

# Dynamic Information Acquisition and Strategic Trading

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## Abstract

We allow a strategic trader to choose when she acquires costly information about an asset's payoff, instead of requiring that she make her decision before trading begins. We show that she optimally delays becoming informed. We find that dynamic information acquisition yields novel predictions. First, the trader acquires information less often when the trading horizon is very short or very long, and the probability of acquisition can decrease with the volatility of public information. Second, volatility and price impact can jump and evolve stochastically, even when underlying shocks are homoskedastic. Finally, the average pricing error when the asset pays off can be larger when the trader acquires information.

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# 1 Introduction

Investors' incentives to acquire private information change over time and with current economic conditions. For instance, a falling real estate market can lead investors to acquire loan-level data on their mortgage-backed securities in order to revalue their positions. Rising oil prices can trigger research into whether airlines are hedged against fuel price increases. A consolidation wave in a particular industry can lead market participants to investigate remaining firms as potential targets. Following [Grossman and Stiglitz \(1980\)](#), a large literature has studied how investors choose to acquire information, and what their decisions imply for financial markets. However, despite the inherently dynamic nature of the information acquisition decision, the existing literature has treated it as a static problem by requiring that investors make their information choices before the start of trading.

We study the dynamic information acquisition decision of a strategic trader in a [Kyle \(1985\)](#) setting. In contrast to prior work, we allow her to choose the timing of information acquisition in response to the evolution of a public signal. Our analysis yields a number of novel insights. Importantly, we find that the optimal acquisition decision exhibits delay and does not follow a naive "NPV" rule. The trader acquires private information less often when the trading horizon is very short or very long, and information collection can decrease with the volatility of public news. Price impact and return volatility can be stochastic and exhibit jumps, even when shocks to fundamentals and noise trading are homoskedastic. Finally, contrary to intuition, the average pricing error at the time the asset pays off need not be smaller when the trader has become informed. Overall, our results suggest that incorporating dynamics into the information acquisition decisions of investors is important for understanding properties of price informativeness, price impact, and volatility.

Our model builds on the continuous time [Kyle \(1985\)](#) model in [Back and Baruch \(2004\)](#), which provides a tractable starting point for our analysis. There is a single risky asset, traded by a risk-neutral, strategic trader and a mass of noise traders. We introduce a publicly observable signal, which may or may not be payoff relevant, that evolves stochastically over time. A risk-neutral market maker competitively sets the price of the risky asset, conditional on the public signal and aggregate order flow. The asset payoff (and consequently, the relevance of the signal) is publicly revealed at a random time that is exponentially distributed. Unlike [Back and Baruch \(2004\)](#), the strategic trader is not endowed with private information. Instead, she can choose to pay a fixed cost to privately learn whether the signal is payoff relevant.

The key feature of our setting is that the decision to become informed need not be made

at the initial date before trading begins. Rather, the trader can choose to investigate the payoff relevance of the public signal at any point in time. Appealing to standard results on optimal stopping, we characterize the trader’s information acquisition strategy and show that it follows a cutoff rule: she chooses to acquire information only when the public signal exceeds a threshold.<sup>1</sup> Intuitively, the ability to decide *when* to acquire information endows the trader with a call option on the expected profits from being privately informed, and she chooses to exercise the option only when the uncertainty about the asset payoff is sufficiently high. Moreover, we show that optimal information acquisition exhibits delay — the strategic trader chooses to wait beyond the threshold that would be prescribed by an “NPV” rule. As such, the standard assumption that investors can only choose to acquire information at the initial date is restrictive.

Consistent with the intuition from real option decisions, we show that the benefit from waiting to acquire information increases in the cost of information and the volatility of the public signal, but decreases in the prior uncertainty about the payoff relevance of the signal. We also find that the acquisition boundary is higher when the asset payoff is expected to be revealed very soon or in the distant future. When the payoff is expected to be revealed quickly, the value from being informed is very low since there is little time over which to profit at the expense of noise traders, and so the acquisition boundary is high. However, as the expected trading horizon increases, there are two offsetting effects. On the one hand, the value from being informed increases with the horizon since the trader expects her information advantage to last longer. On the other hand, the cost of waiting decreases with the horizon, since the likelihood that the payoff is revealed before acquisition is low. We show that initially the first effect dominates, while eventually the second one does. As a result, the trader is less likely to acquire information when the trading horizon is very long or very short.

Next, we characterize the likelihood of information acquisition. Standard intuition suggests that an increase in the volatility of the public signal leads to an increase in the probability that it hits the acquisition boundary before the asset payoff is revealed (i.e., the option ends up sufficiently far “in the money” that exercise is optimal). However, in our model, signal volatility has an offsetting effect. An increase in volatility tends to increase the value of waiting, and so decreases the probability that information is acquired (i.e., increases the optimal acquisition boundary). We show that this effect dominates when volatility is high, and as a result, the probability of information acquisition can be hump-shaped in signal volatility.

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<sup>1</sup>We assume that the trader and the market maker share a common prior about the payoff relevance of the public signal. Furthermore, we restrict to acquisition strategies that depend only on public information and assume that acquisition is publicly observable. This implies that conditional on acquisition, the trading equilibrium is analogous to the one in [Back and Baruch \(2004\)](#).

The dynamic nature of the trader’s information acquisition decision, and the subsequent non-linear filtering problem that the market maker solves, leads to novel price dynamics in our model. Before acquisition, since the market maker’s beliefs about payoff relevance are not affected by the order flow, the price sensitivity to the public signal is constant — as a result, return volatility depends only on the signal volatility. After acquisition, the price responds not only to shocks in the public signal, but also to the order flow, from which the market maker learns about the payoff relevance of the public signal. As we show, information acquisition triggers a jump in instantaneous volatility and price impact, and following acquisition, both evolve stochastically. Notably, these results are not driven by stochastic volatility of fundamentals or noise trading, but arise endogenously due to the trader’s acquisition decision and the market maker’s learning problem.<sup>2</sup>

Finally, we characterize the average absolute price change at the time the asset payoff is publicly announced. Intuitively, one might expect that this announcement effect is smaller when the strategic trader is informed, since the price is more informative about the asset payoff in this case.<sup>3</sup> We show that this need not be the case when information acquisition is endogenous. To see why, note that fixing the conditional (public) uncertainty about the asset’s value, the announcement effect is smaller when the trader is informed — this implies that if the strategic trader is exogenously endowed with information, the standard intuition holds. However, there is an offsetting effect when information acquisition is endogenous: the strategic trader only chooses to acquire information when uncertainty is sufficiently high. As we show, for any fixed prior on the relevance of the public signal, this effect dominates when the acquisition boundary is sufficiently high. As a result, when the cost of information acquisition is sufficiently high, the public signal volatility is sufficiently high, or the expected trading horizon is sufficiently extreme (i.e., sufficiently short or sufficiently long), the expected announcement effect is larger when there is information acquisition.

Our paper relates to the large literature on asymmetric information models with endogenous information acquisition that was initiated by [Grossman and Stiglitz \(1980\)](#). While a number of papers extend the static model of [Grossman and Stiglitz \(1980\)](#) to allow for dynamic trading (e.g., [Mendelson and Tunca \(2004\)](#), [Avdis \(2016\)](#)), to allow traders to condition their infor-

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<sup>2</sup>Although not the focus of their analysis, a similar effect arises in [Back and Baruch \(2004\)](#). However, our result is in contrast to [Collin-Dufresne and Fos \(2016\)](#), where stochastic volatility and price impact are driven by stochastic volatility in noise trading.

<sup>3</sup>For instance, as [Back \(1992\)](#) establishes, the corresponding announcement effect must be zero conditional on the strategic trader being informed in the analogous, finite horizon model where the announcement is perfectly anticipated. When the announcement is stochastic, but the strategic trader is exogenously endowed with information, as in [Back and Baruch \(2004\)](#), the announcement effect is smaller when the strategic trader is informed.

mation acquisition decision on a public signal (e.g., [Foster and Viswanathan \(1993\)](#)), to allow traders to pre-commit to receiving signals at particular dates (e.g., [Back and Pedersen \(1998\)](#), [Holden and Subrahmanyam \(2002\)](#)), or to incorporate a sequence of one-period information acquisition decisions ([Veldkamp \(2006\)](#)), the information acquisition decision remains essentially static — investors make their information acquisition decision before the start of trade. To the best of our knowledge, however, our model is the first to allow for dynamic information acquisition in that the strategic trader can choose to become privately informed at any point of time. Moreover, our analysis implies that allowing for dynamic information acquisition has economically important consequences.

## 2 Model

### 2.1 Model setup

Our framework is based on the continuous time, [Kyle \(1985\)](#) model in [Back and Baruch \(2004\)](#). Fix a probability space  $(\Omega, \mathcal{F}, \mathbb{P})$  on which is defined the standard Brownian motion  $(W_Z, W_N)$  and independent random variables  $\xi$  and  $T$ . Let  $\mathcal{F}_t$  denote the augmentation of the filtration  $\sigma(\{W_{Zt}, W_{Nt}\})$ . The random variable  $\xi \in \{0, 1\}$  is binomial with probability  $\alpha = \Pr(\xi = 1)$ , and  $T$  is exponentially distributed with rate  $r$ . There are two assets: a risky asset and a risk-free asset with interest rate normalized to zero. The risky asset pays off  $v$  at random time  $T$ , where

$$v = \xi N_T. \tag{1}$$

The public news process  $N_t$  is a geometric Brownian motion

$$dN_t = \sigma_N N_t dW_{Nt} \tag{2}$$

where  $\sigma_N > 0$  and the initial value  $N_0 > 0$  is constant.<sup>4</sup> Given this specification, the news process is only informative about the payoff of the risky asset if  $\xi = 1$ .

There is a single, risk-neutral strategic trader who can pay a fixed cost  $c$  at any time  $\tau$  to determine whether an information event has occurred (i.e., to observe the realization of  $\xi$ ).

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<sup>4</sup>The assumptions that the public signal is perfectly informative about  $N_t$  and that  $N_t$  has zero drift are without loss of generality. In the more general case,  $N_t$  is replaced with  $\mathbb{E}[N_T | \mathcal{F}_t^P]$  in the pricing rule and trading strategy and the rest of the analysis is essentially unchanged. It is also straightforward to generalize to a general continuous, positive martingale for the news process, but at the expense of closed-form solutions to the optimal acquisition problem.

Let  $X_t$  denote the cumulative holdings of the trader, and suppose the initial position  $X_0 = 0$ . Further, suppose  $X_t$  is absolutely continuous and let  $\theta(\cdot)$  be the trading rate (so  $dX_t = \theta(\cdot)dt$ ).<sup>5</sup> There are noise traders who hold  $Z_t$  shares of the asset at time  $t$ , where

$$dZ_t = \sigma_Z dW_{Zt}, \quad (3)$$

with  $\sigma_Z > 0$  a constant.

Finally, there is a competitive, risk neutral market-maker who sets the price of the risky asset. This market maker observes the order flow  $Y_t = X_t + Z_t$  and sets the price equal to the conditional expected payoff given the public information set. Formally, let  $\{\mathcal{F}_t^P\}$  denote the augmentation of the filtration  $\sigma(\sigma(\{N_t, Y_t\}) \cup \sigma(\{\mathbf{1}_{\{T \leq t\}}\}))$ . Thus, the price at time  $t < T$  is given by

$$P_t = \mathbb{E} [v | \mathcal{F}_t^P]. \quad (4)$$

Let  $\mathcal{T}$  denote the set of  $\{\mathcal{F}_t^P\}$  stopping times. We require that the trader's information acquisition time satisfies  $\tau \in \mathcal{T}$ . That is, we require the acquisition time to depend only on public information up to that point. Let  $\mathcal{F}_t^I$  denote the augmentation of the filtration  $\sigma(\mathcal{F}_t^P \cup \sigma(\xi))$ . Thus,  $\mathcal{F}_t^I$  represents the trader's information set, post-information acquisition. We require the trader's pre-acquisition trading strategy to be adapted to  $\mathcal{F}_t^P$  and her post-acquisition strategy to be adapted to  $\mathcal{F}_t^I$ .

## 2.2 Financial market equilibrium & optimal information acquisition

We begin by characterizing the equilibrium in the financial market, given an information acquisition time  $\tau$ . To ensure that the trader's expected profit is well-defined, we must rule out trading strategies that first incur infinite losses by driving the price to zero or  $N_t$  and then reap infinite profits. Formally, given a price process  $P_t$  (which will in general depend on  $\theta$  through the order flow) a trading strategy  $\theta$  is *admissible* if it satisfies the measurability restrictions given above (i.e., does not depend on  $\xi$  before the moment of information acquisition) and

$$\mathbb{E} \int_0^T (\theta_u(N_T \xi - P_u))^- du < \infty, \quad (5)$$

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<sup>5</sup>Back (1992) shows that it is optimal for the trader to follow strategies of this form.

where  $x^- = \max\{0, -x\}$ . Note that this admissibility condition is identical to that of [Back and Baruch \(2004\)](#) in the case that  $\tau = 0$  and  $N_t \equiv 1$ .

Our definition of equilibrium in the financial market is standard and follows [Back and Baruch \(2004\)](#).

**Definition 1.** *Fix an information acquisition time  $\tau \in \mathcal{T}$ . An equilibrium in the financial market consists of an admissible trading strategy  $\theta_t$  and a price process  $P_t$  such that, given the trading strategy the price process satisfies (4) and, given the price process, the trading strategy is admissible and maximizes the expected profit*

$$\mathbb{E} \left[ \int_0^T \theta_u (\xi - P_u) du \right].$$

The next result characterizes the financial market equilibrium in our setting.

**Proposition 1.** *Fix an information acquisition time  $\tau \in \mathcal{T}$ . There exists an equilibrium in the trading game in which the price of the risky asset is given by  $P_t = N_t p_t$ , where*

$$p_t \equiv \Pr(\xi = 1 \mid \mathcal{F}_t^P) = \begin{cases} \alpha & 0 \leq t < \tau \\ \Phi \left( \Phi^{-1}(\alpha) e^{r(t-\tau)} + \sqrt{\frac{2r}{\sigma_Z^2}} \int_\tau^t e^{r(t-s)} dY_s \right) & \tau \leq t < T \\ \xi & t = T \end{cases}. \quad (6)$$

Prior to information acquisition, the trader does not trade (i.e.,  $\theta \equiv 0$ ), and conditional on information acquisition, her strategy depends only on  $p$  and is given by

$$\theta_{\xi=1}(p) = \frac{\sigma_Z^2 \lambda(p)}{p}, \quad \text{and} \quad \theta_{\xi=0}(p) = -\frac{\sigma_Z^2 \lambda(p)}{1-p}.$$

In this equilibrium, conditional on becoming informed, the trader's value function is given by

$$J(\xi, p_t, N_t) = \begin{cases} N_t \int_{p_t}^1 \frac{1-a}{\lambda(a)} da & \text{if } \xi = 1 \\ N_t \int_0^{p_t} \frac{a}{\lambda(a)} da & \text{if } \xi = 0 \end{cases}, \quad (7)$$

where  $\lambda(p) = \sqrt{\frac{2r}{\sigma_Z^2}} \phi(\Phi^{-1}(1-p))$ .

Our equilibrium characterization naturally extends the equilibrium in [Back and Baruch \(2004\)](#) to (i) accommodate the news process  $N_t$  and (ii) account for the possibility that the strategic trader is uninformed before  $\tau$ . Before information acquisition, the strategic trader

does not trade,<sup>6</sup> and consequently, the order-flow is uninformative and the market-maker does not update his beliefs about  $\xi$ . As a result, before  $\tau$  the price  $P_t = \alpha N_t$  evolves linearly with  $N_t$ . Conditional on information acquisition, the strategic trader optimally trades according to  $\theta_\xi$  characterized in the proposition. Since  $\theta_1 \neq \theta_0$ , the order flow provides a noisy signal about  $\xi$  to the market maker. The market maker's conditional expectation about  $\xi$ , given by  $p_t$ , depends on the cumulative (weighted) order-flow between since the acquisition date (i.e.,  $\int_\tau^t e^{r(t-s)} dY_s$ ), and consequently, so does the price  $P_t$ .

Given the value function in Proposition 1, we can characterize the optimal information acquisition decision.

**Proposition 2.** *The strategic trader optimally acquires information the first time  $N_t$  hits the optimal exercise boundary  $N^* = \frac{\beta}{\beta-1} \frac{c}{K}$ , where*

$$K = \sqrt{\frac{\sigma_Z^2}{2r}} \phi(\Phi^{-1}(1-\alpha)), \text{ and } \beta = \frac{1+\sqrt{1+8r/\sigma_N^2}}{2}. \quad (8)$$

Moreover, the optimal exercise boundary  $N^*$  increases in  $c$  and  $\sigma_N$ , decreases in  $\sigma_Z$ , is U-shaped in  $\alpha$  (minimized at  $\alpha = 0.5$ ), and is U-shaped in  $r$ .

As we show in the proof of the above, the expected profit immediately prior to acquiring information at any date  $t$  (i.e., the value function the instant before  $\xi$  is observed) is given by

$$\hat{U}(N_t) \equiv \mathbb{E}_t[\alpha J(1, \alpha, N_t) + (1-\alpha) J(0, \alpha, N_t)] = KN_t. \quad (9)$$

Note that the value function given information acquisition at date  $t$  is higher when there is more noise in the order flow (i.e., higher  $\sigma_Z$ ), when there is more prior uncertainty about whether  $N_t$  is informative (i.e., when  $\alpha$  is closer to 0.5), and when the information advantage is expected to be longer lived (i.e., when  $r$  is smaller).

The standard approach in the literature restricts the strategic trader to make her information choices before trading begins. In this case, she follows a naive “NPV” rule — she only acquires information if the value from becoming informed is higher than the cost i.e.,  $\hat{U}(N_0) \geq c$ . As the following corollary highlights, the resulting information acquisition decision is effectively a static one.

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<sup>6</sup>Under the posited price function, the pre-acquisition trading strategy is indeterminate. Any strategy that uses only public information earns zero expected profit in this region. Given such a trading strategy, it also remains optimal for the market maker to set  $p_t = \alpha$ . Without loss of generality, we focus on the case in which the trader does not trade before time  $\tau$ . In the presence of transaction costs, this would be the optimal strategy.



**Corollary 1.** *If the strategic trader is restricted to acquiring information at  $t = 0$ , she optimally acquires information only if  $N_0 \geq N_0^*$ , where  $N_0^* = \frac{c}{K}$ . Moreover, the optimal exercise boundary  $N_0^*$  increases in  $c$ , decreases in  $\sigma_Z$ , is U-shaped in  $\alpha$  (minimized at  $\alpha = 0.5$ ), and increases in  $r$ .*

With dynamic information acquisition, the optimal time to acquire information is characterized by the following problem:

$$U(n) \equiv \sup_{\tau \in \mathcal{T}} \mathbb{E} \left[ \mathbf{1}_{\{\tau < T\}} (\hat{U}(N_\tau) - c) \mid N_t = n \right] = \sup_{\tau \in \mathcal{T}} \mathbb{E} \left[ e^{-r\tau} (KN_\tau - c)^+ \mid N_t = n \right]. \quad (10)$$

This problem is analogous to characterizing the optimal exercise time for a perpetual American call option.<sup>7</sup> Notably, the optimal information acquisition decision exhibits delay: information is not acquired when  $KN_t = c$ , as would be implied by the static NPV rule. The intuition for this effect is analogous to that for investment delay in a real options problem. At any point in time, the trader can exercise her “option” to acquire information and use that information to profit at the expense of the noise traders. However, by waiting and observing the news process she learns additional information about the asset payoff (and therefore her ultimate profits) on which she can condition her decision. Since acquiring information irreversibly sacrifices the ability to wait, it is optimal to acquire only when doing so is sufficiently profitable to overcome this opportunity cost. Moreover, the option to wait is more valuable (and hence  $N^*$  is higher) when the volatility of the news process (i.e.,  $\sigma_N$ ) is higher.

A key difference between the static acquisition boundary of Corollary 1 and the dynamic acquisition boundary of Proposition 2 is how they respond to the expected trading horizon. In the static case, the exercise boundary is increasing in  $r$ . Recall that increasing  $r$  increases the likelihood that the payoff is revealed sooner i.e., it decreases the expected trading horizon. This naturally decreases the value from acquiring information, since the trader has a shorter window over which to exploit her informational advantage.

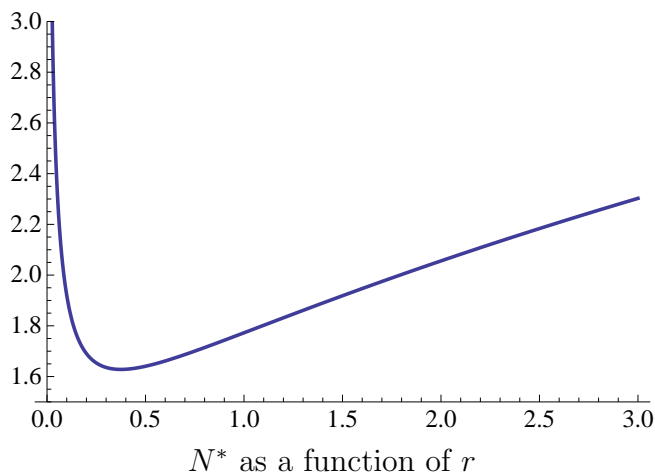
With dynamic information acquisition, the trader also accounts for the cost of waiting to acquire information. Specifically, as the trading horizon increases (i.e.,  $r$  decreases), the expected value from acquiring information at any date (i.e.,  $U(N_t)$ ) increases. However, she is also willing to wait longer to acquire this information, since the cost of waiting (the probability the value will be revealed before she acquires information) also decreases. Initially, the first effect dominates, which leads the exercise boundary to decrease as the trading horizon increases. Eventually, however, the second effect dominates, and the exercise boundary increases with the

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<sup>7</sup>Hence, appealing to standard results, we establish that the optimal stopping time is a first hitting time for the  $N_t$  process and show that the given  $N^*$  is a solution to this problem.

Figure 1: Exercise Boundary  $N^*$

Unless otherwise specified, parameters are set to  $\sigma_Z = \sigma_N = 1$ ,  $c = 0.25$  and  $\alpha = 0.5$ .



horizon. As Figure 1 illustrates, this implies that the exercise boundary is non-monotonic in  $r$  with dynamic information acquisition: the trader is less likely to acquire information when the asset payoff is expected to be revealed too quickly or too slowly.

### 3 Predictions

#### 3.1 Likelihood of information acquisition

The likelihood of information acquisition depends on two forces. First, the cost of information may be too high relative to the value of acquiring it: given  $c$ , the trader might never find it optimal to acquire the information. Second, even if the (relative) cost of acquisition is not too high, the asset payoff may be revealed before the strategic trader chooses to acquire information. The following results characterize how these effects interact to determine the likelihood of information acquisition.

In what follows, it is useful to define  $T_N$  as the first time  $N_t \geq N^*$ . Then, the time at which information is acquired can be expressed as

$$\tau = T_N 1_{\{T_N \leq T\}} + \infty \times 1_{\{T_N > T\}}, \quad (11)$$

where, as before,  $\tau = \infty$  corresponds to no information acquisition. To avoid the trivial case, assume  $N_0 < N^*$ . We begin with the following observation.

**Lemma 1.** *Suppose  $N_0 < N^*$ . For  $0 \leq t < \infty$ , the probability that  $T_N \in [t, t + dt]$  is given by*

$$\Pr(T_N \in [t, t + dt]) = \frac{\left(\log\left(\frac{N^*}{N_0}\right)\right)}{\sigma_N \sqrt{2\pi t^3}} \exp\left\{-\frac{\left(\frac{1}{\sigma_N} \log\left(\frac{N^*}{N_0}\right) + \frac{1}{2}\sigma_N^2 t\right)^2}{2t}\right\} dt. \quad (12)$$

*The probability that  $T_N$  is not finite is given by  $\Pr(T_N = \infty) = 1 - \frac{N_0}{N^*}$ .*

The result follows from applying results on the first hitting time of a Brownian motion with drift. Since information acquisition is costly and the news process is a martingale, there is a positive probability that the boundary is never hit, even if  $T \equiv \infty$ . Since the above expression is increasing in the boundary  $N^*$ , the probability of information acquisition decreases in the cost  $c$  and volatility  $\sigma_N$ , increases in volatility of noise trading  $\sigma_Z$  and uncertainty about  $\xi$  (i.e., is hump-shaped in  $\alpha$ ), and is hump-shaped in  $r$ .

The next result accounts for the possibility that the payoff is revealed before the information is acquired (i.e.,  $T_N > T$ ).

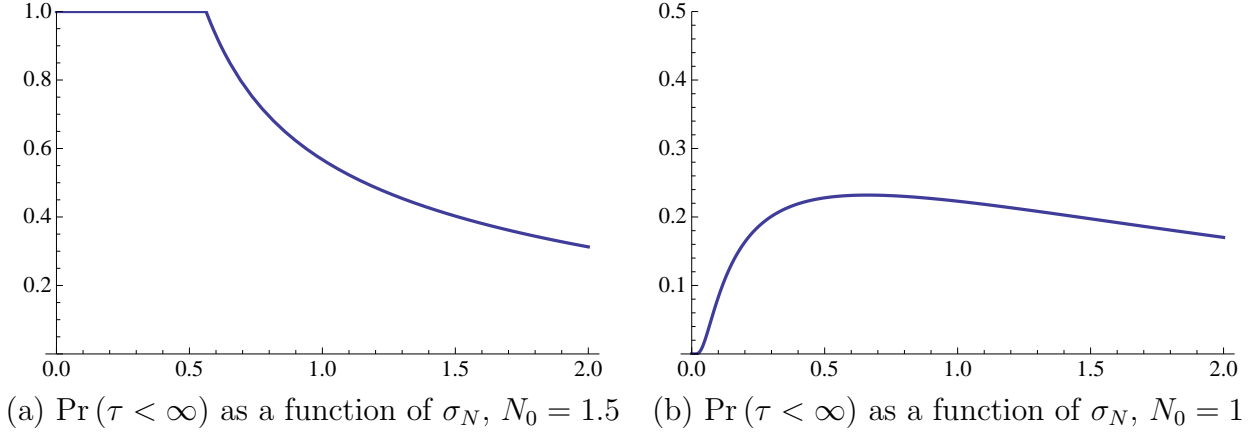
**Proposition 3.** *Suppose  $N_0 < N^*$ . The probability that information is acquired is  $\Pr(\tau < \infty) = \left(\frac{N_0}{N}\right)^\beta$ . The probability is decreasing in  $c$ , increasing in  $N_0$  and  $\sigma_Z$ , hump-shaped in  $\alpha$  (around  $\frac{1}{2}$ ), and hump-shaped in  $r$ . When  $c \leq N_0 K$ , the probability is decreasing in  $\sigma_N$ ; when  $c > N_0 K$ , it is hump-shaped in  $\sigma_N$ .*

Not surprisingly, accounting for the possibility that the payoff is revealed before  $N_t$  hits  $N^*$  reduces the likelihood of information acquisition (i.e.,  $\Pr(\tau < \infty) < \Pr(T_N < \infty)$ , since  $N_0 < N^*$  and  $\beta > 1$ ). More interestingly, it also changes the effect of the volatility  $\sigma_N$  of the news process on the likelihood of acquisition. Increasing  $\sigma_N$  has two effects: (i) it increases the acquisition boundary (i.e.,  $N^*$  increases in  $\sigma_N$ ), and (ii) fixing the boundary, it increases the likelihood that  $N_t$  will hit the boundary by any given time (i.e.,  $N_t$  is more volatile). Appealing to the analogy with an American call option, the above result highlights that when the option starts in the money (i.e.,  $c \leq N_0 K$ ), the first effect dominates and the probability of acquisition (i.e., the probability the option is exercised) decreases in  $\sigma_N$ . However, when the option is initially out of the money (i.e.,  $c > N_0 K$ ), then for low values of  $\sigma_N$ , the second effect dominates the first and the probability of acquisition initially increases in  $\sigma_N$ .

Figure 2 presents an example of this non-monotonic effect of  $\sigma_N$  on the probability of information acquisition. In panel (a),  $N_0$  is sufficiently high so that  $N_0 K \geq c$ , and so the probability of information acquisition is decreasing in  $\sigma_N$ . In panel (b),  $N_0$  is low enough so that the probability of information acquisition initially increases and then decreases in  $\sigma_N$ .

Figure 2: Probability that information is acquired  $\Pr(\tau < \infty)$ .

Unless otherwise specified, parameters are set to  $\sigma_Z = 1$ ,  $c = 0.25$ ,  $r = 1.5$ ,  $\alpha = 0.5$ .



### 3.2 Price dynamics

The expression for the price in Proposition 1 immediately implies that price impact of order flow before information acquisition is zero, but jumps to  $\lambda(p_\tau)$  when information is acquired. Moreover, price impact evolves stochastically post-acquisition, since it is driven by the evolution of the market maker's beliefs  $p_t$ .

The following result characterizes return volatility in our model.

**Proposition 4.** *The instantaneous variance of returns is*

$$\nu_t \equiv \begin{cases} \sigma_N^2 & 0 \leq t < \tau \\ \sigma_N^2 + \left(\frac{\lambda^*(p_t)}{p_t}\right)^2 \sigma_Z^2 & \tau \leq t < T \end{cases}$$

*Conditional on information acquisition, volatility is stochastic and exhibits the “leverage” effect i.e., the instantaneous covariance between returns and variance of returns is negative ( $\text{cov}\left(\nu_t, \frac{dp_t}{p_t}\right) \leq 0$ ).*

The above result highlights that return volatility is higher conditional on information acquisition. Conditional on no acquisition, price changes are driven purely by changes in the news process. However, conditional on the strategic trader being informed, the market maker also conditions on order flow to update his beliefs about the asset payoff, and as a result, return volatility is driven by two sources of variation.

In contrast to the standard Kyle (1985) model, our model generates stochastic return volatil-

ity and price impact, even though fundamentals (i.e.,  $N_t$ ) and noise trading (i.e.,  $Z_t$ ) are homoskedastic. This is a consequence of the non-linearity in the filtering problem of the market maker, and is in contrast to models where the (conditionally linear) filtering problem amplifies stochastic volatility in an underlying process (e.g., in [Collin-Dufresne and Fos \(2016\)](#), return volatility amplifies stochastic volatility in noise trading).<sup>8</sup> Moreover, conditional on information acquisition, return volatility also exhibits the “leverage effect” (see [Black \(1976\)](#) and the subsequent literature) — the instantaneous variance increases when returns are negative, and vice versa — even though there is no leverage (debt) in the underlying risky asset.

Despite the large empirical literature documenting the importance of stochastic volatility and jumps in volatility, there are relatively few theoretical explanations for how these patterns arise. Our model provides an explanation for both, but it does not rely on jumps or stochastic volatility in fundamentals. Instead, volatility jumps (and becomes stochastic) when the public news process triggers private information acquisition by the strategic trader. Our analysis suggests that further understanding the interaction between public news and private information can provide new insights into what drives empirically observed patterns in volatility.

### 3.3 Announcement effects

Next, we turn to the absolute price change at the time the payoff of the risky asset is announced. In finite horizon models where the announcement is perfectly anticipated (e.g., [Back \(1992\)](#)), the informed trader’s optimal strategy ensures that the price change at announcement is zero. While this is no longer the case with a stochastic announcement date, the intuition from these models would suggest that the announcement effect is smaller on average if information is acquired than if it is not. However, as the next result highlights, this is not always the case.

**Proposition 5.** *The expected absolute price jump on announcement, conditional on information acquisition is*

$$\mathbb{E} [|\xi N_T - P_{T-}| | \tau < \infty] = 2N^* h(\alpha), \quad (13)$$

where  $h(\alpha)$  is characterized in the Appendix, and fully illustrated by the plot in [Figure 3](#). The expected absolute price jump on announcement, conditional on no information acquisition is

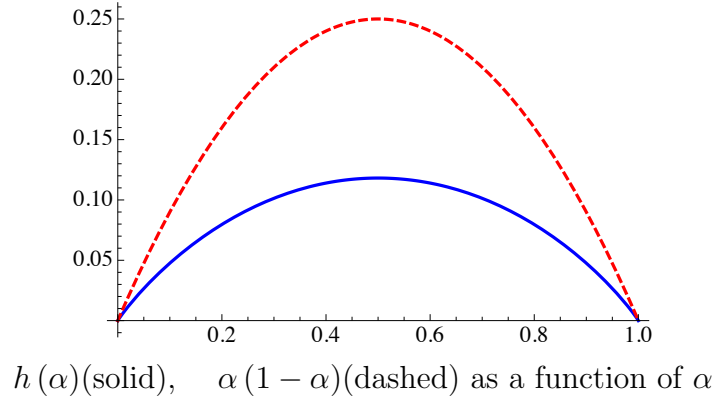
$$\mathbb{E} [|\xi N_T - P_{T-}| | \tau = \infty] = 2\alpha(1 - \alpha) N^* \frac{\frac{N_0}{N^*} - \left(\frac{N_0}{N^*}\right)^\beta}{1 - \left(\frac{N_0}{N^*}\right)^\beta}.$$

---

<sup>8</sup>Similar results obtain in the continuous-time models of [Back and Baruch \(2004\)](#), [Li \(2013\)](#), [Back, Crotty, and Li \(2014\)](#), [Back, Crotty, and Li \(2015\)](#), and the discrete time model of [Banerjee and Green \(2015\)](#).

Figure 3:  $h(\alpha)$  and  $\alpha(1 - \alpha)$

The figure plots  $h(\alpha)$  (solid) and  $\alpha(1 - \alpha)$  (dashed) as a function of  $\alpha$ .



Fixing  $\alpha \in (0, 1)$  and the other parameters, the announcement effect is larger with information acquisition when:  $N_0$  is sufficiently small,  $c$  is sufficiently high,  $\sigma_N^2$  is sufficiently high,  $\sigma_Z^2$  is sufficiently low, or  $r$  is sufficiently extreme (i.e., sufficiently low, or sufficiently high).

The proposition characterizes conditions under which a potentially surprising result holds: the announcement effect is larger with information acquisition than without. In a setting where the strategic trader is exogenously endowed with information, the standard intuition holds — the announcement effect conditional on an informed trading is smaller than the announcement effect conditional on no informed trading. To see why, note that in this case, the announcement effect can be expressed as

$$\mathbb{E} [|\xi N_T - P_{T-}|] = N_0 \mathbb{E} [|\xi - p_T|] = 2N_0 \mathbb{E} [p_T(1 - p_T)]. \quad (14)$$

When the strategic trader is not informed,  $p_T = \alpha$ . When the strategic trader is informed, however, Jensen's inequality implies that  $\mathbb{E} [p_T(1 - p_T)] \leq \alpha(1 - \alpha)$ . Intuitively, the market-maker's posterior beliefs are more precise when the strategic trader is informed, and as a result, the price reflects the asset payoff more accurately.

When information acquisition is endogenous, however, there is an offsetting effect at work. Recall that the strategic trader only acquires information when the news process is sufficiently high ( $N_t \geq N^*$ ). This implies that the expected level of  $N_T$ , conditional on information acquisition, is higher since  $\mathbb{E} [N_T | \tau < \infty] = N^* \geq N_0$ . Intuitively, the strategic trader only chooses to acquire information when the prior uncertainty about fundamentals is sufficiently high. This offsetting effect dominates when the initial news level  $N_0$  is sufficiently small or the

optimal exercise boundary  $N^*$  is sufficiently large, and as a result, the announcement effect conditional on information acquisition is higher in these cases.

## 4 Conclusions

We consider a dynamic Kyle (1985) model in which a strategic trader can choose *when* to acquire information about the payoff of a risky asset in response to the evolution of a public signal. We characterize explicitly the trader’s optimal information acquisition and trading strategies, as well as the pricing rule of the market maker. We show that the optimal acquisition strategy is a cutoff rule – the trader acquires information only when the public signal is sufficiently extreme. Intuitively, acquisition occurs when conditional (public) uncertainty is sufficiently high; however, the optimal decision does not follow a naive “NPV” rule. Instead expected net trading profits are strictly positive upon acquisition.

In contrast to standard models where the information acquisition decision is static, our model has a number of novel implications. As optimal information acquisition does not follow the NPV rule, a more volatile public news process need not increase private information production. Rather, higher news volatility may lead the trader to wait longer to acquire information than she otherwise would. Information is less likely to be acquired when trading opportunities are very short- or long-lived. The dynamic nature of information acquisition also leads to jumps and time-variation in volatility, with increases in volatility corresponding to the acquisition and subsequent incorporation of private information by the trader. Perhaps surprisingly, we show the average price jump when the payoff is realized need not be smaller when the trader is informed, since the trader only chooses to acquire information when uncertainty is sufficiently high.

Our results have implications for empirical work. For instance, the model highlights the importance of conditioning jointly on both measures of informed trading (e.g, price impact, spreads, etc.) and the precision of public information (e.g, the dispersion in analyst forecasts) when interpreting the size of announcement effects. Smaller (absolute) announcement returns need not be associated with the presence of more informed trading in the market. Our results also suggest a role for incorporating announcement frequency or horizon when studying informed trading around unanticipated announcements. Specifically, extremely high frequency or extremely low frequency private information is less likely to be acquired by traders, and therefore, less likely to be impounded into prices. Furthermore, the model provides a potential explanation for jumps in price impact and volatility without relying on corresponding jumps

in the underlying fundamentals, and suggests that such jumps are intimately linked with the interaction of public news and private information acquisition.

Our analysis implies that allowing for dynamic information acquisition has important consequences for understanding how markets generate and transmit information. While our model is stylized, it provides a natural benchmark. Studying the effect of competition among traders over when to acquire information, what types of information the trader chooses to investigate, or how our implications change when traders can choose the precision of their private signals (i.e., how intensively to investigate the firm), are natural next steps. It would also be interesting to study how our analysis changes when the public signal is endogenized (e.g., in the form of strategic disclosure by firms or regulators). We leave these questions for future work.



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# Appendix A - Proofs

**Proof of Proposition 1.** To establish the equilibrium in the Proposition, we need to show:

(i) the proposed price function is rational, and (ii) the informed trader's strategy is optimal.

Fix any  $\tau \in \mathcal{T}$ .

## Rationality of pricing function

Consider the set  $\{t : t < \tau\}$  on which the trader has not acquired information. Then, because  $\{N_t\}$ ,  $\{Z_t\}$  and  $\xi$  are independent, and under the proposed trading strategy  $Y_t = Z_t$  for  $t < \tau$ , it is immediate that

$$\mathbb{E}[\xi N_T | \mathcal{F}_t^P] = \mathbb{E}[\xi | \mathcal{F}_t^P] \mathbb{E}[N_T | \mathcal{F}_t^P] = \alpha \mathbb{E}[N_T | \mathcal{F}_t^P].$$

Since  $T$  is almost surely finite and is independent of the process  $N_t$  we have  $\mathbb{E}[N_T | \mathcal{F}_t^P] = N_t$ , and so  $\mathbb{E}[\xi N_T | \mathcal{F}_t^P] = \alpha N_t$ .

Now, consider the set  $\{t : \tau \leq t < T\}$  on which the trader has acquired information and the asset payoff has not yet occurred. Up to the addition of the news process, the problem now resembles that considered in [Back and Baruch \(2004\)](#), and we can adapt the proof offered there. Specifically, consider the pricing rule from [Back and Baruch \(2004\)](#), adapted for the fact that information is acquired at time  $\tau$ ,

$$dp_t = \lambda(p) dY_t, \quad p_\tau = \alpha,$$

where  $\lambda(p)$  is given in the statement of the Proposition. (Later we will show that this pricing rule can be written in the explicit form in eq. (6).) Note that the proposed trading strategy depends only on  $\xi$  and  $p$ , this pricing rule depends only on the order flow, and  $\{N_t\}$  is independent of  $\xi$  and  $\{Z_t\}$ , so  $(\xi, \{p_t\})$  is independent of  $\{N_t\}$ , and therefore

$$\mathbb{E}[\xi N_T | \mathcal{F}_t^P] = \mathbb{E}[\xi | \mathcal{F}_t^P] \mathbb{E}[N_T | \mathcal{F}_t^P] = \mathbb{E}[\xi | \{Y_s\}_{s \leq t}] N_t,$$

where the final equality follows since  $\mathbb{E}[N_T | \mathcal{F}_t^P] = N_t$ . Furthermore, since  $Y_t = Z_t$  for  $t < \tau$  under the proposed trading strategy and  $\xi$  is independent of  $\{Z_t\}$  it follows that  $\mathbb{E}[\xi | \{Y_s\}_{s \leq t}] = \mathbb{E}[\xi | \{Y_s\}_{\tau \leq s \leq t}]$ .

Recall that as of time  $\tau$ , the informed trader begins trading according to the strategy  $\theta_\xi(p)$  and the order flow becomes informative. The market maker's conditional expectation is simply

equal to her prior  $\alpha$  since before this time only noise traders have been active. It follows that starting at time  $\tau$  the market maker's filtering problem becomes identical to that of the market maker in [Back and Baruch \(2004\)](#). Hence, their Theorem 1 implies that for  $t \geq \tau$  the pricing rule

$$dp_t = \lambda(p)dY_t, \quad p_\tau = \alpha,$$

satisfies  $p_t = \mathbb{E}[\xi | \{Y_s\}_{s \geq \tau}]$ .

To complete the proof of the rationality of the proposed price, it suffices to show that the explicit form of  $p(\cdot)$  for  $\tau \leq t < T$  in eq. (6) satisfies  $dp_t = \lambda(p)dY_t$ . Applying Ito's Lemma to the function  $f(p) = \sqrt{\frac{\sigma_Z^2}{2r}} \Phi^{-1}(p)$  to the above process for  $p_t$  gives

$$\begin{aligned} df(p_t) &= \frac{1}{2} \sigma_Z^2 \lambda^2(p_t) \frac{\frac{2r}{\sigma_Z^2} f(p_t)}{\lambda^2(p_t)} dt + \frac{1}{\lambda(p_t)} \lambda(p_t) dY_t \\ &= r f(p_t) dt + dY_t. \end{aligned}$$

Now applying Ito's lemma to the function  $e^{-rt} f(p_t)$  and integrating allows one to express

$$f(p_t) = f(p_\tau) e^{rt} + \int_\tau^t e^{r(t-s)} dY_s.$$

Note that  $f(p_\tau) = \sqrt{\frac{\sigma_Z^2}{2r}} \Phi^{-1}(\alpha)$ , so returning to the explicit form of the function  $f(p)$  and inverting it follows that

$$p_t = \Phi \left( \Phi^{-1}(\alpha) e^{r(t-\tau)} + \sqrt{\frac{2r}{\sigma_Z^2}} \int_\tau^t e^{r(t-s)} dY_s \right).$$

## Optimality of trading strategy

Next, we demonstrate the optimality of the proposed trading strategy, taking as given the acquisition time  $\tau$ . This analysis closely follows the proof in [Back and Baruch \(2004\)](#). Define  $V(p) \equiv \int_p^1 \frac{1-a}{\lambda(a)} da$  and consider the proposed post-acquisition value function for the case  $\xi = 1$

$$J(\xi, p_t, N_t) = N_t V(p).$$

We begin by showing that the given  $J$  characterizes the value function for  $t \geq \tau$ . Consider  $\{t : \tau \leq t < T\}$  and suppose  $\xi = 1$  (the case for  $\xi = 0$  is symmetric). Direct calculation on the

function  $V$  yields

$$V' = \frac{p-1}{\lambda} \quad (15)$$

$$rV = \frac{1}{2}\sigma_Z^2\lambda^2V'', \quad (16)$$

which coincides with eq. (1.15) and (1.16) in [Back and Baruch \(2004\)](#).

Let  $\theta_t$  denote an arbitrary admissible trading strategy. Following [Back and Baruch \(2004\)](#), let  $\hat{p}_t$  denote the process defined by  $\hat{p}_s = \alpha$  for  $s \leq \tau$  and  $d\hat{p}_t = \lambda(\hat{p})dY_t$  for  $t > \tau$  and  $0 < \hat{p}_t < 1$ , with  $Y_t$  generated when the trader follows the given arbitrary trading strategy. In order to condense notation, in this section, we denote  $\mathbb{E}[\cdot|\mathcal{F}_t^P] = \mathbb{E}_t[\cdot]$ . Since  $\theta$  is admissible, we know that

$$\mathbb{E}_\tau \left[ \int_\tau^T N_u(1-p_u)\theta_u^- du \right] = \mathbb{E}_\tau \left[ \int_\tau^\infty e^{-r(u-\tau)} N_u(1-\hat{p}_u)\theta_u^- du \right] < \infty,$$

from which it follows that

$$\int_\tau^\infty e^{-r(u-\tau)} N_u(1-\hat{p}_u)\theta_u^- du < \infty$$

almost surely, and therefore that the integral

$$\int_\tau^\infty e^{-r(u-\tau)} N_u(1-\hat{p}_u)\theta_u du$$

is well-defined, though is possibly infinite.

Let  $\hat{T} = \inf\{t \geq \tau : \hat{p} \in \{0, 1\}\}$ . Applying Ito's lemma to  $e^{-r(t-\tau)}J$  yields

$$\begin{aligned} & e^{-r(t\wedge\hat{T}-\tau)}J(1, \hat{p}_{t\wedge\hat{T}}, N_{t\wedge\hat{T}}) - J(1, \hat{p}_\tau, N_\tau) \\ &= \int_\tau^{t\wedge\hat{T}} e^{-r(u-\tau)}N \left( -rV(\hat{p}_u) + \lambda\theta V'(\hat{p}_u) + \frac{1}{2}\sigma_Z^2\lambda^2V'' \right) du \\ & \quad + \sigma_Z \int_\tau^{t\wedge\hat{T}} e^{-r(u-\tau)}N\lambda V'(\hat{p}_u)dW_{Zu} + \sigma_N \int_\tau^{t\wedge\hat{T}} e^{-r(u-\tau)}NV(\hat{p}_u)dW_{Nu} \\ &= - \int_\tau^{t\wedge\hat{T}} e^{-r(u-\tau)}N_u\theta_u(1-\hat{p}_u)du - \sigma_Z \int_\tau^{t\wedge\hat{T}} e^{-r(u-\tau)}N_u(1-\hat{p}_u)dW_{Zu} \\ & \quad + \sigma_N \int_\tau^{t\wedge\hat{T}} e^{-r(u-\tau)}N_uV(\hat{p}_u)dW_{Nu} \end{aligned} \quad (17)$$

where the last equality uses eq. (15) and (16). Since  $V \geq 0$ , the above implies

$$\int_{\tau}^{t \wedge \hat{T}} e^{-r(u-\tau)} N_u \theta_u (1 - \hat{p}_u) du \leq N_{\tau} V(\alpha) + x(t), \quad (18)$$

where we define  $x(t) = \sigma_N \int_{\tau}^{t \wedge \hat{T}} e^{-r(u-\tau)} N_u V(\hat{p}_u) dW_{N_u} - \sigma_Z \int_{\tau}^{t \wedge \hat{T}} e^{-r(u-\tau)} N_u (1 - \hat{p}_u) dW_{Z_u}$ . The integrands in the stochastic integrals are locally bounded and hence the integrals are local martingales (Thm. 29, Ch. 4, Protter (2003)). It follows that  $x(t)$  is itself a local martingale (Thm. 48, Ch. 1, Protter (2003)).

Let  $\hat{\tau}_n$  be a localizing sequence of stopping times for  $x(t)$ . That is,  $\hat{\tau}_{n+1} \geq \hat{\tau}_n$ ,  $\hat{\tau}_n \rightarrow \infty$ , and  $x(t \wedge \hat{\tau}_n)$  is a martingale for each  $n$ . Because  $x(t)$  is a local martingale such a sequence exists (e.g., because  $x(t)$  is continuous we can take  $\hat{\tau}_n = \inf\{t : |x(t)| \geq n\}$ ). Further considering the sequence  $n \wedge \hat{\tau}_n$ , eq. (18) implies

$$\int_{\tau}^{n \wedge \hat{\tau}_n \wedge \hat{T}} e^{-r(u-\tau)} N_u \theta_u (1 - \hat{p}_u) du \leq N_{\tau} V(\alpha) + x(n \wedge \hat{\tau}_n).$$

Applying Fatou's lemma,<sup>9</sup> along with this inequality, yields

$$\begin{aligned} \mathbb{E}_{\tau} \left[ \int_{\tau}^{\hat{T}} e^{-r(u-\tau)} N_u \theta_u (1 - \hat{p}_u) du \right] &\leq \liminf_{n \rightarrow \infty} \mathbb{E}_{\tau} \left[ \int_{\tau}^{n \wedge \hat{\tau}_n \wedge \hat{T}} e^{-r(u-\tau)} N_u \theta_u (1 - \hat{p}_u) du \right] \\ &\leq N_{\tau} V(\alpha) + \liminf_{n \rightarrow \infty} \mathbb{E}_{\tau} [x(n \wedge \hat{\tau}_n)] \\ &\leq N_{\tau} V(\alpha). \end{aligned}$$

Note that for  $\hat{T} < \infty$  we have  $\hat{p}_{\hat{T}} = 1$  since  $\hat{p}_{\hat{T}} = 0$  would imply a violation of the admissibility condition. To establish this, note that eq. (17) implies

$$-\mathbb{E}_{\tau} \left[ \int_{\tau}^{n \wedge \hat{\tau}_n \wedge \hat{T}} e^{-r(u-\tau)} N_u \theta_u (1 - \hat{p}_u) du \right] = \mathbb{E}_{\tau} \left[ e^{-r(t \wedge \hat{T} - \tau)} N_{t \wedge \hat{T}} V(\hat{p}_{t \wedge \hat{T}}) - N_{\tau} V(\alpha) \right] - J(1, \hat{p}_{\tau}, N_{\tau}),$$

and therefore

$$-\mathbb{E}_{\tau} \left[ \int_{\tau}^{\hat{T}} e^{-r(u-\tau)} N_u \theta_u (1 - \hat{p}_u) du \right]$$

---

<sup>9</sup>The typical formulation of Fatou's Lemma requires that the integrands  $f_n$  be weakly positive. However, if  $f_n^-$  is bounded above by an integrable function  $g$ , considering  $f_n + g$  in Fatou's lemma delivers the result. Here, due to the admissibility condition we can take  $g = N_u(1 - p_u)\theta_u^-$ .

$$\begin{aligned}
&\geq \limsup_{n \rightarrow \infty} \mathbb{E}_\tau \left[ - \int_\tau^{n \wedge \hat{\tau}_n \wedge \hat{T}} e^{-r(u-\tau)} N_u \theta_u (1 - \hat{p}_u) du \right] \\
&= \limsup_{n \rightarrow \infty} \mathbb{E}_\tau \left[ e^{-r(n \wedge \hat{\tau}_n \wedge \hat{T} - \tau)} N_{n \wedge \hat{\tau}_n \wedge \hat{T}} V(\hat{p}_{n \wedge \hat{\tau}_n \wedge \hat{T}}) - N_\tau V(\alpha) \right] - J(1, \hat{p}_\tau, N_\tau) \\
&\geq \mathbb{E}_\tau \left[ e^{-r(\hat{T} - \tau)} N_{\hat{T}} V(\hat{p}_{\hat{T}}) \right] - J(1, \hat{p}_\tau, N_\tau) \\
&= \infty,
\end{aligned}$$

where the first line applies the ‘reverse’ Fatou’s Lemma, the second line uses the equality in the previous displayed equation, the third line applies Fatou’s Lemma and the final line follows because  $V(0) = \infty$ . Furthermore,  $\hat{p}_u = \hat{p}_{\hat{T}} = 1$  for all  $u \geq \hat{T}$  since 1 is an absorbing state. It follows that

$$\mathbb{E}_\tau \left[ \int_\tau^\infty e^{-r(u-\tau)} N_u \theta_u (1 - \hat{p}_u) du \right] = \mathbb{E}_\tau \left[ \int_\tau^{\hat{T}} e^{-r(u-\tau)} N_u \theta_u (1 - \hat{p}_u) du \right] \leq N_\tau V(\alpha). \quad (19)$$

Furthermore, this inequality is trivially true for  $\hat{T} = \infty$ , so it holds regardless of the behavior of  $\hat{T}$ . It follows that

$$N_\tau V(\alpha) \geq \mathbb{E}_\tau \left[ \int_\tau^\infty e^{-r(u-\tau)} N_u \theta_u (1 - \hat{p}_u) du \right] = \mathbb{E}_\tau \left[ \int_\tau^T N_u \theta_u (1 - p_u) du \right],$$

since  $\hat{p} = p$  for  $t \leq T$ . Hence  $N_\tau V(\alpha)$  is an upper bound on the post-acquisition value function.

To establish the optimality of the trader’s post-acquisition strategy and the expression for the value function, it remains to show that the expected profits generated by the strategy attain the bound  $N_\tau V(\alpha)$ . (We show below that the trader’s overall strategy both pre- and post-acquisition is admissible.) Compute the trader’s expected profit at time  $\tau$ . We have

$$\begin{aligned}
\mathbb{E}_\tau \left[ \int_\tau^T \theta_1(p_u) N_u (1 - p_u) du \right] &= \int_\tau^\infty \mathbb{E}_\tau \left[ \mathbf{1}_{\{t \leq T\}} \theta_1(p_u) N_u (1 - p_u) \right] du \\
&= \int_\tau^\infty \mathbb{E}_\tau [N_u] \mathbb{E}_\tau \left[ \mathbf{1}_{\{t \leq T\}} \theta_1(p_u) (1 - p_u) \right] du \\
&= N_\tau \int_\tau^\infty \mathbb{E}_\tau \left[ \mathbf{1}_{\{t \leq T\}} \theta_1(p_u) (1 - p_u) \right] du \\
&= N_\tau \mathbb{E}_\tau \left[ \int_\tau^T \theta_1(p_u) (1 - p_u) du \right],
\end{aligned}$$

where the first equality applies Fubini’s theorem which is permissible because the integrand is positive, the second equality uses the fact that  $N$  is independent of  $T$  and  $\{p_u\}$ , the next-to-last equality follows because  $N$  is a martingale, and the final equality applies Fubini’s theorem

again. [Back and Baruch \(2004\)](#) establish that under the given trading strategy and pricing rule,  $V(\alpha) = \mathbb{E}_\tau \left[ \int_\tau^T \theta_1(p_u)(1 - p_u) du \right]$ . Hence,

$$N_\tau V(\alpha) = \mathbb{E}_\tau \left[ \int_\tau^T \theta_1(p_u) N_u (1 - p_u) du \right],$$

which establishes the optimality of the post-acquisition trading strategy.

Let  $\hat{J}(p, N)$  denote the expected profit, pre acquisition. We need to characterize this function and establish that the overall posited trading strategy, involving no trade prior to acquisition, is optimal. Under the given trading strategy, we have

$$\begin{aligned} \hat{J}(p_t, N_t) &= \mathbb{E} \left[ \mathbf{1}_{\{\tau < T\}} \int_\tau^T \theta_\xi(p_u) N_u (\xi - p_u) du \right] \\ &= \mathbb{E} \left[ \mathbf{1}_{\{\tau < T\}} J(\xi, p_\tau, N_\tau) | \mathcal{F}_t^P \right] \end{aligned}$$

Let  $\check{\theta}$  be any admissible trading strategy that is adapted to  $\mathcal{F}_t^P$  and  $\hat{\theta}$  any admissible strategy that is adapted to  $\mathcal{F}_t^I$ . Then  $\theta = \mathbf{1}_{\{t < \tau\}} \check{\theta} + \mathbf{1}_{\{t \geq \tau\}} \hat{\theta}$  is an arbitrary admissible strategy that obeys the restriction that the trader does not observe  $\xi$  until time  $\tau$ . The expected profits from following this strategy are

$$\begin{aligned} &\mathbb{E}_0 \left[ \mathbf{1}_{\{\tau < T\}} \int_0^\tau \check{\theta}_u N_u (\xi - p_u) du + \mathbf{1}_{\{\tau < T\}} \int_\tau^T \hat{\theta}_u N_u (\xi - p_u) du + \mathbf{1}_{\{\tau \geq T\}} \int_0^T \check{\theta}_u N_u (\xi - p_u) du \right] \\ &= \mathbb{E}_0 \left[ \mathbf{1}_{\{\tau < T\}} \int_0^\tau \check{\theta}_u N_u (\xi - \alpha) du + \mathbf{1}_{\{\tau < T\}} \int_\tau^T \hat{\theta}_u N_u (\xi - p_u) du + \mathbf{1}_{\{\tau \geq T\}} \int_0^T \check{\theta}_u N_u (\xi - \alpha) du \right] \\ &= \mathbb{E}_0 \left[ \mathbf{1}_{\{\tau < T\}} \int_\tau^T \hat{\theta}_u N_u (\xi - p_u) du \right] \\ &= \mathbb{E}_0 \left[ \mathbf{1}_{\{\tau < T\}} \mathbb{E} \left[ \int_\tau^T \hat{\theta}_u N_u (\xi - p_u) du | \mathcal{F}_\tau^I \right] \right] \\ &\leq \mathbb{E}_0 \left[ \mathbf{1}_{\{\tau < T\}} J(\xi, p_\tau, N_\tau) \right] \\ &= \hat{J}(\alpha, N_0), \end{aligned}$$

where the second line uses  $p_u = \alpha$  for  $u < \tau$ , the third line takes expectations over  $\xi$ , the fourth line uses the law of iterated expectations, and the final line follows since it was shown above that as of time  $\tau$ , our posited trading strategy achieves higher expected profit than any other admissible strategy.  $\square$

**Proof of Proposition 2.** Let  $\hat{U}(N_t)$  denote the value of acquiring information when the



news process is equal to  $N_t$ . Using the expression for the post-acquisition value function in Proposition 1, we have

$$\hat{U}(N_t) = N_t \left( \alpha \int_{\alpha}^1 \frac{1-a}{\lambda(a)} da + (1-\alpha) \int_0^{\alpha} \frac{a}{\lambda(a)} da \right) \equiv N_t K.$$

Make the change of variables  $x = \Phi^{-1}(1-a)$  in the integrals in the expression for  $U(N_t)$

$$\begin{aligned} K &= \alpha \sqrt{\frac{\sigma_Z^2}{2r}} \int_{\alpha}^1 (1-a) \frac{1}{\phi(\Phi^{-1}(1-a))} da + (1-\alpha) \sqrt{\frac{\sigma_Z^2}{2r}} \int_0^{\alpha} a \frac{1}{\phi(\Phi^{-1}(1-a))} da \\ &= -\alpha \sqrt{\frac{\sigma_Z^2}{2r}} \int_{\Phi^{-1}(1-\alpha)}^{-\infty} \Phi(x) dx - (1-\alpha) \sqrt{\frac{\sigma_Z^2}{2r}} \int_{\infty}^{\Phi^{-1}(1-\alpha)} (1-\Phi(x)) dx \\ &= \alpha \sqrt{\frac{\sigma_Z^2}{2r}} \int_{-\infty}^{\Phi^{-1}(1-\alpha)} \Phi(x) dx + (1-\alpha) \sqrt{\frac{\sigma_Z^2}{2r}} \int_{\Phi^{-1}(1-\alpha)}^{\infty} (1-\Phi(x)) dx. \end{aligned}$$

Now integrate by parts

$$\begin{aligned} K &= \alpha \sqrt{\frac{\sigma_Z^2}{2r}} \int_{-\infty}^{\Phi^{-1}(1-\alpha)} \Phi(x) dx + (1-\alpha) \sqrt{\frac{\sigma_Z^2}{2r}} \int_{\Phi^{-1}(1-\alpha)}^{\infty} (1-\Phi(x)) dx \\ &= \alpha \sqrt{\frac{\sigma_Z^2}{2r}} \left( - \int_{-\infty}^{\Phi^{-1}(1-\alpha)} x \phi(x) dx + x \Phi(x) \Big|_{-\infty}^{\Phi^{-1}(1-\alpha)} \right) \\ &\quad + (1-\alpha) \sqrt{\frac{\sigma_Z^2}{2r}} \left( \int_{\Phi^{-1}(1-\alpha)}^{\infty} x \phi(x) dx + x(1-\Phi(x)) \Big|_{\Phi^{-1}(1-\alpha)}^{\infty} \right) \\ &= \alpha \sqrt{\frac{\sigma_Z^2}{2r}} \left( - \int_{-\infty}^{\Phi^{-1}(1-\alpha)} x \phi(x) dx + (1-\alpha) \Phi^{-1}(1-\alpha) \right) \\ &\quad + (1-\alpha) \sqrt{\frac{\sigma_Z^2}{2r}} \left( \int_{\Phi^{-1}(1-\alpha)}^{\infty} x \phi(x) dx - \alpha \Phi^{-1}(1-\alpha) \right) \\ &= \sqrt{\frac{\sigma_Z^2}{2r}} \int_{-\infty}^{\Phi^{-1}(1-\alpha)} -x \phi(x) dx = \sqrt{\frac{\sigma_Z^2}{2r}} \phi(\Phi^{-1}(1-\alpha)), \end{aligned}$$

since  $\int -x \phi(x) dx = \int \phi'(x) dx = \phi(x)$ .

The pre-acquisition value function under optimal stopping is

$$U(p, n) \equiv \sup_{\tau \in \mathcal{T}} \mathbb{E} \left[ \mathbf{1}_{\{\tau < T\}} (KN_{\tau} - c) \mid p_t = p, N_t = n \right] = \sup_{\tau \in \mathcal{T}} \mathbb{E} \left[ e^{-r\tau} (KN_{\tau} - c)^+ \mid p_t = p, N_t = n \right],$$

where the second equality follows because  $T$  is independently exponentially distributed and it suffices to consider only the positive part of  $KN_{\tau} - c$  since the trader can always guarantee herself zero profit by not acquiring. Note that this problem is similar to pricing a perpetual

American call option on an asset with price process  $KN_t$  that follows a geometric Brownian motion and with strike price  $c$ . Hence, standard results (Peskir and Shiryaev (2006), Chapter 4) imply that the optimal stopping time is a first hitting time of the  $N_t$  process,

$$T_N = \inf\{t > 0 : N_t \geq N^*\},$$

where  $N^* > 0$  is a constant to be determined. Furthermore, given that  $\{N_u\}$  is independent of  $\{p_u\}$  it follows that the value function does not depend on  $p$ , so we suppress that argument in the function  $U$ .

The value function and optimal  $N^*$  solve the following free boundary problem

$$\begin{aligned} rU &= \frac{1}{2}\sigma_N^2 N_t^2 U'' \quad \text{for } n < N^* \\ U(N^*) &= KN^* - c \quad \text{'value matching'} \\ U'(N^*) &= K \quad \text{'smooth pasting'} \\ U(n) &> (n - c)^+ \quad \text{for } n < N^* \\ U(n) &= (n - c)^+ \quad \text{for } n > N^* \\ U(0) &= 0. \end{aligned}$$

To determine the solution in the continuation region  $n < N^*$ , consider a trial solution of the form  $U(n) = An^\beta$ . Substituting and matching terms in the differential equation yields

$$r = \frac{1}{2}\sigma_N^2 \beta(\beta - 1), \quad \beta = \frac{1}{2} \pm \frac{1}{2} \sqrt{1 + \frac{8r}{\sigma_N^2}}$$

and the boundary condition at  $N = 0$  requires that one take the positive root

$$\beta = \frac{1}{2} + \frac{1}{2} \sqrt{1 + \frac{8r}{\sigma_N^2}}.$$

Applying the above conjecture to the value-matching and smooth pasting conditions implies:

$$N^* = \frac{\beta}{\beta - 1} \frac{c}{K}, \quad A = \frac{K}{\beta} \left( \frac{\beta}{\beta - 1} \frac{c}{K} \right)^{1-\beta} = \frac{c}{\beta - 1} \frac{1}{(N^*)^\beta},$$

and the resulting function satisfies  $U(n) > n - c$  in the continuation region, which establishes the result. The comparative statics with respect to  $c$ ,  $\sigma_N$ ,  $\sigma_Z$ , and  $\alpha$  are immediate from the

explicit expression for  $N^*$ . Moreover, since

$$\frac{\partial}{\partial r} N^* = \frac{c}{\sigma_Z^2 \phi(\Phi^{-1}(1-\alpha))} \frac{4\sqrt{2} \left( \sqrt{r} - 2 \sqrt{\frac{r}{\frac{8r}{\sigma_N^2} + 1}} \right)}{\left( \sigma_N - \sqrt{\sigma_N^2 + 8r} \right)^2} \quad (20)$$

we know that  $N^*$  is decreasing in  $r$  when  $r < \frac{3}{8}\sigma_N^2$ , but increasing otherwise.  $\square$

**Proof of Lemma 1.** Note that

$$\begin{aligned} N_t \geq N^* &\iff \log(N_t) \geq \log(N^*) \\ &\iff -\frac{1}{2}\sigma_N t + W_{Nt} \geq \frac{1}{\sigma_N}(\log(N^*/N_0)), \end{aligned}$$

so that the first time that  $N_t$  hits  $N^*$  is the first time that a Brownian motion with drift  $-\frac{1}{2}\sigma_N$  hits  $\frac{1}{\sigma_N}(\log(\frac{N^*}{N_0}))$ . It follows from [Karatzas and Shreve \(1998\)](#) (Chapter 3.5, Part C, p.196-197) that for  $N_0 < N^*$  the density of  $T_N$  is

$$\Pr(T_N \in [t, t+dt]) = \frac{\left(\log\left(\frac{N^*}{N_0}\right)\right)}{\sigma_N \sqrt{2\pi t^3}} \exp\left\{-\frac{\left(\frac{1}{\sigma_N} \log\left(\frac{N^*}{N_0}\right) + \frac{1}{2}\sigma_N t\right)^2}{2t}\right\} dt.$$

Moreover, since  $\frac{1}{\sigma_N}(\log(\frac{N^*}{N_0})) > 0$  but the drift of the Brownian motion is  $-\frac{1}{2}\sigma_N < 0$ , it follows from [Karatzas and Shreve \(1998\)](#) (p.197) that  $\Pr(T_N = \infty) > 0$ . Specifically, note that

$$\Pr(T_N < \infty) = \int_0^\infty \frac{\left(\log\left(\frac{N^*}{N_0}\right)\right)}{\sigma_N \sqrt{2\pi t^3}} \exp\left\{-\frac{\left(\frac{1}{\sigma_N} \log\left(\frac{N^*}{N_0}\right) + \frac{1}{2}\sigma_N t\right)^2}{2t}\right\} dt = \frac{N_0}{N^*}, \quad (21)$$

which implies  $\Pr(T_N = \infty) = 1 - \frac{N_0}{N^*}$ .  $\square$

**Proof of Proposition 3.** Given the definition of  $\tau$ , we have that for  $0 \leq t < \infty$ ,

$$\Pr(\tau \in [t, t+dt]) = \frac{\Pr(\tau \in [t, t+dt] | T_N \leq T) \Pr(T_N \leq T)}{\Pr(\tau \in [t, t+dt] | T_N > T) \Pr(T_N > T)} \quad (22)$$

$$= \Pr(T_N \in [t, t+dt] | T_N \leq T) \Pr(T_N \leq T) \quad (23)$$

$$= \Pr(T_N \in [t, t+dt]) \Pr(T \geq t) \quad (24)$$

$$= e^{-rt} \Pr(T_N \in [t, t+dt]). \quad (25)$$

Integrating gives us

$$\Pr(\tau < \infty) = \int_0^\infty e^{-rt} \frac{\left(\log\left(\frac{N^*}{N_0}\right)\right)}{\sigma_N \sqrt{2\pi t^3}} \exp\left\{-\frac{\left(\frac{1}{\sigma_N} \log\left(\frac{N^*}{N_0}\right) + \frac{1}{2}\sigma_N t\right)^2}{2t}\right\} dt \quad (26)$$

$$= \frac{e^{-\frac{\log(N^*/N_0)\sqrt{8r+\sigma_N^2}}{2\sigma_N}}}{\sqrt{N^*/N_0}} = \left(\frac{N_0}{N^*}\right)^\beta \quad (27)$$

The comparative statics for  $c$ ,  $N_0$ ,  $\sigma_Z$  and  $\alpha$  follow from plugging in the expressions for  $N^*$  and  $\beta$ . To establish the comparative statics for  $\sigma_N$ , first note that since  $\lim_{\sigma_N \rightarrow 0} \beta = \infty$ ,  $\lim_{\sigma_N \rightarrow \infty} \beta = 1$ , and  $N^* = \frac{\beta}{\beta-1} \frac{c}{K}$ ,

$$\lim_{\sigma_N \rightarrow \infty} \Pr(\tau < \infty) = 0 \quad (28)$$

$$\lim_{\sigma_N \rightarrow 0} \Pr(\tau < \infty) = \begin{cases} 0 & \text{if } c > N_0 K \\ 1 & \text{if } c \leq N_0 K \end{cases}. \quad (29)$$

Let

$$\zeta \equiv \frac{\partial}{\partial \beta} (\log(\Pr(\tau < \infty))) = \frac{\partial}{\partial \beta} \left(\frac{N_0}{N^*}\right)^\beta = \log\left(\frac{N_0}{N^*}\right) + \frac{1}{\beta-1} \quad (30)$$

which implies  $\lim_{\sigma_N \rightarrow 0} \zeta = \lim_{\beta \rightarrow \infty} \zeta = \log\left(\frac{N_0 K}{c}\right)$ ,  $\lim_{\sigma_N \rightarrow \infty} \zeta = \lim_{\beta \rightarrow 1} \zeta = \infty$ , and

$$\frac{\partial \zeta}{\partial \sigma_N} = \frac{\partial \zeta}{\partial \beta} \frac{\partial \beta}{\partial \sigma_N} = -\frac{1}{\beta(1-\beta)^2} \frac{\partial \beta}{\partial \sigma_N} > 0. \quad (31)$$

Since  $\frac{\partial}{\partial \sigma_N} \log(\Pr(\tau < \infty)) = \zeta \frac{\partial \beta}{\partial \sigma_N}$ , we have the following results:

- When  $c \leq N_0 K$ , since  $\zeta \geq 0$  for  $\sigma_N \rightarrow 0$  and  $\frac{\partial \zeta}{\partial \sigma_N} > 0$  we have  $\zeta > 0$  for all  $\sigma_N$ , which in turn implies  $\frac{\partial}{\partial \sigma_N} \log(\Pr(\tau < \infty)) < 0$  for all  $\sigma_N$ .
- When  $c > N_0 K$ ,  $\zeta$  crosses zero once, from below, as  $\sigma_N$  increases, which implies  $\frac{\partial}{\partial \sigma_N} \log(\Pr(\tau < \infty)) = 0$  at exactly this one point. In this case,  $\Pr(\tau < \infty)$  is hump-shaped.

Similarly, for  $r$ ,  $\frac{\partial}{\partial r} \log(\Pr(\tau < \infty)) = \zeta \frac{\partial \beta}{\partial r} - \frac{\beta}{2r}$ . We have  $\frac{\partial \zeta}{\partial r} = -\frac{1}{\beta(\beta-1)^2} \frac{\partial \beta}{\partial r} - \frac{1}{2r} < 0$ . Since  $\frac{\partial \beta}{\partial r} = \frac{1}{\sigma_N^2(\beta-\frac{1}{2})} > 0$  this implies  $\frac{\partial}{\partial r} \log(\Pr(\tau < \infty))$  crosses zero at most once as  $r$  increases and from above if it does so. Consider the limit as  $r$  tends to zero,

$$\begin{aligned} \lim_{r \rightarrow 0} \frac{\partial}{\partial r} \log(\Pr(\tau < \infty)) &= \lim_{r \rightarrow 0} \left(\zeta \frac{\partial \beta}{\partial r} - \frac{\beta}{2r}\right) \\ &= \lim_{r \rightarrow 0} \frac{2r\zeta - \sigma_N^2 \beta \left(\beta - \frac{1}{2}\right)}{2\sigma_N^2 r \left(\beta - \frac{1}{2}\right)}. \end{aligned} \quad (32)$$

If it can be shown that the numerator in eq. (32) has a finite, positive limit it will follow that the overall limit is  $\infty$ . Considering the numerator, we have

$$\begin{aligned} \lim_{r \rightarrow 0} (2r\zeta - \sigma_N^2 \beta (\beta - \frac{1}{2})) &= 2 \lim_{r \rightarrow 0} r \left( \frac{1}{\beta-1} - \log \frac{\beta}{\beta-1} - \log \sqrt{2r} \right) - \frac{1}{2} \sigma_N^2 \\ &= \sigma_N^2 - 2 \lim_{r \rightarrow 0} \frac{\frac{1}{\beta(\beta-1)}}{\frac{1}{r^2}} - \frac{1}{2} \sigma_N^2 \\ &= \frac{1}{2} \sigma_N^2 - 2 \lim_{r \rightarrow 0} \frac{2r}{(2\beta - 1) \frac{\partial}{\partial r} \beta} = \frac{1}{2} \sigma_N^2 \end{aligned}$$

where the second equality applies l'Hôpital's rule to the three different terms and uses the fact  $\frac{\partial}{\partial r} \beta \rightarrow \frac{2}{\sigma_N^2}$  as  $\beta \rightarrow 1$ . The third equality rearranges the expression in the remaining limit to place  $r^2$  in the numerator and uses l'Hôpital's rule again. Returning to eq. (32), this implies  $\lim_{r \rightarrow 0} \frac{\partial}{\partial r} \log(\mathbb{P}(\tau < \infty)) = \infty$ .

Now, consider  $\lim_{r \rightarrow \infty} \frac{\partial}{\partial r} \log(\Pr(\tau < \infty))$ . We have

$$\lim_{r \rightarrow \infty} \zeta = \lim_{r \rightarrow \infty} \left( \frac{1}{\beta-1} - \log \frac{\beta}{\beta-1} - \log \sqrt{2r} \right) = \lim_{\beta \rightarrow \infty} \left( \frac{1}{\beta-1} - \log \frac{\beta}{\beta-1} \right) - \lim_{r \rightarrow \infty} \log \sqrt{2r} = -\infty.$$

Because  $\frac{\partial}{\partial r} \beta > 0$ , it follows that  $\lim_{r \rightarrow \infty} \frac{\partial}{\partial r} \log(\Pr(\tau < \infty)) = -\infty$ , which completes the proof.  $\square$

**Proof of Proposition 4.** Using the expression for the asset price in Proposition 1,

$$dP_t = \begin{cases} \alpha \sigma_N N_t dW_{Nt} & 0 \leq t < \tau \\ \sigma_N N_t p(Y_t) dW_{Nt} + N_t \lambda^*(p_t) \sigma_Z dW_{Yt} & \tau \leq t < T \end{cases},$$

where  $W_{Yt} \equiv Y_t/\sigma_Z$  is a standard Brownian motion under the public filtration and is independent of  $W_{Nt}$ . Hence,

$$\frac{dP_t}{P_t} = \begin{cases} \sigma_N dW_{Nt} & 0 \leq t < \tau \\ \sigma_N dW_{Nt} + \frac{\lambda^*(p_t)}{p_t} \sigma_Z dW_{Yt} & \tau \leq t < T \end{cases}.$$

Letting  $\nu_t$  denote the instantaneous variance of the return process gives:

$$\nu_t \equiv \begin{cases} \sigma_N^2 & 0 \leq t < \tau \\ \sigma_N^2 + \left( \frac{\lambda(p_t)}{p_t} \right)^2 \sigma_Z^2 = \sigma_N^2 + 2r \left( \frac{\phi(\Phi^{-1}(p_t))}{p_t} \right)^2 & \tau \leq t \end{cases} \quad (33)$$

Let  $f(p) \equiv \phi(\Phi^{-1}(p))$ , and note that  $f_p = -\Phi^{-1}(p)$  and  $f_{pp} = -\frac{1}{f}$ . Conditional on information acquisition, note that by Ito's Lemma, we have:

$$d\nu_t = \nu_p dp_t + \frac{1}{2} \nu_{pp} (\lambda(p_t))^2 \sigma_Z^2 dt = \nu_p dp_t + r f(p)^2 \nu_{pp} dt, \quad (34)$$

where  $\nu_p = 4r \left(\frac{f}{p}\right) \left(\frac{f_p p - f}{p^2}\right) < 0$ , and

$$\nu_{pp} = 4r \left(\frac{f_p p - f}{p^2}\right)^2 + 4r \left(\frac{f}{p}\right) \left(\frac{p^2(f_{pp} p + f_p - f_p) - 2p(f_p p - f)}{p^4}\right). \quad (35)$$

Since  $\nu_p < 0$ , the above implies that conditional on information acquisition, instantaneous return variance  $\nu_t$  and returns are negatively related i.e.,  $\text{cov}\left(\frac{dp_t}{p_t}, d\nu_t\right) < 0$ .  $\square$

**Proof of Proposition 5.** For the no acquisition case,

$$\mathbb{E}[\xi N_T - P_{T-} | T_N > T] = \mathbb{E}[N_T | \xi - \alpha | T_N > T] = 2\alpha(1 - \alpha) \mathbb{E}[N_T | T_N > T] \quad (36)$$

Next, note that

$$\mathbb{E}[N_T] = \Pr(T_N < T) \mathbb{E}[N_T | T_N < T] + \Pr(T_N \geq T) \mathbb{E}[N_T | T_N \geq T] \quad (37)$$

$$\Rightarrow \mathbb{E}[N_T | T_N > T] = \frac{N_0 - \Pr(T_N < T) N^*}{\Pr(T_N \geq T)} \quad (38)$$

$$= \frac{N_0 - \left(\frac{N_0}{N^*}\right)^\beta N^*}{1 - \left(\frac{N_0}{N^*}\right)^\beta} \quad (39)$$

since  $\mathbb{E}[N_T] = N_0$ ,  $\mathbb{E}[N_T | T_N < T] = N^*$  and  $\Pr(T_N < T) = \left(\frac{N_0}{N^*}\right)^\beta$ . This produces the desired expression.

Conditional on information acquisition, the expected announcement effect is

$$\mathbb{E}[\xi N_T - P_{T-} | \tau < \infty] = \mathbb{E}[N_T | \xi - p(Y_T) | T_N < T] \quad (40)$$

$$= 2\mathbb{E}[N_T p(Y_T) (1 - p(Y_T)) | T_N < T] \quad (41)$$

$$= 2\mathbb{E}[\mathbb{E}_{T_N}[N_T p(Y_T) (1 - p(Y_T)) | T_N < T] | T_N < T] \quad (42)$$

$$= 2\mathbb{E}[N_{T_N} \mathbb{E}_{T_N}[p(Y_T) (1 - p(Y_T)) | T_N < T] | T_N < T] \quad (43)$$

$$= 2N^* \mathbb{E}[p(Y_T) (1 - p(Y_T)) | T_N < T] \quad (44)$$

the first and second equalities use the law of iterated expectations, the third equality uses the

fact that conditional on  $\sigma(\mathcal{F}_{T_N}^P \cup \{T_N < T\})$ ,  $N_T - N_{T_N}$  and  $Y_T$  are independent, the fourth equality uses the fact that  $N$  is a martingale, and the final equality uses  $N_{T_N} = N^*$ .

Suppose  $\tau \in [t, t + dt]$ . Given the characterization of  $p_t$  in Proposition 1, we can express  $p_s$  for  $s \geq t$  as  $p_s = \Phi\left(\frac{\sqrt{2r}}{\sigma_Z} z_s\right)$ , where

$$z_s | \{\tau \in [t, t + dt]\} \sim N\left(\Phi^{-1}(\alpha) e^{r(s-t)}, \frac{\sigma_Z^2}{2r} (e^{2r(s-t)} - 1)\right). \quad (45)$$

Next, note that for  $w \sim N(0, 1)$ , we have

$$\mathbb{E}[\Phi(a + bw)[1 - \Phi(a + bw)]] = \Phi\left(\frac{a}{\sqrt{1+b^2}}\right) - \left[\Phi\left(\frac{a}{\sqrt{1+b^2}}\right) - 2T^o\left(\frac{a}{\sqrt{1+b^2}}, \frac{1}{\sqrt{1+2b^2}}\right)\right] \quad (46)$$

from Owen (1980) 10,010.8 and 20,010.4, where  $T^o(a, b)$  is the Owen T function. Let  $\tilde{z}_s \equiv \frac{z_s - e^{r(s-t)} z_0}{\sqrt{\frac{\sigma_Z^2}{2r}(e^{2r(s-t)} - 1)}} \sim N(0, 1)$ , and note that  $p(z_s) = \Phi(a + b\tilde{z}_s)$ . This implies

$$G(t, s) \equiv \mathbb{E}_t[p_s(1 - p_s) | \tau \in [t, t + dt], s > t] = 2T^o\left(\Phi^{-1}(\alpha), \frac{1}{\sqrt{2e^{2r(s-t)} - 1}}\right). \quad (47)$$

Since the stopping time  $T$  is exponentially distributed, we have

$$\begin{aligned} & \mathbb{E}_t[p(Y_T)(1 - p(Y_T)) | T > t, \tau \in [t, t + dt]] \\ &= e^{-rt} \int_{s=t}^{\infty} r e^{-r(s-t)} \mathbb{E}_t[p(Y_s)(1 - p(Y_s)) | \tau \in [t, t + dt]] ds \end{aligned} \quad (48)$$

$$= \int_0^{\infty} r e^{-rs} G(0, s) ds \quad (49)$$

$$= 2 \int_0^{\infty} r e^{-rs} T^o\left(\Phi^{-1}(\alpha), \frac{1}{\sqrt{2e^{2rs} - 1}}\right) ds \quad (50)$$

$$= 2 \int_0^{\infty} e^{-x} T^o\left(\Phi^{-1}(\alpha), \frac{1}{\sqrt{2e^{2x} - 1}}\right) dx, \text{ where } x = rs \quad (51)$$

$$\equiv h(\alpha) \quad (52)$$

This implies that

$$\begin{aligned} & \mathbb{E}[p(Y_T)(1 - p(Y_T)) | \tau < T] \\ &= \int_0^{\infty} \mathbb{E}_t[p(Y_T)(1 - p(Y_T)) | T > t, \tau \in [t, t + dt]] \Pr(\tau \in [t, t + dt] | T > \tau) dt \end{aligned} \quad (53)$$

$$= h(\alpha) \int_0^{\infty} \Pr(\tau \in [t, t + dt] | T > \tau) dt = h(\alpha) \quad (54)$$

which implies  $\mathbb{E} [|\xi N_T - P_{T-}| | \tau < T] = 2N^*h(\alpha)$ .

Note that the announcement effect is bigger conditional on no acquisition if and only if:

$$2N^*h(\alpha) < 2\alpha(1-\alpha)N^*\frac{\frac{N_0}{N^*} - \left(\frac{N_0}{N^*}\right)^\beta}{1 - \left(\frac{N_0}{N^*}\right)^\beta} \Leftrightarrow \frac{h(\alpha)}{\alpha(1-\alpha)} < \frac{\frac{N_0}{N^*} - \left(\frac{N_0}{N^*}\right)^\beta}{1 - \left(\frac{N_0}{N^*}\right)^\beta} \quad (55)$$

$$\Leftrightarrow \frac{h(\alpha)}{\alpha(1-\alpha)} \left(1 - \left(\frac{N_0}{N^*}\right)^\beta\right) < \frac{N_0}{N^*} - \left(\frac{N_0}{N^*}\right)^\beta \quad (56)$$

$$\Leftrightarrow \frac{h(\alpha)}{\alpha(1-\alpha)} < \frac{N_0}{N^*} - \left(\frac{N_0}{N^*}\right)^\beta \left(1 - \frac{h(\alpha)}{\alpha(1-\alpha)}\right) \quad (57)$$

For a fixed  $\alpha$ , since

$$\frac{N_0}{N^*} = N_0^{\frac{\beta-1}{\beta c}} K = \frac{N_0^{\frac{1}{2}} \left(1 + \sqrt{1 + 8\frac{r}{\sigma_N^2}}\right) - 1}{c \frac{1}{2} \left(1 + \sqrt{1 + 8\frac{r}{\sigma_N^2}}\right)} \frac{\sigma_Z}{\sqrt{2r}} \phi\left(\Phi^{-1}(\alpha)\right), \quad (58)$$

implies that  $\frac{N_0}{N^*} \rightarrow 0$  when  $r \rightarrow 0$ ,  $r \rightarrow \infty$ ,  $\sigma_N \rightarrow \infty$ ,  $c \rightarrow \infty$  or  $\sigma_Z \rightarrow 0$ . Moreover, since  $\beta > 1$  and  $\frac{N_0}{N^*} < 1$ , we have  $\left(\frac{N_0}{N^*}\right)^\beta \rightarrow 0$  when  $\left(\frac{N_0}{N^*}\right) \rightarrow 0$ . Now, fix  $\alpha$  and pick a  $\delta$  such that  $0 < \delta < \frac{h(\alpha)}{\alpha(1-\alpha)}$ . Then, the above implies that for sufficiently extreme  $r$ , sufficiently large  $\sigma_N$ , sufficiently large  $c$  or sufficiently small  $\sigma_Z$ ,  $\frac{N_0}{N^*} - \left(\frac{N_0}{N^*}\right)^\beta \left(1 - \frac{h(\alpha)}{\alpha(1-\alpha)}\right) < \delta$ , and so the announcement effect is bigger conditional on acquisition.  $\square$