theta|\mathbf{h}_{1:T}, \mathbf{y}_{1:T}, \mathbf{m}_{1:T}$

Consider again the kernel in (25), which depends on ϵ_t^*, $\tilde{u}_t$, and $\tilde{\eta}_{jt}$, where:

$$\epsilon_t^* = (M_t^{(\beta)})^{-0.5} H_t^{-0.5} A(y_t - \Pi_y(L) y_{t-1} - \Pi_m(L) \ln m_{t-1} - \phi \ln m_t) \tag{28}$$

$$\tilde{u}_t = \ln m_t - \delta_y(L) y_{t-1} - \delta_m(L) \ln m_{t-1} - \psi \epsilon_t^* \tag{29}$$

$$\tilde{\eta}_{jt} = \ln h_{jt} - \alpha_j - \delta_j \ln h_{jt-1}, \quad j = 1, ..., n. \tag{30}$$

We need to express (25) as a function of the data, the states, and the coefficients θ only (i.e., there must be no unobservable shocks). We can derive an expression for the first equation coefficients θ_1 by using (28). An expression for the second equation coefficients θ_2 can also be derived using the expression for $\tilde{u}_t$ in (29). However, this is not sufficient to get to an expression without any unobservable shocks appearing, because the term $\psi\epsilon_t^*$ would still be there, and therefore a further substitution for ϵ_t^* using (28) is required. This happens because, conditional on θ_2, the shock u_t is observable, but the shock $\tilde{u}_t = u_t - \psi\epsilon_t^*$ is not observable. These considerations lead to:

$$p(y_t, m_t, h_t|\theta) =$$

$$m_t^{-1} \Pi_{j=1}^n m_t^{-0.5\beta_j} h_{jt}^{-1.5} \exp\left\{ -\frac{1}{2} \left(\begin{array}{c} (y_t - \Pi X_t - \phi \ln m_t)' A' (M_t^{(\beta)})^{-1} \\ \times H_t^{-1} A (y_t - \Pi X_t - \phi \ln m_t) \end{array} \right) \right\} \tag{31a}$$

$$\times \frac{1}{\sqrt{\sigma_{\tilde{u}}^2}} \exp\left\{ -\frac{1}{2\sigma_{\tilde{u}}^2} \left(\begin{array}{c} \ln m_t - \delta_y(L) y_{t-1} - \delta_m(L) \ln m_{t-1} \\ -\psi (M_t^{(\beta)})^{-0.5} H_t^{-0.5} A (y_t - \Pi X_t - \phi \ln m_t) \end{array} \right)^2 \right\} \tag{31b}$$

$$\times \Pi_{j=1}^n \frac{1}{\sqrt{\sigma_{\tilde{\eta}_i}^2}} \exp\left\{ -\frac{(\ln h_{jt} - \alpha_j - \delta_j \ln h_{jt-1})^2}{2\sigma_{\tilde{\eta}_i}^2} \right\}, \tag{31c}$$

where

$$\Pi X_t = \Pi_y(L) y_{t-1} + \Pi_m(L) \ln m_{t-1}.$$

Note that in the density above, the coefficients of the second equation, θ_2, only appear in the kernel (31b), and the coefficients of the third equation, θ_3, only appear in the kernel (31c). This means that — conditionally on θ_1 (and on the states and data) — (31b) can be used as the posterior kernel for θ_2 and (31c) as the posterior kernel for θ_3.

In particular, since (31b) is a Normal-Inverse Gamma kernel, we have that $\theta_2|y_t, m_t, h_t, \theta_1, \theta_3$ can be drawn via a Gibbs step based on using equation (2) as a linear regression model. Also, note that $p(\theta_2|y_t, m_t, h_t, \theta_1, \theta_3) \propto p(\theta_2|y_t, m_t, h_t, \theta_1)$. Similarly, since (31c) is a (product of) Normal-Inverse Gamma kernel(s), it follows that $\theta_3|y_t, m_t, h_t, \theta_1, \theta_2$ can be drawn via a Gibbs step based on using equation (5) as a linear regression model. Also note that $p(\theta_3|y_t, m_t, h_t, \theta_1, \theta_2) \propto p(\theta_3|h_t)$.

We are now left with the coefficients θ_1. These coefficients are challenging because — when $\psi \neq 0$ —they appear in both (31a) and (31b). The posterior density $p(\theta_1|y_t, m_t, h_t, \theta_1, \theta_2)$ is proportional to the product of the prior and the posterior kernels (31a) and (31b):

$$
\begin{aligned}
&p(\theta_1|y_t, m_t, h_t, \theta_2, \theta_3) \propto \\
&p(\theta_1) \times \Pi_{j=1}^{n} m_t^{-0.5\beta_j} \exp\left\{-\frac{1}{2}\left(\begin{array}{c} (y_t - \Pi X_t - \phi \ln m_t)' A' (M_t^{(\beta)})^{-1} \\ \times H_t^{-1} A (y_t - \Pi X_t - \phi \ln m_t) \end{array}\right)\right\} \\
&\times \exp\left\{-\frac{1}{2\sigma_{\tilde{u}}^2}\left(\begin{array}{c} \ln m_t - \delta_y(L) y_{t-1} - \delta_m(L) \ln m_{t-1} \\ -\psi (M_t^{(\beta)})^{-0.5} H_t^{-0.5} A (y_t - \Pi X_t - \phi \ln m_t) \end{array}\right)^2\right\}
\end{aligned}
$$

which is not a known distribution. Therefore, this calls for a Random Walk Metropolis step with acceptance probability

$$
a = \min\left(1, \frac{p(\theta_1^{cand}|\mathbf{y}_{1:T}, \mathbf{m}_{1:T}, \mathbf{h}_{1:T}, \theta_2, \theta_3)}{p(\theta_1^{pres}|\mathbf{y}_{1:T}, \mathbf{m}_{1:T}, \mathbf{h}_{1:T}, \theta_2, \theta_3)}\right). \tag{33}
$$

In order to improve the mixing of the algorithm, this step is blocked in several sub-steps involving the coefficients in θ_1. Specifically, we draw separately ϕ, A, $\Pi_y(L)$, $\Pi_m(L)$, $\beta_j, j = 1, ..., n$.

3.2 Efficiency and convergence of the algorithm

We have verified the correctness, convergence, and mixing properties of the estimation algorithm in our empirical applications. Detailed results are reported in section 1 of the supplementary appendix. Specifically, Figure 1 of the supplementary appendix reports results from Geweke's (2004) test of correctness of the posterior sampler. Figure 2 of the supplementary appendix reports a summary set of diagnostics that all support convergence and good mixing of the MCMC algorithm. The Monte Carlo experiments described in Section 4 further test the algorithm using simulated data.

3.3 Priors

In the empirical application, we demean the variables and omit intercepts, to reduce the dimensionality of the parameter space. The priors on ϕ, A, $\Pi_y(L)$, $\Pi_m(L)$ are flat. For the loadings β_j, $j = 1, ..., n$ we elicit a Gaussian prior $\beta_j \sim N(1, 10)$, which is very diffuse (but still proper). For the equation for m_t, we elicit an independent Gaussian prior for each coefficient in the polynomials $\delta_y(L)$ and $\delta_m(L)$, with standard deviation 1 and mean 0, with the only exception being the parameter associated with $\ln m_{t-1}$, whose prior mean is set at 0.5. Following Jacquier, Polson, and Rossi (2004) we assume a conjugate prior for ψ and $\sigma_{\tilde{u}}^2$, with $\psi|\sigma_{\tilde{u}}^2 \sim N(0, \sigma_{\tilde{u}}^2 I_n)$ and $\sigma_{\tilde{u}}^2 \sim IG\left(\frac{v_0}{2} = 2, \frac{S_0}{2} = 0.05\right)$. For the equations for h_t, we elicit an independent Gaussian–Inverse Gamma prior, with $\delta_j \sim N(0.99, 0.1^2)$, $\alpha_j \sim N(0, 0.1^2)$, and $\sigma^2_{\tilde{\eta}_j} \sim IG\left(\frac{v_0}{2} = 3, \frac{S_0}{2} = 0.05\right)$, $j = 1, ..., n$. To ensure stationarity of the idiosyncratic volatility process, we also impose prior (and posterior) truncation for the parameter δ_j using the rejection sampling approach of Cogley and Sargent (2005). For the initial conditions of the states we use a Gaussian prior $\ln h_{j0} \sim N(\mu_{h_0}, \sigma^2_{h_0})$ with $\mu_{h_0} = \ln 0.20$ and $\sigma^2_{h_0} = 10$. These priors are the same across all the presented empirical applications. In the Monte Carlo exercise, to get an accurate read on bias in the coefficient estimates, we use a slightly different, less informative prior for some coefficients, namely $\beta_j \sim N(1, 100)$ and $\sigma^2_{\tilde{u}} \sim IG\left(\frac{v_0}{2} = 2, \frac{S_0}{2} = 0.0015\right)$.

3.4 Impulse responses

We will use the model to compute the impulse response functions (IRFs) to an uncertainty shock of size $\sqrt{\sigma^2_{\tilde{u}}}$. Following Hamilton (1994, page 10) we obtain responses by simulating the model in the window $t+1, ..., t+H$ under a baseline and a shocked scenario. The baseline scenario is constructed as follows. Let $i = 1, ..., M$ be the index of the posterior draws from the MCMC algorithm. We generate M paths of the system by starting from the initial condition $y_t = \ln m_t = 0$. In the baseline scenario, we set all of the shocks to 0 in all periods (this can be thought of as the steady state). In the alternative scenario, we set $\tilde{u}_{t+1} = \sqrt{\sigma^2_{\tilde{u}}}$, i.e., we give a shock of dimension $\sqrt{\sigma^2_{\tilde{u}}}$ to the system in the first period of the window $t+1, ..., t+H$. The impulse response functions (IRF) for each draw are then computed as the difference between the shocked and the baseline scenario: $\left\{ y^i_{t+h,shocked} - y^i_{t+h,baseline} \right\}_{i=1}^{M}$, $h = 1, \ldots, H$.

4 Illustrative example and Monte Carlo evaluation

Empirical results based on the general model can be found in Section 5. This section contains an illustrative application and a Monte Carlo evaluation based on a simpler bivariate version of the model. Specifically, we consider model (1)-(2) with 1 lag, 1 economic variable, and either Specification 1 ($\beta = 0$) or Specification 2 ($\beta \neq 0$) for the volatilities:

$$y_t = \Pi_y y_{t-1} + \Pi_m \ln m_{t-1} + \phi \ln m_t + \sqrt{m_t^{\beta} h_t} \epsilon_t^* \quad (34a)$$

$$\ln m_t = \delta_y y_{t-1} + \delta_m \ln m_{t-1} + \psi \epsilon_t^* + \tilde{u}_t \quad (34b)$$

$$\ln h_t = \alpha + \delta \ln h_{t-1} + \tilde{\eta}_t. \quad (34c)$$

4.1 Illustrative empirical example

We start with evaluating empirically the relationship between GDP growth and uncertainty in the U.S. We define y_t as the quarter-on-quarter GDP growth rate and $\ln m_t$ as the JLN measure of macro uncertainty, with quarterly data ranging from 1960Q3 to 2017Q2. Details on the data will be provided in Section 5.1. The posterior means of the parameters (with standard deviations in square brackets) are:

$$y_t = \underbrace{\underset{[0.0667]}{0.245}}_{\Pi_y} y_{t-1} + \underbrace{\underset{[2.3255]}{3.009}}_{\Pi_m} \ln m_{t-1} + \underbrace{\underset{[2.4622]}{-5.302}}_{\phi} \ln m_t + \sqrt{m_t^{\beta} h_t}\epsilon_t^*; \ \beta = \underset{[1.2991]}{3.660} \tag{35a}$$

$$\ln m_t = \underbrace{\underset{[0.0033]}{-0.0021}}_{\delta_y} y_{t-1} + \underbrace{\underset{[0.0317]}{0.9325}}_{\delta_m} \ln m_{t-1} + \underbrace{\underset{[0.0036]}{-0.0040}}_{\psi} \epsilon_t^* + \tilde{u}_t; \sigma^2_{\tilde{u}} = \underset{[0.0001]}{0.0013} \tag{35b}$$

$$\ln h_t = \underbrace{\underset{[0.0284]}{-0.044}}_{\alpha} + \underbrace{\underset{[0.0291]}{0.956}}_{\delta} \ln h_{t-1} + \tilde{\eta}_t; \ \sigma^2_{\tilde{\eta}} = \underset{[0.0153]}{0.0236} \ ; \ln h_0 = \underset{[1.0668]}{0.928} \ . \tag{35c}$$

The (log) marginal likelihood computed through Geweke's harmonic mean estimator is -88.49. The restricted Specification 1 (i.e., $\beta = 0$) has a much lower (log) marginal likelihood of -125.53. Therefore, the (log) Bayes factor is 37.04, which provides very strong evidence in favor of Specification 2. Results for Specification 1 are very similar in terms of coefficient estimates. The impact effect of uncertainty on growth as measured by ϕ is negative, and also the effect of growth on uncertainty as measured by ψ is negative, but the value of zero is included in the 5-95% percentiles of the posterior of ψ.

Figure 1 reports the JLN uncertainty measure, m_t, the posterior mean of the latent state h_t, and the overall stochastic volatility term in the GDP growth equation, $\lambda_t = m_t^{\beta} h_t$. The figure highlights the importance of having the h_t term, whose main roles are to further increase the volatility during the recessionary periods of the '70s and early '80s, and to capture the Great Moderation episode.

We now focus on what would happen if we assumed exogeneity of uncertainty, in the sense of setting $\psi = 0$. The results become:

$$y_t = \underbrace{\underset{[0.0657]}{0.240}}_{\Pi_y} y_{t-1} + \underbrace{\underset{[1.5791]}{5.078}}_{\Pi_m} \ln m_{t-1} + \underbrace{\underset{[1.6378]}{-7.545}}_{\phi} \ln m_t + \sqrt{m_t^{\beta} h_t}\epsilon_t^*; \ \beta = \underset{[1.2920]}{3.572}$$

$$\ln m_t = \underbrace{\underset{[0.0033]}{-0.0021}}_{\delta_y} y_{t-1} + \underbrace{\underset{[0.0314]}{0.9335}}_{\delta_m} \ln m_{t-1} + \underbrace{0}_{\psi} \epsilon_t^* + \tilde{u}_t; \ \sigma^2_{\tilde{u}} = \underset{[0.0001]}{0.0013}$$

$$\ln h_t = \underbrace{\underset{[0.0292]}{-0.047}}_{\alpha} + \underbrace{\underset{[0.0299]}{0.953}}_{\delta} \ln h_{t-1} + \tilde{\eta}_t; \ \sigma^2_{\tilde{\eta}} = \underset{[0.0137]}{0.0231} \ ; \ln h_0 = \underset{[0.8740]}{0.909} .$$

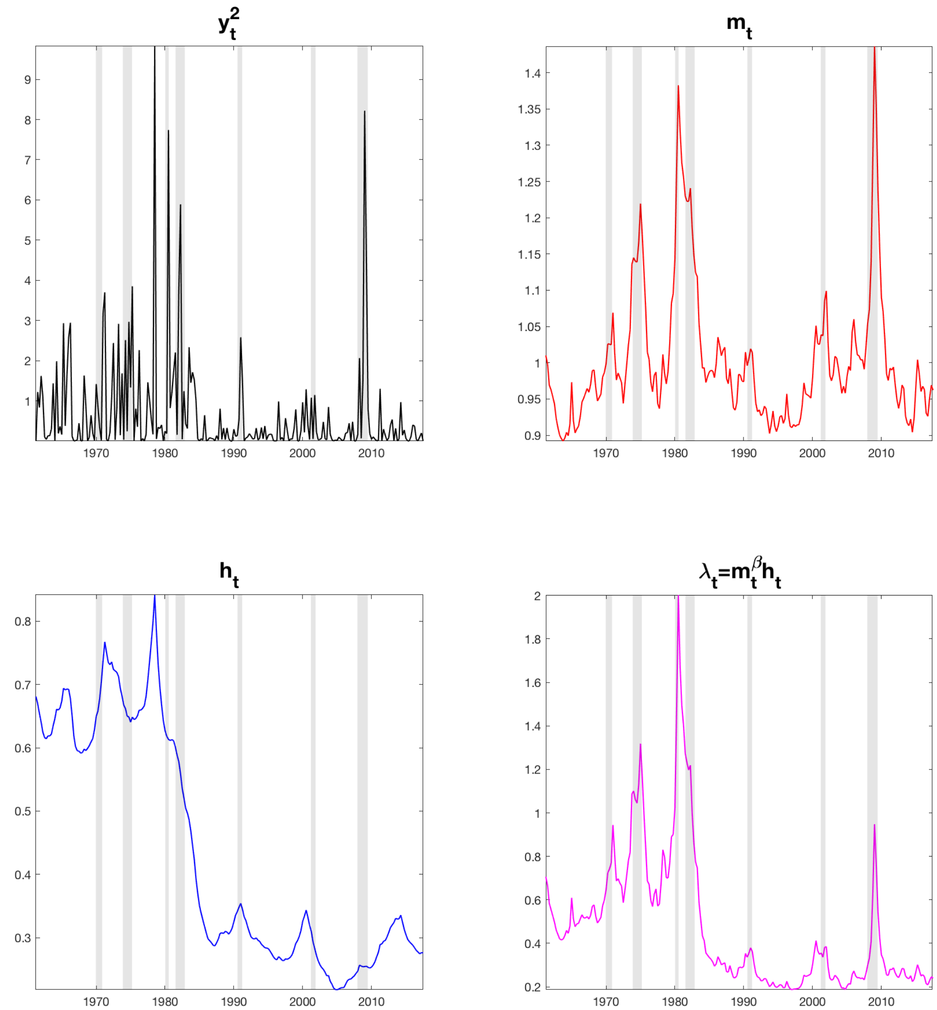

Figure 1: Squared GDP growth rates, uncertainty, idiosyncratic volatility states, and reduced-form volatilities.

Compared to the case with unrestricted ψ, the posterior of ϕ shifts toward more negative values: the posterior mean goes from -5.302 to -7.545 (a decrease of 2.243). Figure 2 shows the entire posterior distributions of ϕ when ψ is either unrestricted or set to zero. Setting $\psi = 0$ leads to an over-estimate of the negative impact of uncertainty on growth. This happens because, in the more general model with $\psi \neq 0$, following an uncertainty shock there is a decrease in growth, and this in turn increases uncertainty, which further decreases growth, increases uncertainty, and so on. If we shut down the feedback effect of growth on uncertainty by setting $\psi = 0$, the overall impact effect of uncertainty on growth, as measured by ϕ, must increase (in absolute value).

Note also that equation (20b) in this example simplifies to $C_{21} = \Pi_m + \phi\delta_m$. This

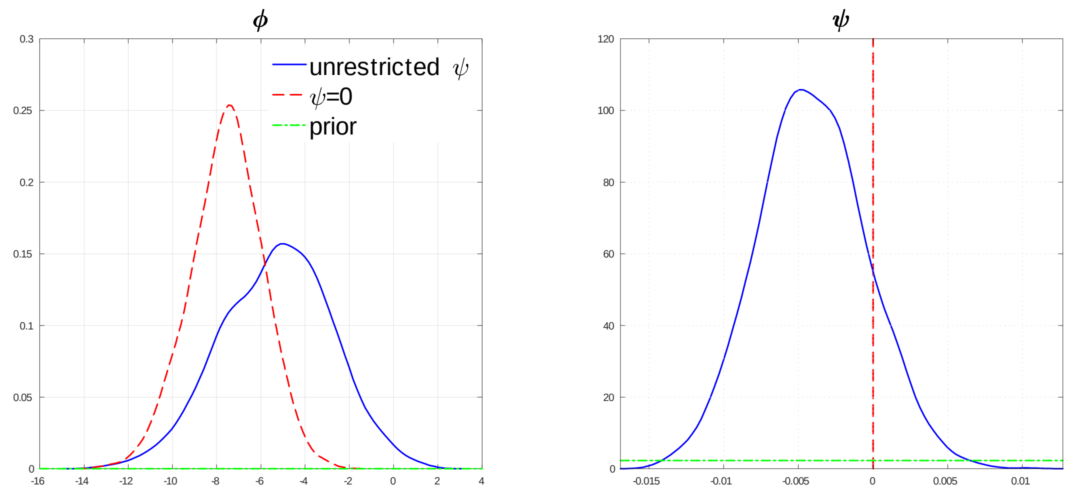

Figure 2: Posterior distributions of ϕ and ψ when ψ is unrestricted (blue solid line) and when ψ is restricted to 0 (red dashed line). Model with JLN macro uncertainty.

equation offers additional insights. Indeed, since the reduced form coefficient C_{21} cannot change, and $\ln m_t$ and $\ln m_{t-1}$ are highly correlated (δ_m is about 0.93), to compensate for the distortion in ϕ, the parameter Π_m increases by roughly the same amount of the decrease in ϕ. Indeed, the median of the posterior of Π_m becomes much larger, increasing from 3.009 to 5.078 (by 2.069), almost entirely offsetting the 2.243 decrease in ϕ. This means that the sum $\phi + \Pi_m$ does not change much when imposing $\psi = 0$, and that under this restriction the model confounds the contemporaneous (ϕ) and the lagged effect (Π_m) of uncertainty on growth.

Moreover, equation (20a) in this example simplifies to $C_{11} = \Pi_y + \phi\delta_y$. Since the product $\phi\delta_y$ is very small, as δ_y is very low relative to ϕ, the coefficient Π_y is virtually unaffected by the exclusion of the feedback channel. Finally, there are virtually no effects on the parameters of the equation for $\ln m_t$ when setting $\psi = 0$. This is not attributable to the insignificance of ψ but rather is a consequence of the fact that ϵ_t^* is uncorrelated with the other regressors, so that its omission does not introduce a distortion.

While Figure 2 makes clear that there is an effect to setting $\psi = 0$, it also shows that the posterior distribution of this parameter comfortably contains the value 0 within the center of its probability mass. This points towards the fact that it might be better, in terms of efficiency, to actually set this coefficient to 0. We can formally compare alternative specifications by using Bayes factors. Marginal likelihoods and Bayes factors computed for the various alternative models are displayed in Table 1.

Table 1: **Marginal Likelihood and Bayes Factors of alternative specifications**

	(log) **ML**	(log) **Bayes Factors vs.**			
		$\psi \neq 0, \beta \neq 0$	$\psi = 0, \beta \neq 0$	$\psi \neq 0, \beta = 0$	$\psi = 0, \beta = 0$
Macro Uncertainty					
Model with $\psi \neq 0, \beta \neq 0$	-88.49	0	-7.59	37.04	20.09
Model with $\psi = 0, \beta \neq 0$	-80.90	7.59	0	44.63	27.68
Model with $\psi \neq 0, \beta = 0$	-125.53	-37.04	-44.63	0	-16.95
Model with $\psi = 0, \beta = 0$	-108.58	-20.09	-27.68	16.95	0
Financial Uncertainty					
Model with $\psi \neq 0, \beta \neq 0$	-160.10	0	-24.33	44.17	25.38
Model with $\psi = 0, \beta \neq 0$	-135.77	24.33	0	68.49	49.71
Model with $\psi \neq 0, \beta = 0$	-204.27	-44.17	-68.49	0	-18.79
Model with $\psi = 0, \beta = 0$	-185.48	-25.38	-49.71	18.79	0

The (log) Bayes factors are simply given by the difference of the (log) marginal likelihoods of two competing models, and therefore the entries in Table 1 are symmetric along the main diagonal. The (log) marginal likelihood of the model with $\psi = 0$ imposed is -80.90, and that of the model with $\psi \neq 0$ is -88.49, which gives a (log) Bayes factor of 7.59, providing strong evidence in favor of the model with $\psi = 0$. The same happens when using Specification 1, which has a (log) Bayes factor of 16.95 in favor of the model with restricted ψ.

Table 1 also shows that the models in which $\beta \neq 0$ (i.e., Specification 2) have stronger support in the data than models in which $\beta = 0$ (i.e., Specification 1).

The empirical evidence is similar when rather than macroeconomic uncertainty we use an index of financial uncertainty. In particular, we repeat the analysis using the Chicago Board Options Exchange (CBOE) S&P 100 Volatility Index, known as VXO, over the same sample 1960Q3 to 2017Q2. In this instance, for purely illustrative purposes, we report the results for Specification 1, obtained by setting $\beta = 0$ so that $\lambda_t = h_t$. The coefficient estimates based on Specification 2 give a virtually identical picture.

Figure 3 reports the entire posterior distributions of ϕ and ψ under the two alternative cases $\psi = 0$ and $\psi \neq 0$ in the estimated model. Starting with the case $\psi \neq 0$, the distribution of ϕ is centered around zero, which implies that financial uncertainty has virtually no effects on growth. As we shall see this result may be due to omitted variables since it will change

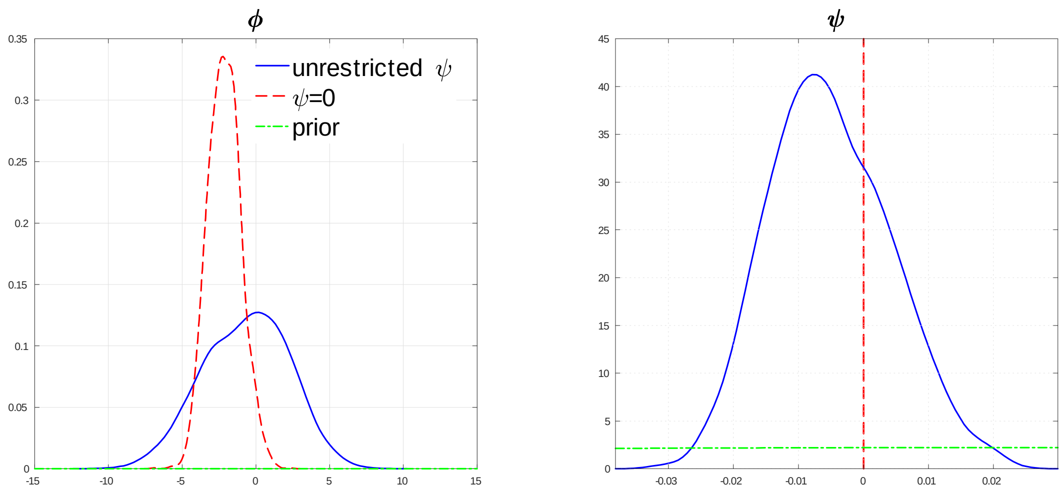

Figure 3: Posterior distributions of ϕ and ψ when ψ is unrestricted (blue solid line) and when ψ is restricted to 0 (red dashed line). Model with VXO financial uncertainty, and with $\beta = 0$.

in the multivariate setting of Section 5.[14] Setting $\psi = 0$ shifts to the left the posterior distribution of ϕ, pushing the posterior mean to from -0.676 to -2.129**,** and also reduces its variance substantially. These results show that if financial uncertainty is treated as exogenous in this simple bivariate model, it can lead to an overestimation of its negative effects on growth. According to Table 1, the Bayes factors prefer the Specification 2 for the volatility ($\beta \neq 0$) and also the exogeneity of financial uncertainty.

4.2 A Monte Carlo evaluation

This section describes a range of Monte Carlo experiments, with the aim of illustrating the main features of the model, checking the estimation algorithm, and assessing the effects of some sources of misspecification. As data generating process (DGP) we use the estimated model (35a)-(35c). With this DGP, we simulate 1000 time series for y_t, m_t, and h_t with $t = 1, ..., 250$. We then estimate the model using 60,000 draws for each set of time series, storing at each replication the posterior mean of each parameter, reported in charts below. This provides the empirical distribution of the point estimates across the MC replications. The prior distributions are overlaid on the results in all of the charts provided.

We have considered a wide range of Monte Carlo designs, and applied them to both

[14] The complexity of the model and identification precludes an analytical evaluation of bias. In general, if the DGP has multiple variables in y_t but the estimated model omits some elements of y_t, the simultaneity between $\ln m_t$ and ϵ_t^* will create bias in the estimates of ψ and ϕ.

Specification 1 and Specification 2. The designs consider the following issues:

1. Designs 1-3: What are the effects of setting either $\psi = 0$ or $\phi = 0$ in the model?
2. Design 4: What is the effect of the heteroskedasticity of the system going to 0?
3. Design 5: What is the role of the Gaussianity assumption for the structural shocks ε_t^*?

The complete set of results is available in section 6 of the supplementary appendix. In what follows, we only present a selection of the results based on Specification 2 and focusing only on three key coefficients: Π_m, ψ, ϕ. Results are in Figure 4. In the figure, the rows of panels correspond to alternative Monte Carlo designs, and the columns of panels provide the results for each coefficient. Specifically, the results consist of the empirical distribution of the point estimates across the MC replications, under two alternative scenarios (identified by a blue solid line and a red dashed line).

Omitted and redundant feedback effects We start with Design 1, in which the DGP features $\psi \neq 0$ and the researcher either does or does not impose the restriction $\psi = 0$. Since the value for ψ resulting from the empirical application is quite low (-0.0038) and not significantly different from 0, this value would not serve well to explore the case of a relevant omitted ψ. Hence, for this design we multiply it by 20, and fix $\psi = -0.0038 \times 20$. All the remaining coefficients are set as in the empirical estimates of equation (35). The results for this MC design are shown in the first line of panels in Figure 4. When the researcher chooses to leave the coefficient ψ unrestricted, the estimated model does not suffer from misspecification, and the resulting posteriors are entirely in line with the true DGP. However, if the researcher imposes the restriction $\psi = 0$, the resulting posteriors are markedly distorted and fail to recover the true values of the coefficients in the DGP. It is interesting to note that such distortions resemble the pattern we have found in the empirical application. Specifically, the posterior mean of the distribution of ϕ moves from -5.41 to -10.79, while that for Π_m shifts from 3.10 to 8.19, so that the sum $\phi + \Pi_m$ is little changed, from -2.31 to -2.6. As we have already emphasized, this is because the reduced-form parameter $C_{21} = \Pi_m + \phi\delta_m$ cannot change, and — since δ_m is equal to 0.93 — the parameter Π_m must increase by roughly the same amount of the decrease in ϕ.

Next, we consider Design 2, in which we use the coefficient values of the estimated model (35a)-(35c) as DGP, with the relevant exception of setting $\psi = 0$. This means that there is no contemporaneous feedback effect of macro variables on uncertainty; that is, uncertainty is exogenous. As before, the researcher can either impose the restriction

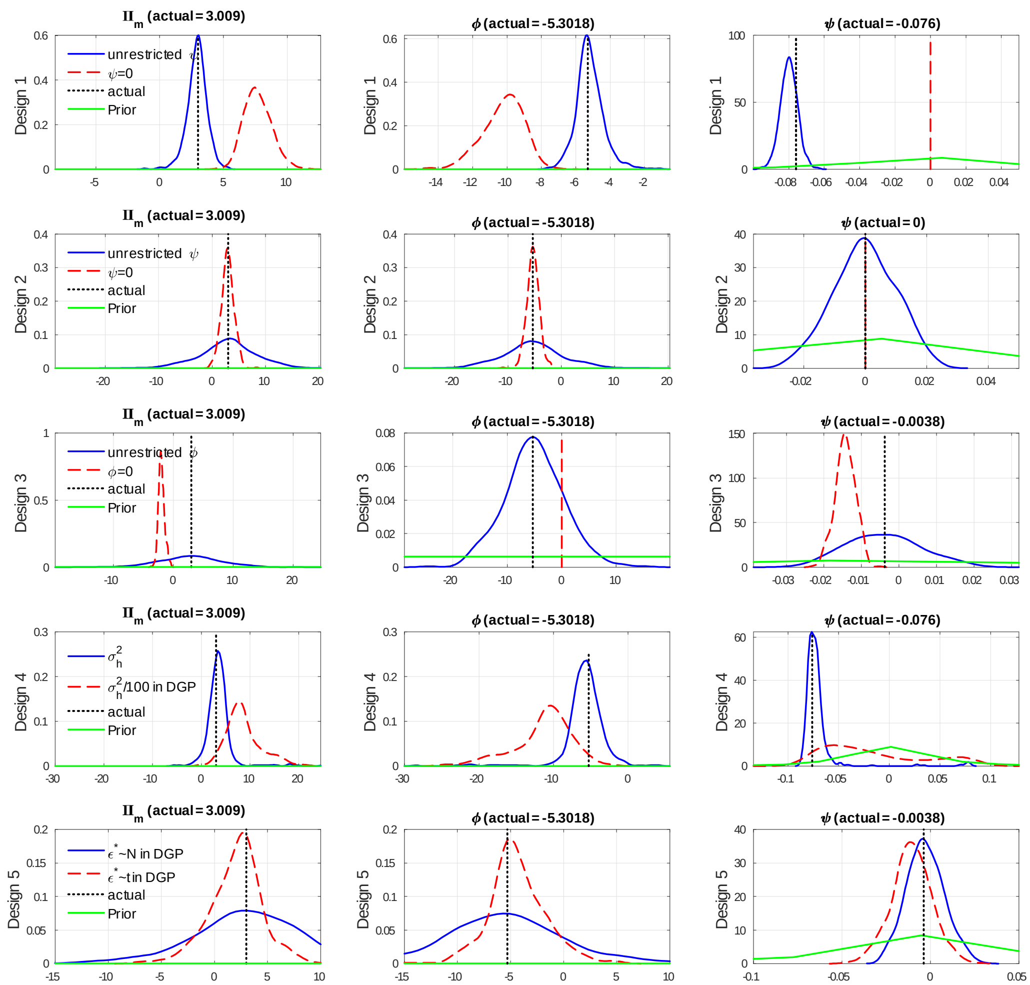

Figure 4: Excerpts from Monte Carlo experiments, designs 1-5. Empirical distributions of posterior means across Monte Carlo replications.

or not in the estimated model. This case resembles the case of redundant variables in the estimated model, which typically leads to inefficient — but yet unbiased — estimators. This is precisely what happens, as illustrated in the second row of Figure 4. In this case, both models recover the correct values for the coefficients, but the model imposing the correct restriction $\psi = 0$ attains more precise estimates.

Design 3 considers the effects of erroneously imposing the restriction $\phi = 0$, i.e., no contemporaneous effects of uncertainty on y_t, while in the true DGP such a restriction does not hold. We use the coefficient values of the estimated model (35a)-(35c) for the DGP, without any modification being required, since ϕ is already high and significant in these estimates. Results from this design are presented in the third row of Figure 4. As one would expect, imposing the restriction $\phi = 0$ distorts the results. In particular, the contemporaneous effect of uncertainty on output (as measured by ϕ) is underestimated (set at $\phi = 0$ rather than estimated as $\phi = -5.41$), and since the reduced-form parameter $C_{21} = \Pi_m + \phi\delta_m$ cannot change, the lagged effect Π_m gets correspondingly over-estimated. Moreover, the distortion in ϕ implies a distortion in ϵ_t^* (it no longer features zero mean), and therefore, ψ gets overestimated (in absolute value).

In light of the three designs described above, the identification problem faced by the standard approach can be rephrased as follows. In the reduced form we have the parameter $C_{21} = \Pi_m + \phi\delta_m$, which is uniquely identified in the likelihood. Recursive identification schemes impose either $\phi = 0$ or $\psi = 0$, which both amount to a potential omitted variable problem. We have that:

- If $\psi = 0$ (uncertainty ordered first), then the estimated ϕ will be more negative than it is in the DGP, and we have an overestimate of Π_m. In this case the identification scheme attributes too much of the impact variation to uncertainty and too little to lagged uncertainty.

- If $\phi = 0$ (uncertainty ordered last), then the estimated ϕ will be less negative than it is in the DGP, and we have an underestimate of Π_m. In this case the identification scheme attributes too little of impact variation to uncertainty and too much to lagged uncertainty.

The approach we propose in this paper solves these issues, since it allows estimating a model that does not put any restrictions on either ψ or ϕ. The results of the Monte Carlo evaluation highlight the importance of properly modeling the endogeneity of uncertainty, and support the interpretation of the empirical findings about the relationship between GDP growth and either macro or financial uncertainty. In addition, there is no tendency to spuriously estimate a significant contemporaneous dependence of uncertainty on macro conditions when none exists in the DGP.

Effects of shutting down the heteroskedasticity As we have discussed, identification in this model comes from the heteroskedasticity, and hence it is natural to ask what happens as heteroskedasticity vanishes. Design 4 answers this question, by letting the variance $\sigma_{\tilde{\eta}}^2$ be close to 0. In this case λ_t will no longer be a random state variable, but rather will converge to a fixed $\bar{\lambda}$, and this will make the model converge to a standard homoskedastic VAR. We simulate the model using Specification 1 (which involves $\beta = 0$) and either $\sigma_{\tilde{\eta}}^2 = 0.023$ or $\sigma_{\tilde{\eta}}^2 = 0.00023$ (the remaining coefficients being as in Design 1). Then, we estimate the model and compare the results. This is illustrated in the fourth row of Figure 4. In this case, as the idiosyncratic volatility's variance becomes very small, the model estimates generally differ widely from the true values. This highlights that identification in our model rests on the time-varying volatility. In the empirical application, we will provide overwhelming evidence of its existence.

Effects of mis-specifying the distributional assumption on ε_t^* Design 5 looks at what happens if the distribution of ε_t^* is misspecified. To do so, we simulate artificial data using the estimated model (35), but we modify the shock to (35a) so that it has a t-distribution with 3 degrees of freedom, as opposed to a Gaussian distribution. Then, we estimate the model and compare the results. This is illustrated in the last row of Figure 4. The figure shows that this type of misspecification does have an impact on the bias of the estimates. This effect is limited in the case at hand (which is calibrated on our empirical application) but in general it would lead to inconsistency in the posterior in large samples. Sensitivity to this assumption is a particular fragility of the stochastic volatility framework, as the same would not happen in either standard SVARs identified by short or run-long restrictions, or under long-span regime switching specifications of time varying volatility. The most natural way to address this issue would be to eliminate the possible misspecification by modeling the shocks as a t-student, which is a feasible but computationally demanding extension of our framework.[15] Moreover, we will provide below evidence in favor of the normality assumption in our empirical application.

5 The economic effects of (endogenous) uncertainty

We now study the relationship between macroeconomic and financial uncertainty and economic variables. We do so using both quarterly and monthly data for the U.S. The data are described in Section 5.1. Section 5.2 discusses the choice of the specification. Section 5.3 provides the results of macro uncertainty shocks. Section 5.4 presents the results of financial uncertainty shocks.

5.1 Data

5.1.1 Uncertainty

Assessing the relationship between uncertainty and economic variables requires choosing a concept and measure of uncertainty. The uncertainty literature features a range of both

[15] A few studies — e.g., Chiu, Mumtaz, and Pinter 2017; Clark and Ravazzolo 2015; Curdia, Del Negro, and Greenwald 2015 — have considered models with both stochastic volatility and fat-tails in the conditional innovations. The evidence from this literature seems mixed, with some but not all finding fat tails to be helpful to model fit and forecasting.

concepts and measures, reflecting the broad definition in the opening paragraph of Bloom (2014, p.153): "Uncertainty is an amorphous concept. It reflects uncertainty in the minds of consumers, managers, and policymakers about possible futures. It is also a broad concept, including uncertainty over the path of macro phenomena like GDP growth, micro phenomena like the growth rate of firms, and non-economic events like war and climate change."

Bloom (2014) goes on to indicate that uncertainty cannot be perfectly measured but can be proxied by a range of measures. One very common proxy is the volatility of stock prices. As noted in Bloom (2009, p.627), stock market volatility "...is strongly linked to other measures of productivity and demand uncertainty." Sources such as Bloom (2014) discuss some reasons why; note that, as a matter of timeliness, asset prices such as stock prices react quickly to news on the economy, including news relating to economic uncertainty. Other examples of studies using stock volatility-based measures include Basu and Bundick (2017) and Caggiano, Castelnuovo, and Groshenny (2014). Baker, Bloom, and Davis (2016) develop an alternative measure of uncertainty associated with economic policies, based on newspaper coverage.

Jurado, Ludvigson, and Ng (2015, p.1177) take a more specific stand on the concept and measure of uncertainty: "At a general level, uncertainty is typically defined as the conditional volatility of a disturbance that is unforecastable from the perspective of economic agents." JLN argue that, for various reasons, common proxies such as stock market volatility need not be tightly linked to such a concept.[16] JLN go on to develop measures of macroeconomic and financial uncertainty based on forecast error variances of large sets of macro indicators and asset returns. Conceptually similar measures have been developed in studies such as Carriero, Clark, and Marcellino (2017) and Jo and Sekkel (2017).

Accordingly, in this paper we consider a range of measures of uncertainty. Our baseline results are based on one measure of macroeconomic uncertainty and one measure of financial uncertainty. For the former, we use the macroeconomic uncertainty measure of JLN, and for the latter, we use the Chicago Board Options Exchange (CBOE) S&P 100 Volatility Index, known as VXO. In estimates omitted in the interest of brevity, we have verified the robustness of our VAR-based results to alternative measures, including the macroeconomic uncertainty measure of Carriero, Clark, and Marcellino (2017) and the financial uncertainty measures of both JLN and Carriero, Clark, and Marcellino (2017).[17] Consistent with other

[16] Berger, Dew-Becker, and Giglio (2018) present evidence that it is realized stock market volatility, as opposed to uncertainty about the future, that leads to economic fluctuations, and develop a structural model to account for the finding.

[17] In unreported results, we also estimated VARs with a news index-based measure of policy uncertainty (using data back to 1960, with the uncertainty measure provided by Professor Bloom from earlier versions of Baker, Bloom, and Davis (2016)). In these estimates, positive shocks to uncertainty have economic effects

discussions in the uncertainty literature, most measures are significantly correlated with others. For example, the uncertainty estimates of JLN (macro, finance) have a correlation of about 0.8 with the corresponding estimates of Carriero, Clark, and Marcellino (2017). The realized volatility of stock returns has a correlation of about 0.9 with the option-implied volatility measured by the VXO series. The correlation of these measures of financial uncertainty with those from JLN and CCM are about 0.7. Correlations of the macro uncertainty indicators of JLN and CCM with measures of financial uncertainty are a little lower, between 0.3 and 0.5 depending on the variable pairing.[18]

5.1.2 Other data

For each measure of uncertainty, we consider VARs with both quarterly and monthly datasets. The first model is a VAR that includes seven quarterly indicators in addition to the uncertainty measure. We will refer to this model as the quarterly VAR. The variables included in the model are reported in Table 2.

Table 2: Variables in the baseline quarterly model

GDP (100*Δ ln)	GDP
Consumption (100*Δ ln)	CONS
Private Investment (100*Δ ln)	INVES
Hours (100*Δ ln)	HOURS
Compensation of employees (100*Δ ln)	COMPE
GDP deflator (100*Δ ln)	PRICE
Federal funds rate (Δ)	FFR
JLN or VXO uncertainty	uncertainty

The seven indicators are essentially those covered in the widely used DSGE model of Smets and Wouters (2007) and in many related analyses, such as Justiniano, Primiceri, and Tambalotti (2011). The set of indicators is also similar to that used by JLN (in their 8 variable VAR specification) to assess the effects of uncertainty shocks (under a recursive identification scheme that amounts to imposing the restriction $\psi = 0$). We use four lags and the sample covers the period 1960Q3 to 2017Q2, for a total of $T = 228$ observations.

similar to those reported for the paper's baseline measures. In this case, news-based policy uncertainty appears to be endogenous, responding contemporaneously to the business cycle. We interpret that pattern as consistent with the newspaper story basis of the uncertainty measure as being a fast-moving variable, like financial indicators.

[18] Policy uncertainty as measured by the Baker, Bloom, and Davis (2016) series we use is a little less connected to these other measures, with correlations ranging from about 0.2 to 0.5.

As mentioned, all variables are demeaned prior to estimation to reduce the computational burden. We obtained the raw macroeconomic data from the FAME database of the Federal Reserve Board of Governors. We obtained the JLN measure of uncertainty from the website of Professor Ludvigson, defining our quarterly series as the within-quarter average of the source monthly series. To obtain the long series on the VXO measure, we followed the precedents of other studies in the literature and spliced realized volatility of S&P 500 returns (monthly standard deviations of daily returns) for 1960-1985 to the monthly VXO series for 1986-2017 from the St. Louis Fed's FRED database. Our quarterly VXO series uses within-quarter averages of the monthly series.

The second model is a monthly VAR that includes nine monthly indicators in addition to the uncertainty measure. We will refer to this model as the monthly VAR. The variables included in the model are reported in Table 3.

Table 3: Variables in the monthly model

All employees: total nonfarm (100*Δ ln)	PAYEM
Industrial production index (100*Δ ln)	IP
Weekly hours: goods-producing (100*Δ ln)	HOURS
Real consumer spending (100*Δ ln)	SPEND
Orders (index/100)	ORDER
Earnings (100*Δ ln)	EARNI
PCE price index (100*Δ ln)	PCEPI
Federal funds rate (Δ)	FFR
S&P 500 (Δ ln)	S&P
JLN or VXO uncertainty	uncertainty

This specification of variables is very similar to those considered in JLN and Bloom (2009), and contains many of the same variables in the VAR of Caldara, et al. (2016).[19] We use four lags (these are sufficient to provide white noise residuals in both the monthly and quarterly applications) and estimate over the sample 1961m7 to 2016m11, for a total of T = 659 observations. As with the quarterly VAR, all variables are demeaned prior to estimation to reduce the computational burden. We obtained the monthly macroeconomic data from the MD-FRED database developed in McCracken and Ng (2016) and made available on the website of the Federal Reserve Bank of St. Louis.

Although the model is estimated with data transformed as indicated in Table 2 and Table 3, for comparability to previous studies, the impulse responses are cumulated and

[19] We obtained very similar results with a model augmented to include a credit spread.

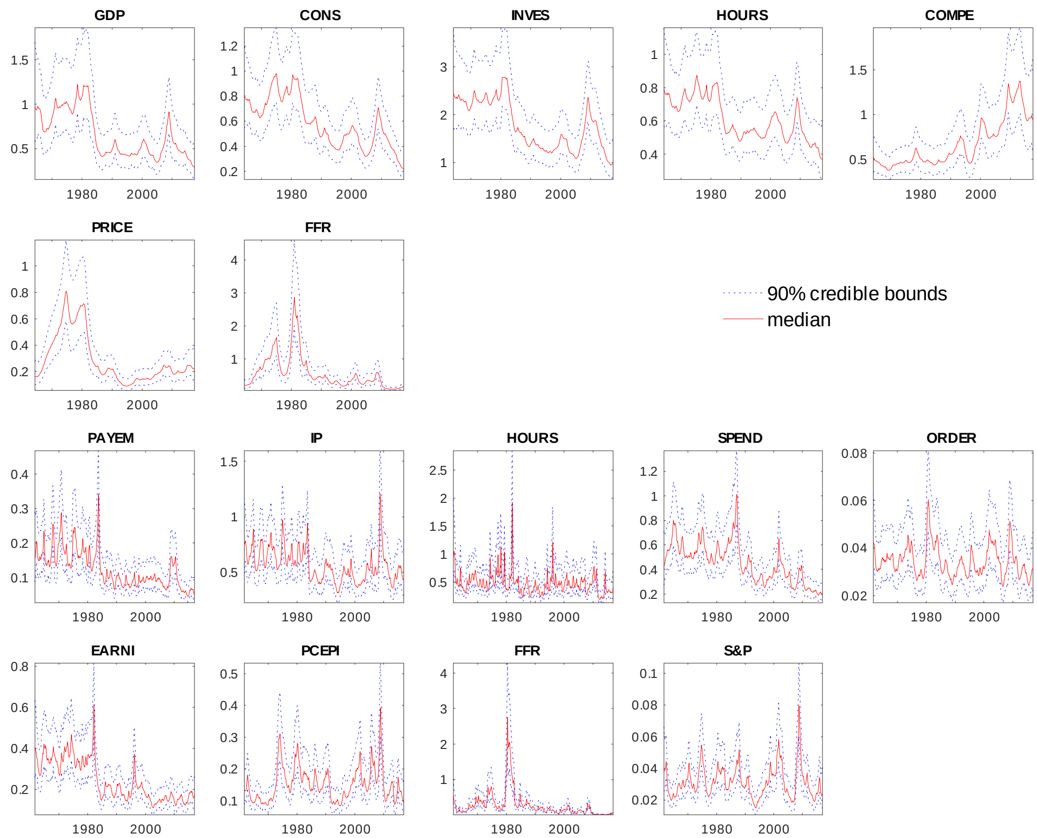

Figure 5: Reduced form volatilities from an unrestricted Bayesian VAR. Rows 1 and 2: quarterly dataset. Rows 3 and 4: monthly dataset.

transformed back to the units typical in the literature. Accordingly, the units of the reported impulse responses are percentage point changes (based on 100 times log levels for variables in logs or rates for variables not in log terms). The fact that the model is estimated using some variables differenced for stationarity (e.g., GDP, consumption, and investment) implies that, for some of these variables, the long-run effects of transitory shocks do not die out.

5.2 Choice of specification

As a preliminary step, we have estimated some unrestricted time varying volatilities for the variables in Table 2 and Table 3. Specifically, we have used the Bayesian VAR with stochastic volatility described in Carriero, Clark and Marcellino (2018b). The data show overwhelming evidence of variation in the conditional volatilities. Figure 5 shows the posterior distributions of the time varying volatilities of these variables.

Moreover, as was the case for the larger data set used in Carriero, Clark, and Marcellino (2018b), also in this data set there is strong evidence of commonality in the volatilities. In particular, the first principal component computed over the resulting estimated volatilities explains 78.36% of the total variance in the quarterly dataset, and 63.45% in the monthly dataset. This first principal component is highly correlated with the macroeconomic uncertainty measure of JLN, with a correlation of 64.47% in the quarterly dataset and of 65.43% in the monthly dataset. The financial uncertainty measure VXO is correlated with higher-order principal components (the 3rd and 6th principal component, with correlations of 25.68% and 49.80% in the quarterly dataset, and the 8th principal component, with a correlation of 61.56% in the monthly dataset).[20]

The analysis of the bivariate model discussed in Section 4.1 and Table 1 has highlighted the fact that the models featuring the uncertainty measures in the conditional variance (i.e., Specification 2) have a markedly higher marginal likelihood with respect to those without (i.e. Specification 1).[21]

We take all the above as evidence that a factor structure is a preferable specification for the volatilities, and for this reason in the remainder of the paper we will report results for Specification 2, which allows non-zero loadings β. However it is worth noting that none of the results we present below are substantially different under Specification 1, which restricts the factor loadings β to 0. This is largely due to the fact that the specification of the conditional variance part of the model mainly enhances the efficiency of the estimates of the conditional mean, but has only a limited effect on the point estimates. The entire analysis has been replicated under Specification 1 and the results are available in section 2 of the supplementary appendix.

Recall that under Specification 2 the total volatility of a variable is follows:

$$\ln \lambda_{jt} = \beta_j \ln m_t + \ln h_{jt},\ j = 1, ..., n,$$

where $\ln m_t$ is common to all volatilities, and $\ln h_{jt}$ is an idiosyncratic stochastic volatility process. The loadings $\beta_j,\ j = 1, ..., n$ measure the proportion with which the common factor impacts on the volatility of variable j. The posterior distributions of the loadings for both datasets and for both macroeconomic and financial uncertainty are displayed in Figure 6. The figure shows that these coefficients generally vary a lot across different variables. The loadings are also generally different from 1.

Finally, in the Monte Carlo section, Design 5, we flagged the potential fragility of our framework to misspecification of the distribution of the shocks. Therefore, a check of the normality of the innovation vector ϵ_t^* is warranted. Jarque-Bera tests performed on the

[20] The financial uncertainty measure constructed by JLN has qualitatively similar correlations.

[21] In principle one could compute the Bayes factors also for these multivariate specifications, and use those to choose a preferred specification. However in practice the parameter space features too many latent states to be able to obtain a reliable estimate of the marginal likelihood.

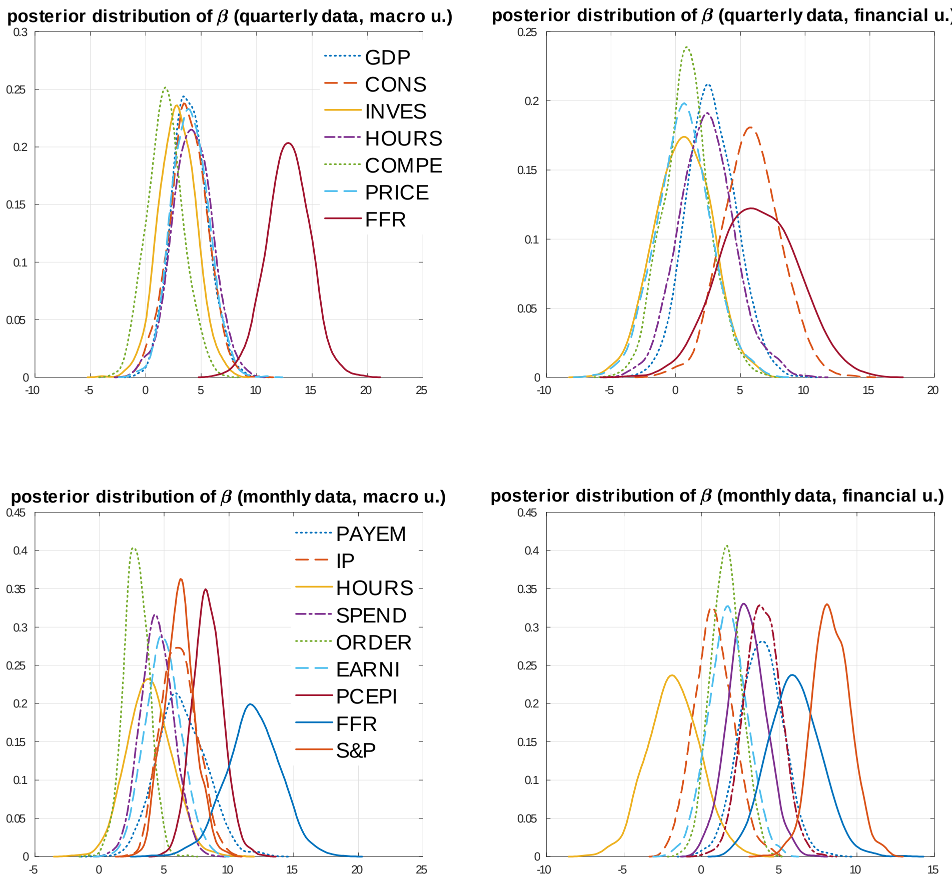

Figure 6: Posterior distributions of the β loadings.

posterior means of the components of ϵ_t^* comfortably support normality for both our specifications and for both datasets. Results can be found in the supplementary appendix's section 5.

5.3 Macroeconomic uncertainty shocks

Figure 7 shows the posterior distributions of the standard effect coefficients ϕ and the feedback coefficients ψ under both the quarterly VAR (upper panels) and the monthly VAR (lower panels). As mentioned in Section 2.3.2, the ϕ and ψ coefficients depicted in Figure 7 (and later in Figure 10) are conditional on the particular ordering of the variables within the macro block. In practice, we have experimented with several alternative orderings in both the monthly and the quarterly model, in both specifications 1 and 2, finding a rather limited impact on the estimated ϕ and ψ coefficients. Section 3 in the supplementary appendix provides these results.

Focusing first on the standard effect coefficients ϕ, which are on the left-hand side panels

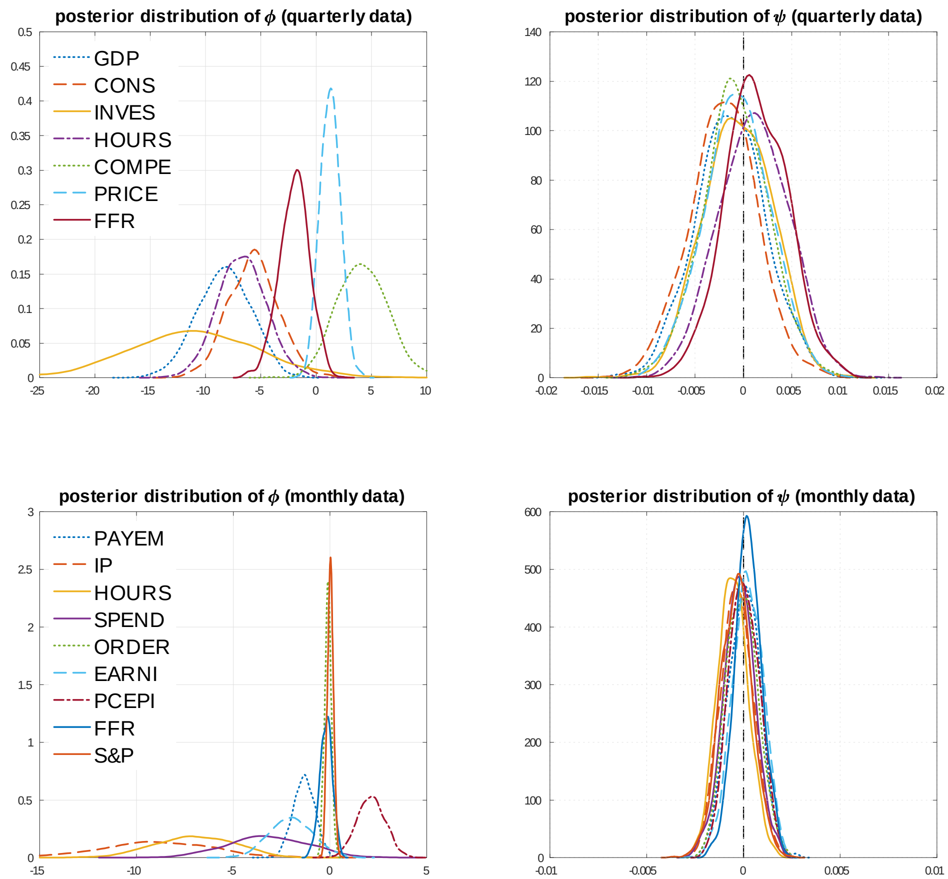

Figure 7: Posterior distributions of the ϕ and ψ coefficients. VARs with macro (JLN) uncertainty. Quarterly VAR in the upper panels, monthly VAR in the lower panels.

of Figure 7, they appear to be largely in line with previous findings about the effects of uncertainty on macroeconomic variables. In particular uncertainty has a large depressive effect on investment, output, consumption, hours, employment, industrial production, consumer spending, and earnings. A shock to uncertainty also leads to a loosening of the federal funds rate, and an increase in inflation as measured by wages, the GDP deflator, and the PCE price index. These results confirm those of several other studies, such as Baker, Bloom, and Davis (2016), Bloom (2009), Carriero, Clark, and Marcellino (2017), Gilchrist, Sim, and Zakrajsek (2014), Jo and Sekkel (2017), and Jurado, Ludvigson, and Ng (2015). They are also in line with the international evidence in Carriero, Clark, and Marcellino (2018a).

Turning our focus to the feedback coefficients ψ, which are on the right-hand side of Figure 7, it is striking that these coefficients do not appear to be statistically different from 0. This is true for both the quarterly dataset and the monthly dataset. The sign of the posterior means is in line with what macroeconomic reasoning would suggest, for example

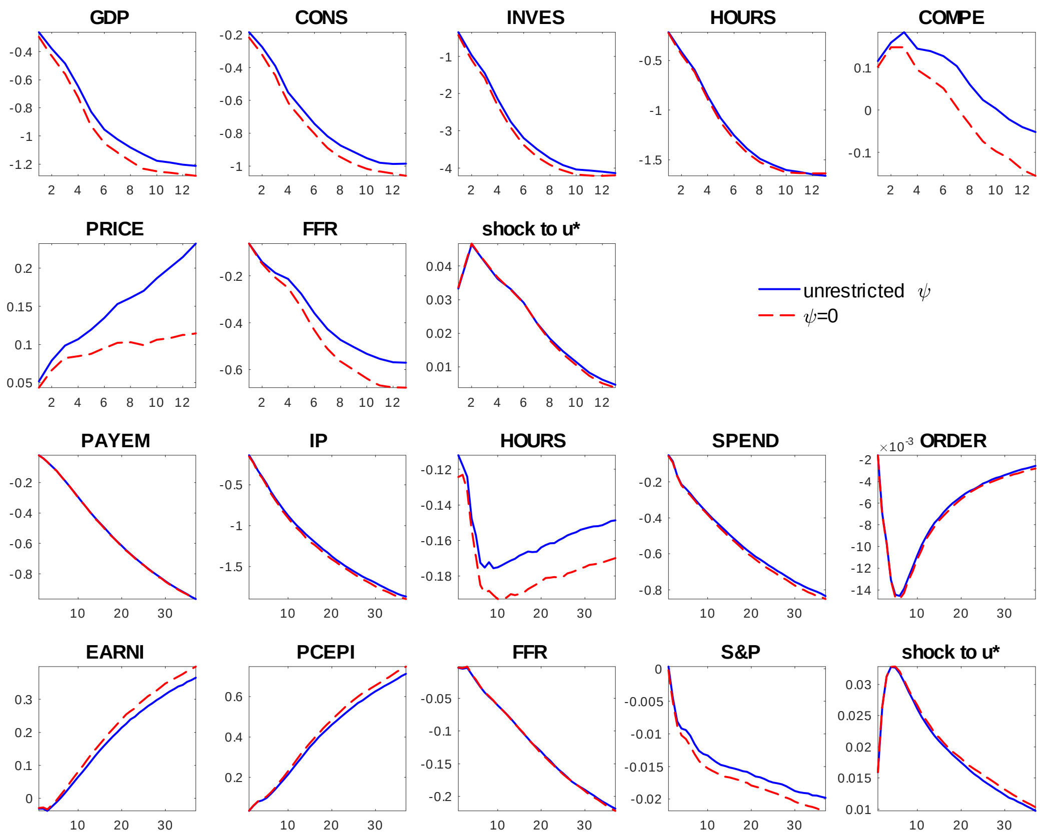

Figure 8: Impulse responses (posterior medians) to a macro uncertainty shock with $\psi = 0$ and $\psi \neq 0$. Rows 1 and 2: quarterly dataset. Rows 3 and 4: monthly dataset

variables such as hours and industrial production tend to reduce uncertainty, whereas the federal funds rate increases macroeconomic uncertainty. However, the overall picture is clearly one in which the contemporaneous effect of macroeconomic variables on uncertainty is feeble, if not entirely absent. It is also worth mentioning that these results are somewhat different from those reported in Figure 2. We attribute this difference to the fact that the simple bivariate model omits several relevant variables, which leads to distortions in the estimated effects of uncertainty on output.

We now consider the consequences that shutting down the feedback coefficients ψ (which capture the immediate response of uncertainty to economic conditions) has on the coefficients ϕ (which capture the immediate response of economic conditions to uncertainty) and on the impulse responses. This amounts to ordering uncertainty first in a VAR identified through a recursive Cholesky scheme. In light of both the results depicted in Figure 7 for the coefficients ψ and the results we obtained with the MC design in which $\psi = 0$ in the DGP, we expect the distortion arising from shutting down the feedback effect to be small.

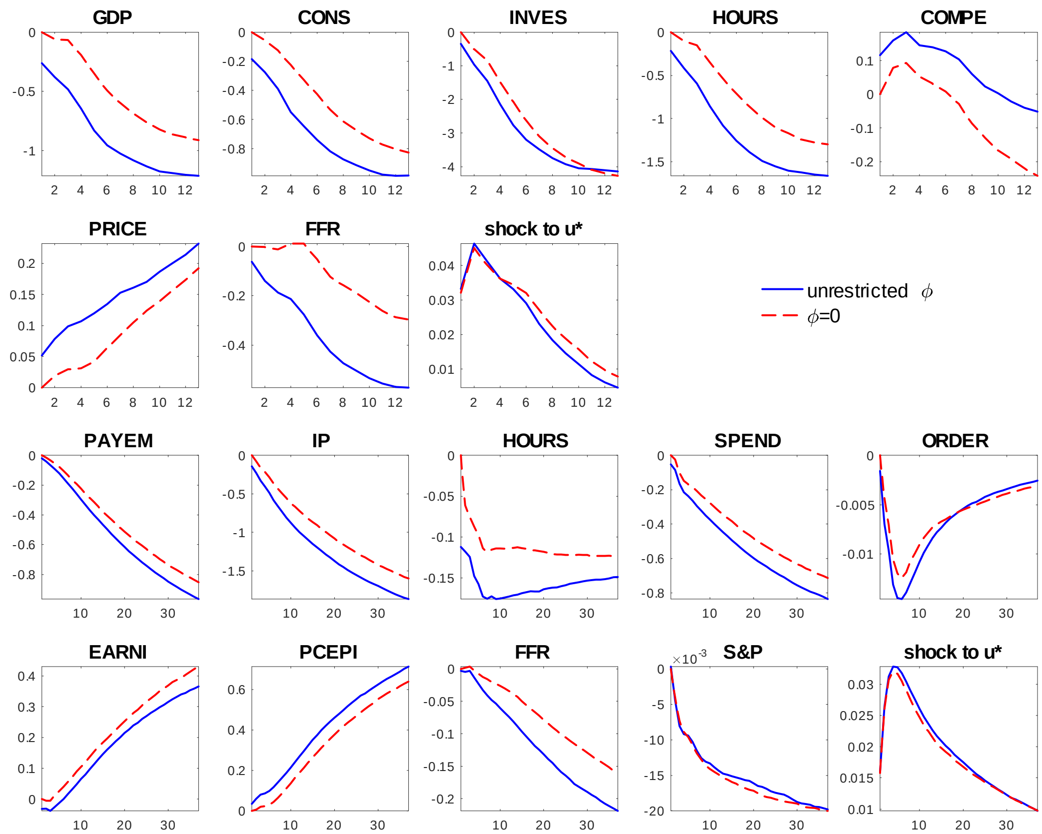

Figure 9: Impulse responses (posterior medians) to a macro uncertainty shock with $\phi = 0$ and $\phi \neq 0$. Rows 1 and 2: quarterly dataset. Rows 3 and 4: monthly dataset

Figure 8 shows the posterior medians of the impulse responses to a macroeconomic uncertainty shock. Graphs in the first two rows provide results for the quarterly dataset, while graphs in the last two rows provide results for the monthly dataset. In the figure, the unrestricted model is denoted by the solid blue lines, while the model with the feedback effects restricted to zero ($\psi = 0$) is denoted by red dashed lines. It turns out that the differences between the two models are very small. An exception seems to be the path of quarterly prices, but it is important to note that this effect is estimated very imprecisely in both models, and that the resulting error bands around these responses (not shown for chart readability) are so wide that any difference between the posterior means is largely insignificant.

The fact that the differences in Figure 8 are barely noticeable, especially for the monthly dataset, might be due to a scaling effect, since the charts plot the time series evolution of the median, or might conceal differences in higher moments, rather than the posterior median. In order to check that this is not the case, we have examined (see section 4 of the supplementary appendix) the entire posterior distribution of the impulse responses at some selected horizons. These distributions show that setting $\psi = 0$ has only a slight effect on impact and in the very short run. After that, the differences between the two models quickly die out, and the distributions of the impulse responses become virtually identical.

It is interesting to also consider the case of shutting down the standard uncertainty effect (the contemporaneous effect of uncertainty on economic conditions), i.e., to set $\phi = 0$. This amounts to ordering uncertainty last in a VAR identified through a recursive Cholesky scheme. Since — as seen in Figure 7 — the coefficients ϕ are broadly different from zero, and considering also the results we obtained with the MC design in which the researcher erroneously imposes $\phi = 0$, we expect the effect from shutting down this channel to be large.

Figure 9 shows the posterior medians of the impulse responses to a macroeconomic uncertainty shock, for the quarterly and monthly datasets (section 4 of the supplementary appendix displays the entire posterior distribution of the impulse responses at some selected horizons). Clearly, shutting down the standard channel produces largely different impulse responses, and the differences do not completely die out even at the 12-quarter- (or 36-month-) ahead horizons. These results, combined with the Monte Carlo evidence we discussed above (in particular, the design in which the researcher erroneously imposes $\phi = 0$), imply that setting $\phi = 0$ — or, equivalently, ordering macroeconomic uncertainty last in a recursive VAR — would very likely lead to distorted estimation of the effects of macro uncertainty shocks on macroeconomic variables, and a confusion between its contemporaneous and lagged effects.

5.4 Financial uncertainty shocks

We now focus on the effects of financial uncertainty shocks. Figure 10 shows the posterior distributions of the standard effect coefficients ϕ (contemporaneous response of economic activity to uncertainty) and the feedback coefficients ψ (contemporaneous response of uncertainty to economic activity) under both the quarterly VAR (upper panels) and the monthly VAR (lower panels).

Focusing first on the standard effect coefficients ϕ, which are on the left-hand side panels of Figure 10, they appear to be largely in line with previous findings about the effects of financial uncertainty on macroeconomic variables. In particular, financial uncertainty has a large depressive effect on investment, output, consumption, hours, employment, and orders. Differently from what happened for macroeconomic uncertainty shocks (see Figure 7), financial uncertainty shocks do not seem to have a significant effect on the federal funds rate, nor on prices as measured by wages, the GDP deflator, and the PCE price index. Instead, financial uncertainty shocks have a strong negative effect on the S&P 500 index,

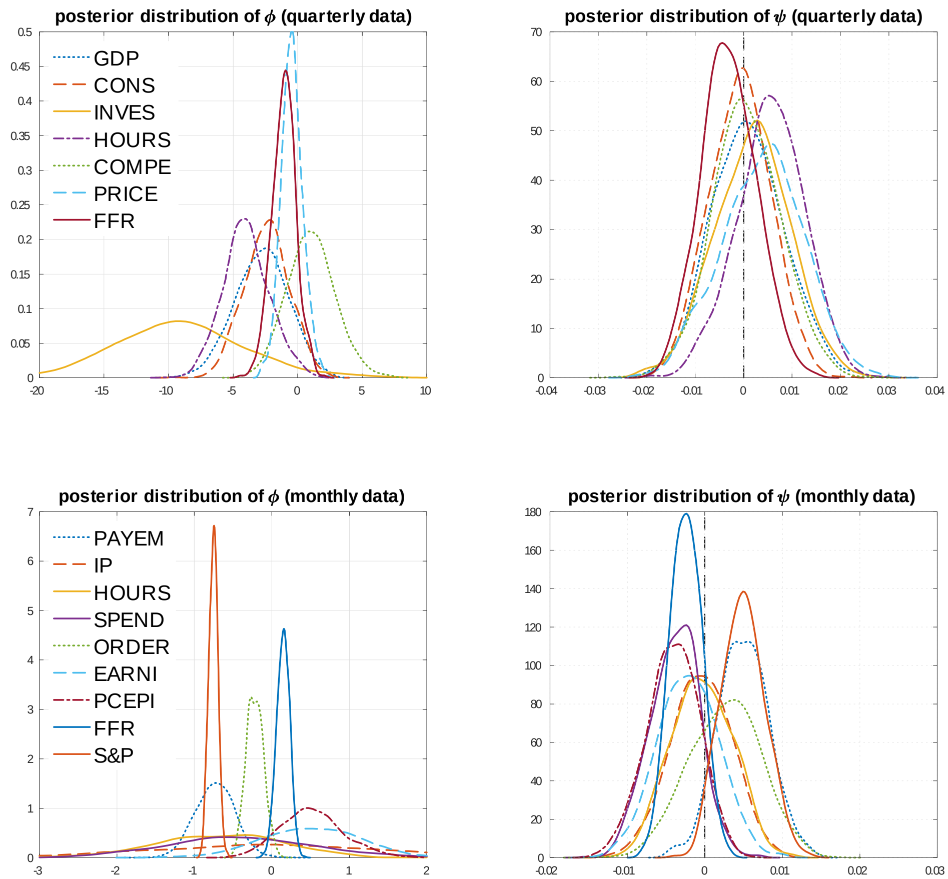

Figure 10: Posterior distributions of the ϕ and ψ parameters. VARs with financial (VXO) uncertainty. Quarterly VAR in the upper panels, monthly VAR in the lower panels.

while macroeconomic uncertainty shocks do not.

Turning to the feedback effect coefficients ψ, it is interesting to note that in the case of financial uncertainty there is more evidence that these coefficients are nonzero, i.e., that financial uncertainty might be endogenous. This pattern is particularly evident in variables such as consumer spending, earnings, inflation, industrial production, and the federal funds rate all featuring negative feedback coefficients, which shows that an increase in these indicators leads to a reduction in financial uncertainty. Some other variables show significantly positive ψ coefficients; in particular, increases in employment and the S&P 500 index seem to increase uncertainty. The latter effect could be related to a risk-return relationship, but the former is somewhat less clear, perhaps linked to worries about a possible overheating of the economy and a monetary policy response. It is worth noting that the effect of the S&P 500 index on financial uncertainty is opposite to that on macro uncertainty. Also, nearly all of the probability mass of the posterior distribution of the parameter associated with the S&P

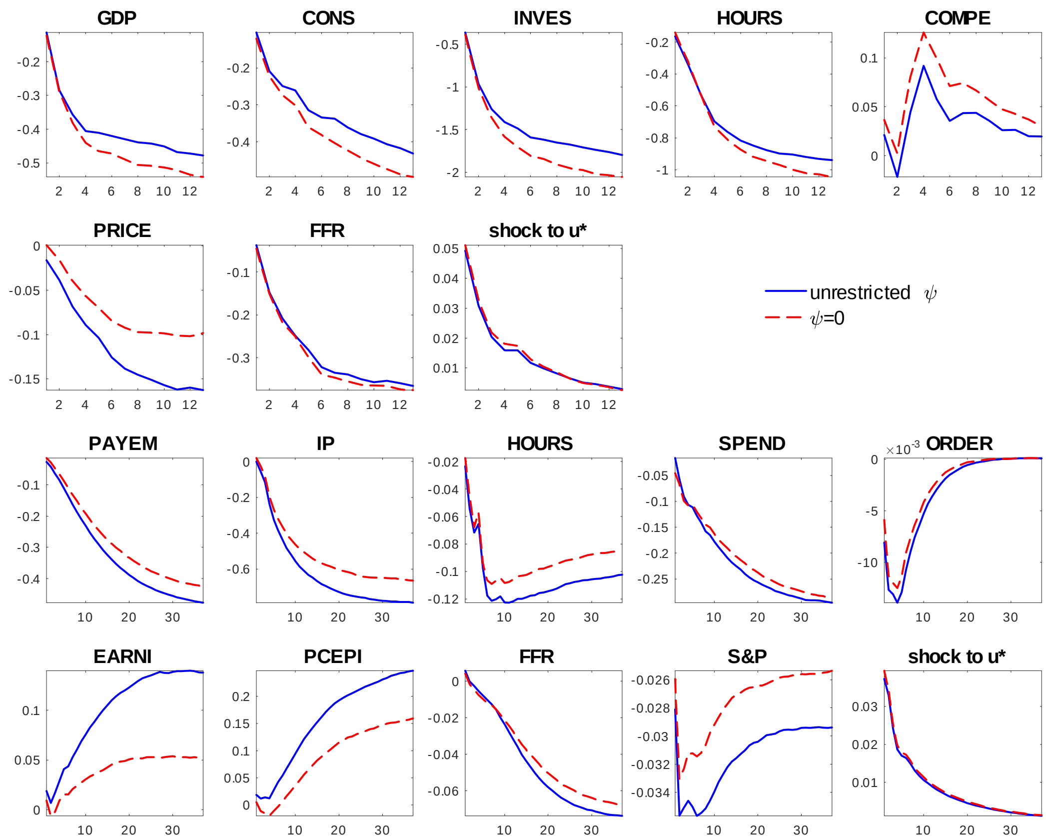

Figure 11: Impulse responses (posterior medians) to a financial uncertainty shock with $\psi = 0$ and $\psi \neq 0$. Rows 1 and 2: quarterly dataset. Rows 3 and 4: monthly dataset

500 index is on positive values, which clearly suggests endogeneity of financial uncertainty, in contrast with the findings of LMN, who find less endogeneity in financial uncertainty with respect to macro uncertainty.[22] Section 7 of the supplementary appendix examines the sources of differences in results. The differences appear to mainly stem from the identifying assumptions in LMN that a large shock to macroeconomic uncertainty was realized in December of 1970, that a large shock to either macroeconomic or financial uncertainty — but not necessarily both — occurred with the failure of Lehman Brothers in September 2008, and that the debt ceiling crisis (with no government shutdown) caused macroeconomic uncertainty to rise in July and August 2011. With adjustments around these identifying assumptions, which are not empirically supported by our analysis, the LMN method yields results qualitatively similar to ours.

[22] We have also removed the S&P 500 index from the monthly VAR model with VXO uncertainty, to control for the possiblity that in its presence, the effects of the financial uncertainty could be diminished. Results were largely unaffected by this change, as documented in section 3 of the supplementary appendix.

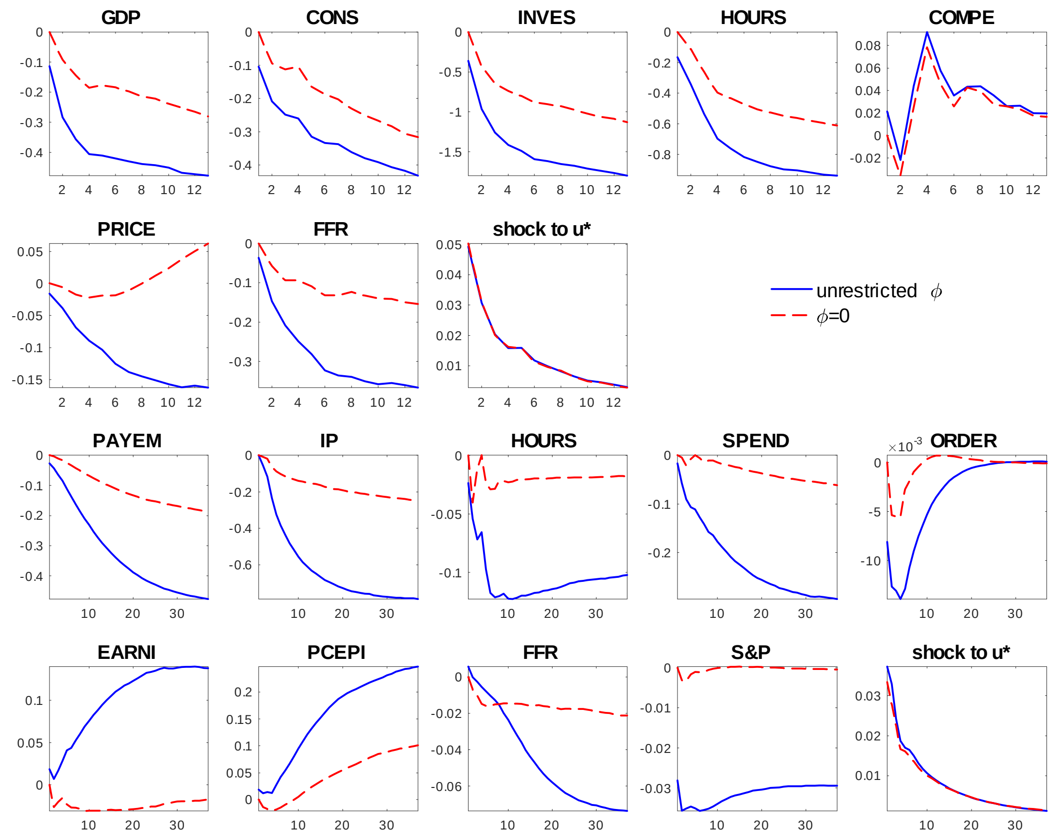

Figure 12: Impulse responses (posterior medians) to a financial uncertainty shock with $\phi = 0$ and $\phi \neq 0$. Rows 1 and 2: quarterly dataset. Rows 3 and 4: monthly dataset

Since for financial uncertainty we do find some evidence of endogeneity (i.e., $\psi \neq 0$) in the monthly dataset, we do expect to find that imposing $\psi = 0$ — i.e., ordering uncertainty first in a recursive VAR — would cause some distortions in the estimated coefficients ϕ (the contemporaneous response of economic activity to uncertainty) and in the impulse responses.

Figure 11 shows the posterior medians of the impulse responses to a financial uncertainty shock. As before, the two top rows of charts provide results for the quarterly dataset, while the two bottom rows provide results for the monthly dataset. In the figure, the unrestricted model is denoted by the solid blue lines, while the model with the feedback effects restricted to zero ($\psi = 0$) is denoted by red dashed lines. As is clear from the figure, there are indeed some differences between the two models, particularly for those variables for which the coefficients ψ are far from zero, such as earnings, inflation, industrial production, and the S&P 500 index. (Additional figures in section 4 of the supplementary appendix report the entire posterior distribution of the impulse responses at some selected horizons for, respectively, quarterly and monthly data.) These results indicate that financial uncertainty can at least in part arise as an endogenous response to some macroeconomic conditions, and that overlooking this channel would lead to a distorted estimate of the effects of financial uncertainty shocks on the economy.

Finally, we repeat the exercise of shutting down the standard channel of transmission, i.e. setting the ϕ coefficients to zero. This amounts to ordering financial uncertainty last in a VAR identified through a recursive Cholesky scheme. Since — as seen in Figure 10 — the coefficients ϕ are broadly different from zero, in Figure 12 we find that shutting down this channel has a large distortionary effect on the impulse responses, which does not completely die out even at the 12-quarter- (or 36-month-) ahead horizons. Therefore, just as happened with macroeconomic uncertainty, setting $\phi = 0$ — or equivalently ordering financial uncertainty last in a recursive VAR — would very likely lead to a distorted estimate of the effects of financial uncertainty shocks on macroeconomic variables, and a confusion between its contemporaneous and lagged effects.

6 Conclusions

Uncertainty is a key variable to understanding economic dynamics, attracting growing interest following the seminal work of Bloom (2009) and the Great Recession of 2007-2009. Several theoretical and empirical papers are by now available on the effects of uncertainty on key economic variables. A general finding from the empirical studies is that uncertainty leads to a deterioration in economic conditions. However, this outcome could be at least partly due to an endogeneity problem. If economic conditions have a contemporaneous effect on uncertainty, ruling it out a priori could result in overestimation of the effects of uncertainty.

In this paper we have developed an econometric model where current and past values of uncertainty affect the current levels of economic variables, and uncertainty is in turn affected by them also contemporaneously. We achieve identification by means of a novel procedure that relies on a particular heteroskedasticity structure, which allows the time-varying conditional variances of the variables to be driven by an uncertainty measure plus an idiosyncratic component, or just a stochastic idiosyncratic component. We provide the relevant conditional posteriors for the states and coefficients of the model, which can be used to estimate the model with a Gibbs sampler.

While the focus of this paper is on uncertainty shocks, the model can be used for any situation in which the researcher wishes to model some of the variables in a vector autoregression as endogenous and there is evidence of time-varying volatility.

Our empirical results point to the conclusion that there is only mild evidence for the endogeneity of uncertainty, and this evidence is limited to financial uncertainty. When looking at macro uncertainty, we found strong evidence that the feedback coefficients ψ are likely close to zero in both quarterly and monthly data, which means that imposing exogenous macroeconomic uncertainty does not do much harm. We found that some ψ coefficients are nonzero in the case of financial uncertainty, which points toward the conclusion that financial uncertainty seems endogenous to some extent.

The contrast in these results for financial uncertainty (which responds endogenously to the business cycle) versus those for macroeconomic uncertainty (which doesn't) raises the important question of what is different in these types of uncertainty that might yield such a pattern. One possibility is that financial uncertainty responds faster than macroeconomic uncertainty to economic news because the former is more closely related to asset prices, which typically react extremely fast to news. In the literature assessing various shocks with structural VARs, it is common to treat financial variables as responding quickly (contemporaneously) to macroeconomic shocks and macroeconomic variables as responding more slowly to the same shocks (e.g., Bernanke, Boivin, and Eliasz (2005)). A related possibility could be that financial uncertainty or volatility measures can reflect financial stresses (as argued in Giglio, Kelly, and Pruitt (2016)) that respond quickly to economic conditions. Although important, this question is beyond the scope of this paper. It is ultimately an economic question, and economic models will need to be developed to answer it. While our model allows us to assess the endogeneity of uncertainty with respect to the business cycle, it is not a structural economic model. We view our results as establishing facts on the endogeneity of uncertainty to be explained in future research developing structural economic models.

Our modeling approach does not put any restrictions on either ψ or ϕ. Still, if a researcher wanted to use a recursive VAR, our results provide two important suggestions for identification. First, ordering macroeconomic uncertainty first is likely to be harmless, not necessarily so for financial uncertainty. Second, ordering either type of uncertainty last is likely to produce misleading results. These findings imply that, to reliably assess financial uncertainty and its macroeconomic effects, it is necessary to depart from a simple recursive ordering and use a more sophisticated approach to identification, such as the one we develop.

7 Appendix

7.1 Details on identification

7.1.1 Derivation of identification relationships

Starting from the model in section 2's equations (1) and (2), substitute (2) into (1) and obtain:

$$\begin{aligned} y_t &= (\Pi_y(L) + \phi\delta_y(L))y_{t-1} + (\Pi_m(L) + \phi\delta_m(L)) \ln m_{t-1} + \phi\tilde{u}_t + (\phi\psi + A^{-1}\Lambda_t^{0.5})\epsilon_t^* \\ \ln m_t &= \delta_y(L)y_{t-1} + \delta_m(L) \ln m_{t-1} + \psi\epsilon_t^* + \tilde{u}_t. \end{aligned}$$

In matrix form:[23]

$$\begin{bmatrix} y_t \\ \ln m_t \end{bmatrix} = \begin{bmatrix} \Pi_y(L) + \phi\delta_y(L) & \Pi_m(L) + \phi\delta_m(L) \\ \delta_y(L) & \delta_m(L) \end{bmatrix} \begin{bmatrix} y_{t-1} \\ \ln m_{t-1} \end{bmatrix} + \begin{bmatrix} \phi\psi + A^{-1}\Lambda_t^{0.5} & \phi \\ \psi & 1 \end{bmatrix} \begin{bmatrix} \epsilon_t^* \\ \tilde{u}_t \end{bmatrix};$$

and the relation between structural and reduced form is:

$$\begin{aligned} \begin{bmatrix} C_{11}(L) & C_{21}(L) \\ C_{12}(L) & C_{22}(L) \end{bmatrix} &= \begin{bmatrix} \Pi_y(L) + \phi\delta_y(L) & \Pi_m(L) + \phi\delta_m(L) \\ \delta_y(L) & \delta_m(L) \end{bmatrix} \\ \begin{bmatrix} \Sigma_{11t} & \Sigma_{12t} \\ \Sigma_{21t} & \Sigma_{22t} \end{bmatrix} &= \begin{bmatrix} (\phi\psi + A^{-1}\Lambda_t^{0.5}) & \phi \\ \psi & 1 \end{bmatrix} \begin{bmatrix} I & 0 \\ 0 & \sigma_{\tilde{u}}^2 \end{bmatrix} \begin{bmatrix} (\phi\psi + A^{-1}\Lambda_t^{0.5})' & \psi' \\ \phi' & 1 \end{bmatrix} \\ &= \begin{bmatrix} (\phi\psi + A^{-1}\Lambda_t^{0.5})(\phi\psi + A^{-1}\Lambda_t^{0.5})' + \phi\phi'\sigma_{\tilde{u}}^2 & \left(\phi\psi + A^{-1}\Lambda_t^{0.5}\right)\psi' + \phi\sigma_{\tilde{u}}^2 \\ \psi\left(\phi\psi + A^{-1}\Lambda_t^{0.5}\right) + \sigma_{\tilde{u}}^2\phi' & \psi\psi' + \sigma_{\tilde{u}}^2 \end{bmatrix}. \end{aligned}$$

By stacking the various blocks in the equations above, we obtain:

$$C_{11}(L) = \Pi_y(L) + \phi\delta_y(L) \tag{36a}$$

$$C_{21}(L) = \Pi_m(L) + \phi\delta_m(L) \tag{36b}$$

$$C_{12}(L) = \delta_y(L) \tag{36c}$$

$$C_{22}(L) = \delta_m(L) \tag{36d}$$

$$\Sigma_{11t} = (\phi\psi + A^{-1}\Lambda_t^{0.5})(\phi\psi + A^{-1}\Lambda_t^{0.5})' + \phi\phi'\sigma_{\tilde{u}}^2 \tag{36e}$$

$$\Sigma_{12t} = \Sigma_{21t}' = \left(\phi\psi + A^{-1}\Lambda_t^{0.5}\right)\psi' + \phi\sigma_{\tilde{u}}^2 \tag{36f}$$

$$\Sigma_{22} = \psi\psi' + \sigma_{\tilde{u}}^2 \tag{36g}$$

The system above is the one reported in section 2.3.2's equation (20). Equations (36a)-(36d) and (36g) are immediately solved for $\Pi_y(L)$, $\Pi_m(L)$, $\delta_y(L)$, $\delta_m(L)$, $\sigma_{\tilde{u}}^2$ under knowledge of

[23] Note that the elements in the square impact matrix after the plus sign also represent the impact effects of shocks to ϵ^* and $\tilde{u}_t$ on y_t and m_t. Hence, while the effects of the uncertainty shocks are stable over time (and equal to ϕ and 1), those of ϵ^* are time-varying.

ϕ and ψ. Moreover we can write (36e) and (36f) as follows:

$$\begin{aligned}
\Sigma_{11t} &= \phi\psi\psi'\phi' + \phi\psi\Lambda_t^{0.5}A^{-1'} + (\phi\psi\Lambda_t^{0.5}A^{-1'})' + A^{-1}\Lambda_t A^{-1\prime} + \phi\phi'\sigma_{\tilde{u}}^2 \\
&= \phi(\psi\psi' + \sigma_{\tilde{u}}^2)\phi' + \phi\psi\Lambda_t^{0.5}A^{-1'} + (\phi\psi\Lambda_t^{0.5}A^{-1'})' + A^{-1}\Lambda_t A^{-1\prime} \\
&= \phi\Sigma_{22}\phi' + \phi\psi\Lambda_t^{0.5}A^{-1'} + (\phi\psi\Lambda_t^{0.5}A^{-1'})' + A^{-1}\Lambda_t A^{-1\prime} \\
\Sigma_{12t} &= \phi\psi\psi' + A^{-1}\Lambda_t^{0.5}\psi' + \phi\sigma_{\tilde{u}}^2 \\
&= \phi(\psi\psi' + \sigma_{\tilde{u}}^2) + A^{-1}\Lambda_t^{0.5}\psi' \\
&= \phi\Sigma_{22} + A^{-1}\Lambda_t^{0.5}\psi',
\end{aligned}$$

which gives the compact system appearing in section 2.3.2's equation (21). To obtain the system for the bivariate case it is sufficient to set $n = 1$.

7.1.2 Identification with a general A matrix

The identification argument reported in the paper hinges on the lower triangularity of the A matrix. Triangularity of the matrix A can be relaxed and a milder sufficient condition is the one reported in Rubio-Ramirez, Waggoner, and Zha (2010), i.e., that every row j in A^{-1} has at least $n - j$ restrictions. This condition ensures that a permutation matrix P' exists and is such that $P'A^{-1} = A^{*-1}$ is lower triangular. Under this condition, substitute A^{-1} with PA^{*-1} in section 2.3.2's equation (21a) to obtain:

$$\begin{aligned}
\Sigma_{11t} &= \Sigma_{22}\phi\phi' + \phi\psi\Lambda_t^{0.5}A^{*-1'}P' + (\phi\psi\Lambda_t^{0.5}A^{*-1'}P')' + PA^{*-1}\Lambda_t A^{*-1'}P' \\
\Sigma_{12t} &= \phi\Sigma_{22} + PA^{*-1}\Lambda_t^{0.5}\psi'.
\end{aligned}$$

Pre- and post- multiplication of both sides by P' and P yields:

$$\begin{aligned}
P'\Sigma_{11t}P &= P'\Sigma_{22}\phi\phi'P + P'\phi\psi\Lambda_t^{0.5}A^{*-1'} + (P'\phi\psi\Lambda_t^{0.5}A^{*-1'})' + A^{*-1}\Lambda_t A^{*-1'} \\
P'\Sigma_{12t}P &= P'\phi\Sigma_{22}P + A^{*-1}\Lambda_t^{0.5}\psi'P,
\end{aligned}$$

which is simply a permutation of the original system in which the matrix A^{*-1} is lower triangular. Since P is fully known (it is only composed of 0 and 1) the system above can be recursively solved using the same argument applied to the case with A triangular.

7.2 Derivation of the joint density of data and states

The starting point is the computation of the joint density of the data and the states $p(y_t, m_t, h_t|\theta)$, which can be obtained via the change of variable theorem. We start by re-writing the shocks as follows:

$$\begin{aligned}
\epsilon_t^* &= (M_t^{(\beta)})^{-0.5} H_t^{-0.5} A(y_t - \Pi_y(L) y_{t-1} - \Pi_m(L) \ln m_{t-1} - \phi \ln m_t) && (40)\\
\tilde{u}_t &= \ln m_t - \delta_y(L) y_{t-1} - \delta_m(L) \ln m_{t-1} - \psi \epsilon_t^* && (41)\\
\tilde{\eta}_{jt} &= \ln h_{jt} - \alpha_j - \delta_j \ln h_{jt-1}, \quad j = 1, ..., n, && (42)
\end{aligned}$$

with:

$$\mathbf{r}_t = \begin{bmatrix} \epsilon_t^* \\ \tilde{u}_t \\ \tilde{\eta}_t \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} I_n & 0 & 0 \\ 0 & \sigma_{\tilde{u}}^2 & 0 \\ 0 & 0 & \Sigma_{\tilde{\eta}} \end{bmatrix} \right),$$

where $\tilde{\eta}_t = (\tilde{\eta}_{1t}, ..., \tilde{\eta}_{nt})'$ and $\Sigma_{\tilde{\eta}}$ is a diagonal matrix with elements $\sigma_{\tilde{\eta}_j}^2, \ j = 1, ..., n$.[24] The vector $\mathbf{r}_t$ is therefore a vector of independent Gaussian (structural) shocks.

Using the shocks (40)-(42) and

$$JN_t = \begin{bmatrix} \partial \epsilon_t^*/\partial y_t & \partial \epsilon_t^*/\partial m_t & \partial \epsilon_t^*/\partial h_t \\ \partial \tilde{u}_t/\partial y_t & \partial \tilde{u}_t/\partial m_t & \partial \tilde{u}_t/\partial h_t \\ \partial \tilde{\eta}_t/\partial y_t & \partial \tilde{\eta}_t/\partial m_t & \partial \tilde{\eta}_t/\partial h_t \end{bmatrix},$$

we can use the change of variable theorem to get:

$$p(y_t, m_t, h_t|\theta) = |JN_t| \times p_G(\epsilon_t^*, \tilde{u}_t, \tilde{\eta}_t). \tag{43}$$

Since $\partial \tilde{\eta}_t/\partial y_t = \partial \tilde{\eta}_t/\partial m_t = 0$ and $\partial \tilde{\eta}_t/\partial h_t = H_t^{-1}$, the determinant simplifies to:

$$|JN_t| = |H_t^{-1}| \left| \begin{pmatrix} \partial \epsilon_t^*/\partial y_t & \partial \epsilon_t^*/\partial m_t \\ \partial \tilde{u}_t/\partial y_t & \partial \tilde{u}_t/\partial m_t \end{pmatrix} \right|,$$

with

$$\begin{aligned}
\partial \epsilon_t^*/\partial y_t &= (M_t^{(\beta)})^{-0.5} H_t^{-0.5} A \\
\partial \tilde{u}_t/\partial y_t &= -\psi \partial \epsilon_t^*/\partial y_t \\
\partial \tilde{u}_t/\partial m_t &= m_t^{-1} - \psi \partial \epsilon_t^*/\partial m_t.
\end{aligned}$$

Hence, the determinant is:

$$\begin{aligned}
|JN_t| &= |H_t^{-1}||\partial \epsilon_t^*/\partial y_t| * |\partial \tilde{u}_t/\partial m_t - \partial \tilde{u}_t/\partial y_t * (\partial \epsilon_t^*/\partial y_t)^{-1} * \partial \epsilon_t^*/\partial m_t| \\
&= |H_t^{-1}||\partial \epsilon_t^*/\partial y_t| * |\partial \tilde{u}_t/\partial m_t + \psi \partial \epsilon_t^*/\partial y_t * (\partial \epsilon_t^*/\partial y_t)^{-1} * \partial \epsilon_t^*/\partial m_t| \\
&= |H_t^{-1}||\partial \epsilon_t^*/\partial y_t| * |m_t^{-1} - \psi \partial \epsilon_t^*/\partial m_t + \psi * \partial \epsilon_t^*/\partial m_t| \\
&= |H_t^{-1}||\partial \epsilon_t^*/\partial y_t| * |m_t^{-1}| \\
&= |H_t^{-1}||(M_t^{(\beta)})^{-0.5} H_t^{-0.5} A| * |m_t^{-1}| \\
&= |H_t^{-1}||(M_t^{(\beta)})^{-0.5} H_t^{-0.5}| * m_t^{-1} \\
&= m_t^{-1} \Pi_{j=1}^n m_t^{-0.5\beta_j} h_{jt}^{-1.5}.
\end{aligned}$$

[24] The assumption that $\tilde{\eta}_t$ are independent can be relaxed without any impact on the identification result.

And the density in (43) is:

$$p(y_t, m_t, h_t|\theta) = m_t^{-1}\Pi_{j=1}^{n} m_t^{-0.5\beta_j} h_{jt}^{-1.5} \underbrace{p_G(\epsilon_t^*)}_{\text{eq. (1)}} \times \underbrace{p_G(\tilde{u}_t)}_{\text{eq. (2)}} \times \underbrace{p_G(\tilde{\eta}_t)}_{\text{eq. (5)}}, \tag{44}$$

where the shocks ϵ_t^*, $\tilde{u}_t$, and $\tilde{\eta}_t$ are those in equations (1), (2), and (5).

7.3 Derivation of the conditional posterior of the states

In this subsection we derive the expression for the conditional posterior of the states in section 3.1's equation (26). To do so we consider the data density (44) for the generic idiosyncratic volatility of variable j at time t (i.e. h_{jt}) and recognize that i) since mutual independence of the shocks ensures that $p_G(\epsilon_t^*) = \Pi_{j=1}^{n} p_G(\epsilon_{jt}^*)$ and $p_G(\tilde{\eta}_t) = \Pi_{j=1}^{n} p_G(\tilde{\eta}_{jt})$, all the terms not involving variable j can be subsumed in the integrating constant; ii) due to the Markov property featured by h_{jt}, all the terms involving time periods beyond $t-1$ or $t+1$ can be ignored.[25] This gives:

$$\begin{aligned} p(h_{jt}|h_{jt-1}, h_{jt+1}, \theta, \mathbf{y}_{1:T}, \mathbf{m}_{1:T}, \mathbf{h}_{\neq j1:T}) \quad \propto \quad & h_{jt}^{-1.5} \exp\left(\frac{-\epsilon_{jt}^{*2} - \epsilon_{jt+1}^{*2}}{2}\right) && \text{(45a)} \\ & \times \exp\left(\frac{-\tilde{u}_t^2 - \tilde{u}_{t+1}^2}{2\sigma_{\tilde{u}}^2}\right) && \text{(45b)} \\ & \times \exp\left(\frac{-\tilde{\eta}_{jt}^2 - \tilde{\eta}_{jt+1}^2}{2\sigma_{\tilde{\eta}}^2}\right), && \text{(45c)} \end{aligned}$$

where we also subsumed $m_t^{-1}\Pi_{j=1}^{n} m_t^{-0.5\beta_j}$ and all of the terms $h_{it}^{-1.5}$ with $i \neq j$ in the integrating constant. Furthermore, the terms ϵ_{jt+1}^{*2} and $\tilde{u}_{t+1}^2$ are also redundant. Define:

$$\begin{aligned} e_t &= H_t^{0.5}\epsilon_t^* = (M_t^{(\beta)})^{-0.5} A(y_t - \Pi_y(L)y_{t-1} - \Pi_m(L)\ln m_{t-1} - \phi \ln m_t) && (46) \\ u_t &= \psi\epsilon_t^* + \tilde{u}_t, && (47) \end{aligned}$$

where in (46) we used $H_t^{0.5}(M_t^{(\beta)})^{-0.5}H_t^{-0.5} = (M_t^{(\beta)})^{-0.5}$, which holds true since both $H_t^{0.5}$ and $M_t^{(\beta)}$ are diagonal. Note that e_t is observable conditioning on m_t and y_t (plus the coefficients in θ_1), while u_t is observable conditioning on m_t and y_t *and* ϵ_t^* (plus the coefficients in θ_2). Using $\epsilon_{jt}^* = h_{jt}^{-0.5}e_{jt}$, the term $\exp(-\epsilon_{jt}^{*2}/2)$ in (45a) can be written as:

$$\exp\left(-\frac{\epsilon_{jt}^{*2}}{2}\right) = \exp\left(-\frac{e_{jt}^2}{2h_{jt}}\right). \tag{48}$$

The term $\exp(-\tilde{u}_t^2/2\sigma_{\tilde{u}}^2)$ in (45b) can be written as:

$$\exp\left(\frac{-\tilde{u}_t^2}{2\sigma_{\tilde{u}}^2}\right) = \exp\left(\frac{-(u_t - \psi\epsilon_t^*)^2}{2\sigma_{\tilde{u}}^2}\right) = \exp\left(\frac{-u_t^2 + 2u_t\psi\epsilon_t^* - \epsilon_t^{*\prime}\psi'\psi\epsilon_t^*}{2\sigma_{\tilde{u}}^2}\right),$$

[25] These initial derivation steps follow the approach of Jacquier, Polson, and Rossi (2004). However, as we stressed in Section 2, our model is more general than theirs.

and, as u_t is observed under the conditioning set, this becomes

$$\exp\left(\frac{-\tilde{u}_t^2}{2\sigma_{\tilde{u}}^2}\right) \propto \exp\left(\frac{2u_t\psi\epsilon_t^* - \epsilon_t^{*\prime}\psi'\psi\epsilon_t^*}{2\sigma_{\tilde{u}}^2}\right),$$

where $\exp(u_t\psi\epsilon_t^*) = \exp(u_t\psi_1\epsilon_{1t}^* + u_t\psi_2\epsilon_{2t}^* + ... + u_t\psi_n\epsilon_{nt}^*) \propto \exp(u_t\psi_j\epsilon_{jt}^*)$. Also, since:

$$\begin{aligned}
& (\psi_1\epsilon_{1t}^* + \psi_2\epsilon_{2t}^* + ... + \psi_n\epsilon_{nt}^*) \times (\psi_1\epsilon_{1t}^* + \psi_2\epsilon_{2t}^* + ... + \psi_n\epsilon_{nt}^*) \\
= \; & \psi_1\epsilon_{1t}^* \times (\psi_1\epsilon_{1t}^* + \psi_2\epsilon_{2t}^* + ... + \psi_n\epsilon_{nt}^*) \\
& +\psi_2\epsilon_{2t}^* \times (\psi_1\epsilon_{1t}^* + \psi_2\epsilon_{2t}^* + ... + \psi_n\epsilon_{nt}^*) \\
& ... \\
& +\psi_j\epsilon_{jt}^* \times (\psi_1\epsilon_{1t}^* + \psi_2\epsilon_{2t}^* + ... + \psi_n\epsilon_{nt}^*) \\
& ... \\
& +\psi_n\epsilon_{nt}^* \times (\psi_1\epsilon_{1t}^* + \psi_2\epsilon_{2t}^* + ... + \psi_n\epsilon_{nt}^*),
\end{aligned}$$

we have that for a given equation j and conditioning on the remaining equations, $\exp(\epsilon_t^{*\prime}\psi'\psi\epsilon_t^*) \propto \exp(\epsilon_{jt}^{*2}\psi_j^2 + 2\psi_j\epsilon_{jt}^*(\psi_1\epsilon_{1t}^* + ... + \psi_{j-1}\epsilon_{j-1t}^* + \psi_{j+1}\epsilon_{j+1t}^* + ... + \psi_n\epsilon_{nt}^*))$. This leads to:

$$\begin{aligned}
\exp\left(\frac{-\tilde{u}_t^2}{2\sigma_{\tilde{u}}^2}\right) & \propto \exp\left(\frac{2u_t\psi_j\epsilon_{jt}^* - (\epsilon_{jt}^{*2}\psi_j^2 + 2\epsilon_{jt}^*\psi_j\psi_{\neq j}\epsilon_{\neq jt}^*)}{2\sigma_{\tilde{u}}^2}\right) \\
& = \exp\left(-\frac{e_{jt}^2\psi_j^2}{2\sigma_{\tilde{u}}^2 h_{jt}} + \frac{e_{jt}\psi_j}{\sqrt{h_{jt}}\sigma_{\tilde{u}}^2}[u_t - \psi_{\neq j}\epsilon_{\neq jt}^*]\right), \quad (49)
\end{aligned}$$

where ψ_j is the j-th element of the vector ψ and $\psi_{\neq j}$ is the vector obtained by removing the element ψ_j from ψ (and a similar convention applies to ϵ_{jt}^* and $\epsilon_{\neq jt}^*$). Finally, by completing the squares, the term $\exp((-\tilde{\eta}_{jt}^2 - \tilde{\eta}_{jt+1}^2)/2\sigma_{\tilde{\eta}}^2)$ in (45c) can be written as:

$$\exp\left(\frac{-\tilde{\eta}_{jt}^2 - \tilde{\eta}_{jt+1}^2}{2\sigma_{\tilde{\eta}}^2}\right) \propto \exp\left(\frac{-(\ln h_{jt}^2 - \mu_{jt})^2}{2s_{\tilde{\eta}_j}^2}\right), \quad (50)$$

where $\mu_{jt} = (\delta_j(\ln h_{jt-1} + \ln h_{jt+1}) + \alpha_j(1-\delta_j))/(\delta_j^2+1)$ and $s_{\tilde{\eta}_j}^2 = \sigma_{\tilde{\eta}_j}^2/(\delta_j^2+1)$.[26]

Using (48), (49), and (50), the density (45) can be written as:

$$\begin{aligned}
& p(h_{jt}|h_{jt-1}, h_{jt+1}, \theta, \mathbf{y}_{1:T}, \mathbf{m}_{1:T}, \mathbf{h}_{\neq j1:T}) \\
\propto \; & h_{jt}^{-0.5}\exp\left(-\frac{e_{jt}^2}{2h_{jt}}\left[1 + \frac{\psi_j^2}{\sigma_{\tilde{u}}^2}\right] + \frac{e_{jt}}{\sqrt{h_{jt}}}\frac{\psi_j}{\sigma_{\tilde{u}}^2}[u_t - \psi_{\neq j}\epsilon_{\neq jt}^*]\right) \times h_{jt}^{-1}\exp\left(\frac{-(\ln h_{jt}^2 - \mu_{jt})^2}{2s_{\tilde{\eta}_j}^2}\right).
\end{aligned}$$

This is the conditional posterior of the states appearing in section 3.1's equation (26).

[26] These conditional moments are slightly different in the first and last periods of the sample: in $t=0$ we have $\mu_{j0} = (\ln h_{j0} - \alpha_j)/\delta_j$ and $s_{\tilde{\eta}_j}^2 = \sigma_{\tilde{\eta}_j}^2/\delta_j^2$. In $t=T$ we have $\mu_{jT} = \alpha_j + \delta_j \ln h_{jT-1}$ and $s_{\tilde{\eta}_j}^2 = \sigma_{\tilde{\eta}_j}^2$.

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