A Disaggregated Economy with Optimal Pricing Decisions*

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Abstract

We develop an analytically-tractable model featuring a disaggregated production economy, with a fully general input-output structure and optimal pricing decisions subject to menu costs. Our framework delivers a novel closed-form aggregation result, which links first-order changes in macroeconomic variables, such as GDP, employment and measured TFP, to microeconomic shocks, the input-output topology and the sector-specific pricing moments. Crucially, we show that relative to the flexible-price efficient benchmark, input-output linkages amplify the productivity and welfare losses associated with menu costs by an order given by a novel centrality measure, which captures a sector's importance as a *supplier of important suppliers*. This generates a powerful amplification of productivity and welfare losses, since input-linkages create two rounds of misallocation: first, within sectors due to the effect on the location adjustment bands; second, across sectors due to the inefficient reallocation of resources towards the key supplier sectors.

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1 Introduction

In modern economies, the production process of goods and services is organized in complex networks of firms. These supply chains serve both as a conduit and amplifier of productivity shocks through the economy. Whether changes in productivity are passed on to other producers and, eventually, households fundamentally depends on the ability of firms to reset their prices. In the presence of even small price adjustment costs, some firms may elect not to reset their prices, thereby operating with zero passthrough of productivity shocks. Previous literature has highlighted that when resetting costs are small, the distortions they induce are relatively limited thanks to strong selection effects.

In this paper, we study the role of production networks in shaping the distortions generated by the presence of menu costs. We build a tractable model in which firms operate in a general Input-Output structure. Each sector is subject to common productivity shocks, and individual firms are also hit by idiosyncratic fluctuations. The key friction in our model is that firms have to set their price before the uncertainty on their productivity unravels. Firms can reset their price optimally upon paying a small *menu cost*.

In the context of this economy, we provide three sets of results. First, we provide a new, analytical aggregation result mapping easily quantifiable objects such as the variance of idiosyncratic shocks, the fraction of adjusting firms, and the Input-Output structure to changes in the labor share, TFP, GDP, and welfare. This result allows us to solve the model analytically only as a function of sectoral variables. We provide closed-form solutions for the non-stochastic steady state and simple analytical formulas for the full non-linear model. To a first order, GDP in our economy behaves exactly like in a flexible price world. However, the presence of pricing frictions generates misallocation. As a consequence, TFP is lower and hours worked are higher than in a frictionless economy.

Second, we show that the welfare costs of sectoral fluctuations are governed by a new sufficient statistic, which we label *supplier-of-supplier centrality*. This statistic can be computed in any Input-Output data as $\mathcal{S} = \Psi^T \Lambda$, where Ψ^T is the transpose of the Leontief Inverse and Λ is a vector of Domar weights. Our sufficient statistic fully pins down the first-order response of the labor share, TFP, GDP, and welfare to sectoral productivity shocks.

At the heart of our model is the idea that the presence of a production network fundamentally alters the interactions between firms choosing whether to reset prices. As a firm's price constitutes a component of other firms' marginal costs, the optimal choice to reset a price significantly affects the likelihood that customers will also reset their price. This interaction implies that a firm's importance in the network is captured by its importance as a supplier to central firms. A natural implication of this new channel is that the pricing decisions of individual firms have a significantly larger aggregate impact as they are transmitted and amplified through the network.

The presence of menu costs generates cross-sectional misallocation due to the optimal choice of some firms not to adjust their prices. Such misallocation and its movements, in turn, affect the aggregate effects of sectoral shocks. We show that the aggregate impact of sectoral productivity fluctuations depends on their effect on the aggregate labor share. We can analytically decompose this into four distinct effects. First, menu costs imply that there are pre-existing distortions (markups). As a consequence, Domar weights respond to sectoral shocks and induce reallocation across industries. Second, as firms are subject to productivity shocks, some firms may optimally choose to hire workers to pay the menu cost. Third, as firms' productivity changes, so does the price of adjusters. This effect propagates through the network. Fourth, as productivity changes, firms adjust the re-pricing decision by changing their inaction region. Importantly, this happens for all firms in the economy, as long as they are connected to the shocked sector through the network. These different effects are summarized by our new sufficient statistic. The *supplier-of-supplier centrality* is as fat-tailed as the Domar weight distribution but is approximately one order of magnitude larger.

Third, we show that, due to the *supplier-of-suppliers centrality*, the welfare cost of the pricing friction is an order of magnitude larger than in the absence of the network. We use the BEA I-O table, matched with sectoral price adjustment frequency, as inputs to pin down the production network and the magnitude of idiosyncratic shocks. We show that the presence of menu costs and the production network significantly alters the aggregate effects of sectoral shocks. Quantitatively, we find that the menu cost effect can be as large as 30% of the direct effect governed by the Domar weight. The bulk of this effect is a direct consequence of the presence of the production network.

The rest of the paper is structured is follows. Section 2 introduces the problem solved by each type of agent, and characterizes the equilibrium. Section 3 specifies the functional forms and solves for the baseline economy with no sectoral or aggregate shocks. Section 4 derives the first-order behaviour of sectoral variables near the baseline. Section 5 performs analytic aggregation, thus obtaining the first order behavior of macroeconomic variables. Section 6 presents calibration of the model as well as the key quantitative results. Section 7 concludes.

2 Model

2.1 Overview

The economy has three types of agents: households, firms, and the government. There is a continuum of identical households who consume and supply labor. Each firm operates in one and only one of N sectors; each sector $i \in [1, ..., N]$ is populated by a continuum $[0, 1]$ of monopolistically competitive firms, where we let Φ_i denote the set of all firms in sector i. There is a constant elasticity of substitution across within-sector varieties, denoted by ϵ, $\epsilon \geq 2$. The government consists of a monetary authority, which sets the money supply M, and a fiscal authority, which sets sector-specific sales taxes and rebates the revenue to households in a lump-sum fashion.

The setting is static in the following sense: at the beginning of the period, households and firms anticipate both idiosyncratic and aggregate shocks (monetary and productivity) to be at their unconditional mean values of one. Then, they discover the realized values of all shocks and adjust their behaviour should the realizations differ from the unconditional means.

2.2 Households

The representative household chooses consumption C and labor supply L to maximize utility:

$$\max_{C,L} U(C, L) \tag{1}$$

subject to the budget constraint:

$$P^C C \leq WL + \sum_i \int_0^1 \Pi_i(j)dj - T \tag{2}$$

where P^C is the consumption price index (to be defined later), W is the nominal wage, $\Pi_i(j)$ is the profits of firm j in sector i, and T is a lump-sum tax levied by the government.

Consumption C is, in turn, an aggregator over consumption of sector-specific varieties:

$$C = \mathcal{C}(C_1, ..., C_N) \tag{3}$$

where $\mathcal{C}(\cdot)$ is homogenous of degree one and non-decreasing in each of the arguments. The household chooses consumption of each of the sector-specific varieties to minimise total expenditure $\sum_i P_i C_i$, subject to the aggregator in (3). The solution to this problem pins down the consumption price index P^C as the minimal cost of a basket that aggregates to $C = 1$: $P^C = \mathcal{P}^C(P_1, ..., P_N)$, where $\mathcal{P}^C$ is homogenous of degree one and non-decreasing in each of the arguments.

Each sector-specific consumption is itself an aggregator over consumptions of varieties bought from the firms in that sector:

$$C_i = \left(\int_0^1 C_i(j)^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}}. \tag{4}$$

The household chooses consumption of each of the firm-specific varieties to minimise total expenditure $\int_0^1 P_i(j) C_i(j) dj$, subject to the aggregator in (4). The latter pins down the demand for each firm-level variety:

$$C_i(j) = \left(\frac{P_i(j)}{P_i} \right)^{-\epsilon} C_i. \tag{5}$$

For the rest of the main text of the paper, we we work with log-linear utility over consumption and labor supply:

Assumption 1 (Golosov-Lucas preferences)**.** *The utility function over consumption and labor supply is log-linear:* $U(C, L) = \log C - L$.

Such a utility function implies the following equilibrium relationship between consumption and labor supply:

$$P^C C = W. \tag{6}$$

2.3 Firms

A firm j in sector i has access to the following production technology:

$$Y_i(j) = Z_i(j) F_i \left[L_i(j), X_{i1}(j), ..., X_{iN}(j) \right] \tag{7}$$

where $F_i(\cdot)$ is homogenous of degree one and non-decreasing in inputs; $L_i(j)$ is labor input, $X_{ik}(j)$ is intermediate inputs bought by firm j in sector i from sector k, $Z_i(j)$ is firm-level productivity, which is decomposed into idiosyncratic and sectoral components according to:

$$Z_i(j) \equiv \zeta_i(j)^{\frac{1}{\epsilon-1}} \times A_i \tag{8}$$

Idiosyncratic productivities have a mean one and are independent across and within sectors; sectoral productivities have a mean one and are independent across sectors.

2.3.1 Firms: cost minimization

Each firm chooses its labor and intermediate inputs to minimize the total variable cost of production $WL_i(j)+\sum_k P_k X_{ik}(j)$ subject to the production technology in (7). The solution to this cost minimization problem pins down the firm-level marginal cost of production, which in equilibrium can be written as:

$$MC_i(j) = \zeta_i(j)^{\frac{1}{1-\epsilon}} \times \mathcal{Q}_i(W, P_1, ..., P_N; A_i) \tag{9}$$

where $\mathcal{Q}_i(\cdot)$ is the marginal cost index, common for all firms within a sector, strictly falls in A_i and is homogenous of degree one and non-decreasing in the prices of all inputs.

2.3.2 Firms: pricing

Price setting is subject to nominal rigidities in the form of a fixed menu cost. In particular, if a firm in sector i wants to set a price different from a pre-determined price of $P_{i,0}$ it needs to purchase υ_i additional units of labor. Formally, profits of a firm j in sector i can be written as:

$$\Pi_i(j) = (1-\tau_i)P_i(j)Y_i(j) - MC_i(j)Y_i(j) - W\upsilon_i\chi_i(j) \tag{10}$$

where

$$\chi_i(j) = \begin{cases} 1 & \text{if} \quad P_i(j) \neq P_{i,0} \\ 0 & \text{if} \quad P_i(j) = P_{i,0} \end{cases} \tag{11}$$

and $(1-\tau_i)$ is a sales tax levied by the government.

Conditional on choosing to adjust the price, the optimal reset price $P_i(j)^*$ maximises profits in (10) subject to the downward sloping demand curve $Y_i(j) = (P_i(j)^*/P_i)^{-\epsilon}Y_i$. It can be shown that:

$$P_i(j)^* = \frac{1}{1-\tau_i}\frac{\epsilon}{\epsilon-1}MC_i(j) \tag{12}$$

As is standard in the New Keynesian literature, we set $1-\tau_i = \frac{\epsilon}{\epsilon-1}$, $\forall i$ to make the optimal reset price competitive, thus removing the non-state contingent distortion associated with the presence of market power.

The loss function associated with the price adjustment decision is given by:

$$\mathcal{L}\left[\zeta_i(j)\right] = \left[\Pi_i(j)|\chi_i(j)=1\right] - \left[\Pi_i(j)|\chi_i(j)=0\right]. \tag{13}$$

Therefore, firm j in sector i should adjust prices if and only if:

$$\mathcal{L}\left[\zeta_i(j)\right] \geq 0. \tag{14}$$

The condition in (14) pins down the adjustment bands (ζ_i^L, ζ_i^H) for a firm in sector i in terms of aggregate variables, such that the firm adjusts iff $\zeta_i(j) > \zeta_i^H$ or $\zeta_i(j) < \zeta_i^L$.

Unfortunately, apart from the special case when $\epsilon = 2$, one cannot characterize the adjustment bands analytically. For the purpose of analytic tractability, we assume that the price adjustment decision is based on an *approximate* loss function $\hat{\mathcal{L}}$, given by a second-order approximation of $\mathcal{L}(\zeta_i(j))$ in $\zeta_i(j)$ around its unconditional mean of 1:

$$\hat{\mathcal{L}}\left[\zeta_i(j)\right] \equiv \mathcal{L}\left[1\right] + \mathcal{L}'\left[1\right](\zeta_i(j)-1) + \frac{1}{2}\mathcal{L}''\left[1\right](\zeta_i(j)-1)^2. \tag{15}$$

The approximate loss function delivers the following closed-form expressions for the adjustment bands (ζ_i^L, ζ_i^H), in terms of sectoral and aggregate variables:

Lemma 1 (Adjustment bands)**.** *Under the approximate loss function $\hat{\mathcal{L}}$, the adjustment bands are:*

$$\zeta_i^L = 1 + \Gamma_1^i - \left[(2+\Gamma_1^i)\Gamma_1^i + \Gamma_2^i\right]^{\frac{1}{2}}, \tag{16}$$

$$\zeta_i^H = 1 + \Gamma_1^i + \left[(2+\Gamma_1^i)\Gamma_1^i + \Gamma_2^i\right]^{\frac{1}{2}}, \tag{17}$$

where Γ_1^i and Γ_2^i are given by:

$$\Gamma_1^i \equiv \frac{\epsilon-1}{\epsilon}(1-\mathcal{Q}_i^{-\epsilon}), \qquad \Gamma_2^i \equiv 2\frac{\epsilon-1}{\mathcal{Q}_i}\left[\frac{\epsilon-1}{\epsilon}\frac{W\upsilon_i}{P_i^\epsilon Y_i} + 1 - \mathcal{Q}_i\right]. \tag{18}$$

The equilibrium adjustment bands depend on two components. The first one, namely Γ_1^i, fully pins down the location of the midpoint of the adjustment bands, or $\frac{\zeta_i^L+\zeta_i^H}{2} = 1+\Gamma_1^i$. Whenever there is a decrease in the sectoral marginal cost $\mathcal{Q}_i$, there is a corresponding fall in Γ_1^i, which shifts the midpoint of adjustment bands to the left. Equivalently, the mass of non-adjusters shifts towards firms with lower idiosyncratic productivity. A fall in Γ_1^i also lowers the width of adjustment bands, or $\frac{\zeta_i^H - \zeta_i^L}{b_i}$, which is also the fraction of firms within a sector that choose not to adjust their price.

At the same time, the width of adjustment bands also depends on the second component Γ_2^i. In particular, a fall in Γ_2^i, for example, due to a rise in sector size Y_i or a reduction in the menu cost υ_i, lowers the width of the bands, or equivalently increases the equilibrium fraction of adjusting firms.

The sectoral price index P_i is obtained by aggregating firm-level prices:

$$P_i^{1-\epsilon} = \int_0^1 P_i(j)^{1-\epsilon} dj$$

$$= \int_{\zeta<\zeta_i^L} P_i(j)^{1-\epsilon} dj + \int_{\zeta_i^L \leq \zeta \leq \zeta_i^H} P_i(j)^{1-\epsilon} dj + \int_{\zeta>\zeta_i^H} P_i(j)^{1-\epsilon} dj \tag{19}$$

Firms which chose to adjust the price set it to $P_i(j)^* = \zeta_i(j)^{\frac{1}{1-\epsilon}} \mathcal{Q}_i$, whereas the non-adjusters set it at the exogneous $P_{i,0}$. We assign $P_{i,0} = 1, \forall i$, as the common price in a symmetric non-stochastic steady-state with flexible prices. The price index is given by

$$P_i^{1-\epsilon} = \mathcal{Q}_i^{1-\epsilon} \left[\int_{\zeta<\zeta_i^L} \zeta_i(j) dj + \int_{\zeta>\zeta_i^H} \zeta_i(j) dj \right] + Pr(\zeta_i^L \leq \zeta \leq \zeta_i^H). \tag{20}$$

Analytically solving for the integrals in the brackets and for the probability of being between the adjustment thresholds requires specifying the distribution from which the idiosyncratic shocks are drawn. For the rest of the main text, we assume they are drawn from a continuous uniform distribution:

Assumption 2 (Idiosyncratic shocks)**.** *Idiosyncratic shocks to productivity of firms in sector i are drawn from a continuous uniform distribution with support* $[1 - \frac{b_i}{2}, 1 + \frac{b_i}{2}], \quad 0 < b_i < 2, \forall i$.

With the additional assumption above, we obtain the final expression for the sectoral price index in terms of the sectoral marginal cost $\mathcal{Q}_i$ and the adjustment bands (ζ_i^L, ζ_i^H):

$$P_i^{1-\epsilon} = \mathcal{Q}_i^{1-\epsilon} \left(1 - \frac{\zeta_i^H - \zeta_i^L}{b_i} \times \underbrace{\frac{\zeta_i^L + \zeta_i^H}{2}}_{\text{Midpoint}} \right) + \underbrace{\frac{\zeta_i^H - \zeta_i^L}{b_i}}_{\text{Width}} \tag{21}$$

The expression above shows that the sectoral price index depends on three components.

First, it rises in the sectoral marginal cost $\mathcal{Q}_i$. Naturally, as $\mathcal{Q}_i$ rises, the optimal reset price rises for every firm, which, *ceteris paribus*, increases the overall sectoral price index. Second, P_i depends on the *width* of adjustment bands $\frac{\zeta_i^H - \zeta_i^L}{b_i}$, which is also the fraction of non-adjusting firms in the sector. As the fraction of non-adjusters rises, the sectoral price index gets closer to the non-adjustment price $P_{i,0} = 1$; as the fraction falls, the sectoral price gets closer to $\mathcal{Q}_i$. Finally, the sectoral price rises in the *midpoint* of the bands $\frac{\zeta_i^L + \zeta_i^H}{2}$. All else equal, a rise in the midpoint implies that the mass of non-adjusters shifts towards firms with larger idiosyncratic productivity. The latter simultaneously implies that the mass of adjusters now has a lower average productivity, which increases the overall sectoral price index.

2.4 Government policy

The government consists of a monetary authority which sets the money supply M, and a fiscal authority which sets sectoral sales taxes $\{\tau_i\}_{i=1}^N$ and reimburses the revenue to households as a lump-sum transfer T.

We introduce money into the model through a cash-in-advance constraint on final nominal demand:

$$P^C C \leq M. \tag{22}$$

Combined with the equilibrium relationship between consumption and labor supply in (6), it follows that in equilibrium, the nominal wage equals money supply: $W = M$.

As the fiscal authority sets $\tau_i = 1 - \frac{\epsilon}{\epsilon - 1}$, it needs to collect the following lump-sum tax from the household in order to balance the fiscal budget:

$$T = \sum_i \int_0^1 \frac{1}{\epsilon - 1} P_i(j) Y_i(j) dj. \tag{23}$$

2.5 Market clearing and equilibrium

In addition to the optimality conditions and policy specifications above, equilibrium in our economy is pinned down by market clearing conditions in the labor market:

$$L = \sum_i \int_0^1 L_i(j) dj + \sum_i \left(1 - \frac{\zeta_i^H - \zeta_i^L}{b_i}\right) \upsilon_i \tag{24}$$

and in the market for each individual good:

$$Y_i(j) = C_i(j) + \sum_k \int_0^1 X_{ki}(j', j) dj', \quad \forall i, \forall j \in \Phi_i. \tag{25}$$

We can now define an equilibrium in our economy:

Definition 1 (Equilibrium)**.** *The equilibrium is a collection of prices $\{P_i(j)|j \in \Phi_i\}_{i=1}^N$, allocations $\left\{Y_i(j), L_i(j), C_i(j), \{X_{ir}(j,j')|j' \in \Phi_r\}_{r=1}^N |j \in \Phi_i\right\}_{i=1}^N$ and wage W, which given the realizations of firm-level productivities $\{\zeta_i(j)|j \in \Phi_i\}_{i=1}^N$, sectoral productivities $\{A_i\}_{i=1}^N$ and money supply M satisfy agent optimization and market clearing conditions.*

2.6 Misallocation in equilibrium

In equilibrium, agents are making decisions that are privately optimal. However, given the distortions introduced by nominal rigidities in the form of menu costs, as well as firms' market power, resources will, in general, be misallocated. In this subsection, we derive the relevant measure of misallocation within each sector and show how input-output linkages create across-sector propagation of misallocation.

For convenience, let $\lambda_i \equiv \frac{P_i Y_i}{P^C C}$ be the (revenue-based) Domar weight (sales share) of sector i. The goods market clearing condition (25) can be used to find a convenient expression for sectoral Domar weights $\{\lambda_i\}_{i=1}^N$. Aggregating across firms and multiplying both sides by the sectoral price:

$$P_i Y_i = P_i C_i + \sum_k \int_0^1 P_i X_{ki}(j') dj' \tag{26}$$

Dividing through by the final nominal demand $P^C C$:

$$\underbrace{\frac{P_i Y_i}{P^C C}}_{\equiv \lambda_i} = \underbrace{\frac{P_i C_i}{P^C C}}_{\equiv \omega_i^c} + \sum_k \int_0^1 \underbrace{\frac{P_i X_{ki}(j')}{MC_k(j') Y_k(j')}}_{\equiv \omega_{ki}} \frac{MC_k(j') Y_k(j')}{P^C C} dj' \tag{27}$$

Defining consumption shares as $\omega_i^c \equiv \frac{P_i C_i}{P^C C}$ and input-output cost shares as $\omega_{ki} \equiv \frac{P_i X_{ki}(j')}{MC_k(j') Y_k(j')}$, and further noting that $\frac{MC_k(j') Y_k(j')}{P^C C} = \frac{MC_k(j') Y_k(j')}{P_k Y_k} \frac{P_k Y_k}{P^C C} = \frac{MC_k(j')}{P_k(j')} \frac{P_k(j') Y_k(j')}{P_k Y_k} \lambda_k = \frac{1}{\mu_k(j')} \frac{P_k(j') Y_k(j')}{P_k Y_k} \lambda_k$, where $\mu_k(j') \equiv \frac{P_k(j')}{MC_k(j')}$ is the firm-level markup, it follows that:

$$\lambda_i = \omega_i^c + \sum_k \omega_{ki} \lambda_k \mu_k^{-1} \tag{28}$$

where

$$\mu_k \equiv \left(\int_0^1 \frac{1}{\mu_k(j)} \frac{P_k(j)Y_k(j)}{P_k Y_k} dj \right)^{-1} \tag{29}$$

is the sectoral sales-weighted harmonic average of firm-level markups. The final step is to perform the analytic aggregation above. Using the downward sloping demand condition $Y_i(j) = (P_i(j)/P_i)^{-\epsilon} Y_i$ it can be shown that:

$$\mu_k^{-1} = \frac{\int_0^1 \frac{1}{\mu_k(j)} P_k(j)^{1-\epsilon} dj}{P_k^{1-\epsilon}}. \tag{30}$$

The sales-weighted harmonic average μ_i is the relevant measure of *within-sector* misallocation in sector i of our economy. In the special case where all firms charge competitive prices ($\mu_i(j) = 1$), our measure of misallocation collapses to the value of one. More generally, the adjusters charge the optimal reset price in (12), implying $[\mu_i(j)|\chi_i(j) = 1] = 1$. The non-adjusters, on the other hand, charge the common price of one, implying that $[\mu_i(j)|\chi_i(j) = 0] = \zeta_i(j)^{\frac{1}{\epsilon-1}} \mathcal{Q}_i^{-1}$. We can write the expression for within-sector misallocation as:

$$\mu_i^{-1} = \left(\underbrace{\int_{\zeta_i(j) \leq \zeta_i^L} \zeta_i(j) dj + \int_{\zeta_i(j) \geq \zeta_i^H} \zeta_i(j) dj}_{\text{Adjusters}} + \overbrace{\int_{\zeta_i^L < \zeta_i(j) < \zeta_i^H} \zeta_i(j)^{\frac{1}{1-\epsilon}} \mathcal{Q}_i^{\epsilon} dj}^{\text{Non-adjusters}} \right) \left(\frac{P_i}{\mathcal{Q}_i} \right)^{\epsilon - 1}. \tag{31}$$

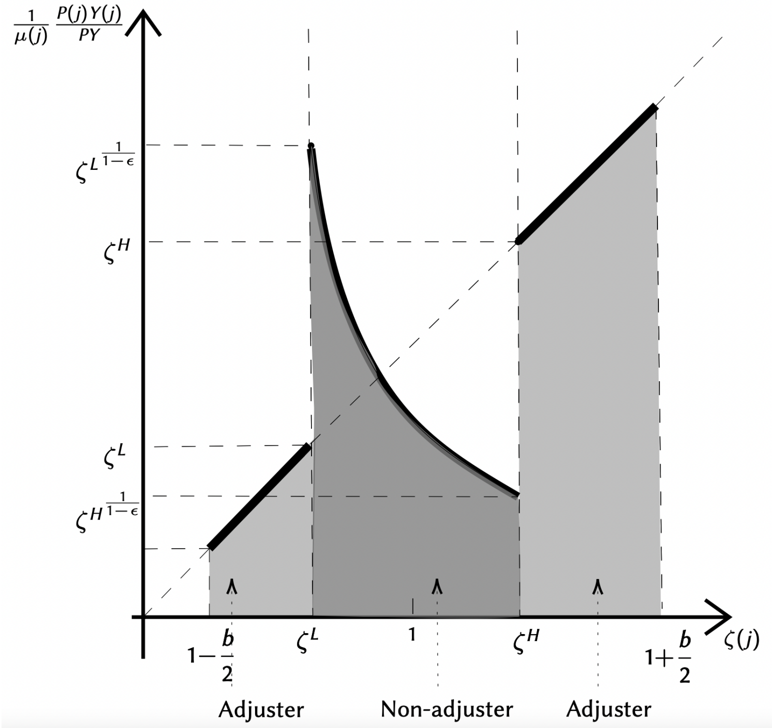
Finally, using the assumption of uniform distribution of idiosyncratic shocks, we can express μ_i^{-1} in terms of sectoral variables only:

$$\mu_i^{-1} = 1 + P_i^{\epsilon-1} \left[\mathcal{Q}_i \frac{(\zeta_i^H)^{\frac{2-\epsilon}{1-\epsilon}} - (\zeta_i^L)^{\frac{2-\epsilon}{1-\epsilon}}}{b_i \left(\frac{2-\epsilon}{1-\epsilon}\right)} - \frac{\zeta_i^H - \zeta_i^L}{b_i} \right]. \tag{32}$$

Figure 1 provides a graphical representation of the link between adjustment decisions and within-sector misallocation. In particular, it plots the integrand in (29), or the sales-adjusted inverse markup, against the firm-level idiosyncratic productivity $\zeta_i(j)$. For adjusting firms, the integrand linearly *rises* in $\zeta_i(j)$, reflecting the fact that their within-sector sales share rises in idiosyncratic productivity. On the other hand, for the non-adjusters, the integrand *falls* in $\zeta_i(j)$, representing the inefficient markups they charge. The area under the three segments in the figure represents within-sector misallocation.

The condition in (28) further establishes the link between misallocation within a specific sector and its propagation to the rest of the economy through input-output linkages. If a

Figure 1: Within-sector misallocation



Notes: *the figure provides a graphical representation of the sectoral sales-weighted harmonic average markup, which represents within-sector misallocation.*

sector i's customer sectors have a lot of misallocation (corresponding to high values of μ_k^{-1}), then $i's$ sales share becomes inefficiently high. This reflects the need to supply additional resources to the customer sectors to compensate for their suboptimal allocation of resources across firms. Naturally, once the sector becomes inefficiently large, this makes their suppliers inefficiently large as well. This *upstream* propagation of misallocation can be summarized by writing the equilibrium sales share in vector form:

$$\boldsymbol{\lambda} = (I - \hat{\Omega}^T)^{-1}\boldsymbol{\omega}^C \tag{33}$$

where $\boldsymbol{\lambda} = [\lambda_1, ..., \lambda_N]^T, \boldsymbol{\omega}^C = [\omega_1^C, ..., \omega_N^C]^T$ and $[\hat{\Omega}]_{i,j} \equiv \omega_{ij}\mu_i^{-1}$. The Leontief inverse $(I - \hat{\Omega}^T)^{-1}$ succinctly captures the fact that misallocation in any sector propagates upstream to its suppliers, suppliers of its supplier, and so on.

3 Functional forms and baseline equilibrium

3.1 Functional forms

We proceed under the assumption the consumption aggregator $\mathcal{C}(\cdot)$ and the production technology $F_i(\cdot), \forall i$, take the Cobb-Douglas form:

Assumption 3 (Cobb-Douglas aggregation across sectors)**.** *The consumption aggregator $\mathcal{C}(\cdot)$ is given by:*

$$\mathcal{C}(C_1, ..., C_N) = \iota^C \prod_{i=1}^{N} C_i^{\overline{\omega}_i^C}, \tag{34}$$

where $\iota^C \equiv \prod_{i=1}^{N} \overline{\omega}_i^{C^{-\overline{\omega}_i^C}}$ is a normalization term and $\sum_i \overline{\omega}_i^c = 1$, $\overline{\omega}_i^c \geq 0, \forall i$. Similarly, the production technology $F_i(\cdot)$ for a firm j in sector i is given by:

$$F_i[L_i(j), X_{i1}(j), ..., X_{iN}(j)] = \iota_i L_i(j)^{\overline{\eta}_i} \prod_{k=1}^{N} X_{ik}(j)^{\overline{\omega}_{ik}}, \tag{35}$$

where $\iota_i \equiv \overline{\eta}_i^{-\overline{\eta}_i} \prod \overline{\omega}_{ik}^{-\overline{\omega}_{ik}}$ is a normalization term and $\overline{\eta}_i + \sum_i \overline{\omega}_{ik} = 1$, $\overline{\eta}_i$, $\overline{\omega}_{ik} \geq 0, \forall i$.

Under such an assumption regarding sectoral aggregation, the consumption price index and sectoral marginal cost take the following form in equilibrium:

$$P^C = \prod_{i=1}^{N} P_i^{\overline{\omega}^C}, \qquad \mathcal{Q}_i = \frac{1}{A_i} W^{\overline{\eta}_i} \prod_{k=1}^{N} P_k^{\overline{\omega}_{ik}}, \quad \forall i. \tag{36}$$

Moreover, the equilibrium final consumption shares $\omega_i^C \equiv \frac{P_i C_i}{P^C C} = \overline{\omega}_i^C$ and input-output cost shares as $\omega_{ij} \equiv \frac{P_j X_{ij}(j)}{MC_i(j) Y_i(j)} = \overline{\omega}_{ij}$ are all constant.

3.2 Baseline: only firm-level idiosyncratic shocks

Here we characterize the equilibrium of our baseline economy, which is driven exclusively by idiosyncratic productivity shocks. In particular, we evaluate equilibrium setting $M = A_i = 1, \forall i$, so that all productivity shocks are at their unconditional means. Let endogenous variables with bars (e.g. $\overline{x}$) denote equilibrium outcomes in such a baseline economy.

Consider the sectoral price condition (21) in our baseline economy:

$$\overline{P}_i^{1-\epsilon} = \overline{\mathcal{Q}}_i^{1-\epsilon} \left(1 - \frac{\overline{\zeta}_i^H - \overline{\zeta}_i^L}{b_i} \times \left\{ 1 + \frac{\epsilon}{\epsilon - 1} (1 - \overline{\mathcal{Q}}_i^{-\epsilon}) \right\} \right) + \frac{\overline{\zeta}_i^H - \overline{\zeta}_i^L}{b_i} \tag{37}$$

Given that $\overline{Q}_i = \frac{1}{\overline{A}_i} \overline{M}^{\overline{\eta}_i} \prod_{k=1}^{N} \overline{P}_k^{\overline{\omega}_{ik}} = \prod_{k=1}^{N} \overline{P}_k^{\overline{\omega}_{ik}}$, it is easy to verify that

$$\overline{P}_i = \overline{Q}_i = 1, \forall i \tag{38}$$

is consistent with equilibrium in our baseline economy. Further, it follows that $\overline{P}^C = 1$ and from the cash-in-advance constraint we get $\overline{C} = 1$. This means both aggregate and sectoral prices and consumptions in the baseline equilibrium admit the same values as in the equilibrium where *both* idiosyncratic and aggregate shocks are at their unconditional means of one. In other words, when it comes to the effect on consumption and prices, idiosyncratic shocks exactly "wash out".

Further, we can also find an expression for adjustment thresholds in the baseline economy. To do that, define

$$\rho_i \equiv \frac{\overline{W} \upsilon_i}{\overline{P}_i \overline{Y}_i} \tag{39}$$

to be the *normalized* menu cost, given by the ratio of menu cost payment to sectoral revenue in baseline equilibrium. Then, the adjustment bands in our baseline equilibrium are given by:

$$\overline{\zeta}_i^L = 1 - \varepsilon\sqrt{\rho_i}, \qquad\qquad \overline{\zeta}_i^H = 1 + \varepsilon\sqrt{\rho_i}. \tag{40}$$

where $\varepsilon \equiv \sqrt{2\frac{(\epsilon-1)^2}{\epsilon}}$. Notice that the bands are proportional to the *square root* of the normalized menu cost. Therefore, even if the normalized menu cost is second-order, the bands are first-order.

Finally, the within-sector misallocation, given by the sectoral sales-weighted harmonic average markup, in the baseline equilibrium can be computed as $\overline{\mu}_i^{-1} = 1+\left[\frac{(\overline{\zeta}_i^H)^{\frac{2-\epsilon}{1-\epsilon}} - (\overline{\zeta}_i^L)^{\frac{2-\epsilon}{1-\epsilon}}}{b_i\left(\frac{2-\epsilon}{1-\epsilon}\right)} - \frac{\overline{\zeta}_i^H - \overline{\zeta}_i^L}{b_j}\right]$. The latter in turn gives the baseline sales shares as $\overline{\boldsymbol{\lambda}} = (I - \overline{\hat{\Omega}}^T)^{-1}\overline{\boldsymbol{\omega}}^{\boldsymbol{C}}$, where $[\overline{\hat{\Omega}}]_{i,j} \equiv \overline{\omega}_{ij}\overline{\mu}_i^{-1}$. Note that in the baseline equilibrium the sectoral Domar weights are equal to the sectoral output: $\lambda_i = \frac{\overline{P}_i \overline{Y}_i}{\overline{P}^C \overline{C}} = \overline{Y}_i, \forall i$.

4 First-order sectoral behaviour

4.1 Sectoral pricing

We start by deriving the equilibrium first-order movements in sectoral prices, around the baseline equilibrium characterized in the previous section. Denote by $\alpha_i \equiv \frac{\overline{\zeta}_i^H - \overline{\zeta}_i^L}{b_i}$ the baseline sector-specific frequency of non-adjustment. Then a first-order change in the price index of sector i can be written as:

$$d \log P_i = \frac{(1-\alpha_i)}{\alpha_i} \underbrace{d \log \frac{\mathcal{Q}_i}{P_i}}_{\Delta \text{ Real MC}} + \underbrace{\frac{1}{\epsilon - 1} d \left[\frac{\zeta_i^L + \zeta_i^H}{2} \right]}_{\text{Selection effect}}. \tag{41}$$

One can see that the sectoral price change has two components. The first component is the change in the sectoral real marginal cost, scaled by the ratio of fractions of adjusters and non-adjusters. This component is exactly equivalent to the *total* sectoral price change in the case of purely time-dependent pricing (Calvo, 1983). The second component is unique to our state-dependent pricing setup and comes from the change in the location of the midpoint of adjustment bands. The latter represents a change in the identity of adjusters within a sector following a sectoral/aggregate shock, or the *selection* effect. If the average idiosyncratic productivity of adjusters rises following a shock, then the sectoral price index gets an additional downward push, and *vice versa*.

Understanding the selection effect requires deriving the equilibrium movements in adjustment bands following a shock. Below we formally characterize the first-order changes in ζ_i^L and ζ_i^H near the baseline:

Proposition 1 (Adjustment bands)**.** *Near the baseline, the first-order movements in upper and lower adjustment bands of sector i are given by:*

$$d\zeta_i^H = \phi_i^H (\epsilon - 1) d \log \mathcal{Q}_i - \frac{(\epsilon-1)^2}{\sqrt{2\epsilon}} \sqrt{\rho_i} d \log P_i - \frac{\epsilon - 1}{\sqrt{2\epsilon}} \sqrt{\rho_i} d \log \lambda_i$$

and

$$d\zeta_i^L = \phi_i^L (\epsilon - 1) d \log \mathcal{Q}_i + \frac{(\epsilon-1)^2}{\sqrt{2\epsilon}} \sqrt{\rho_i} d \log P_i + \frac{\epsilon - 1}{\sqrt{2\epsilon}} \sqrt{\rho_i} d \log \lambda_i,$$

where

$$\phi_i^H \equiv \left(1 - \frac{1}{\sqrt{2\epsilon}} \sqrt{\rho_i}\right), \qquad \phi_i^L \equiv \left(1 + \frac{1}{\sqrt{2\epsilon}} \sqrt{\rho_i}\right).$$

Having formally solved for the equilibrium changes in adjustment bands, we can immediately deduce the equilibrium movements in their midpoint:

$$d \left[\frac{\zeta_i^L + \zeta_i^L}{2} \right] = (\epsilon - 1) d \log \mathcal{Q}_i. \tag{42}$$

Following an increase in the sectoral marginal cost, the midpoint shifts to the right and *vice* *versa*. The intuition behind the results is as follows. Following an increase in $\mathcal{Q}_i$, there is a corresponding one-for-one increase in the optimal reset price of every firm in the sector. The latter pushes the distance between the non-adjustment price ($P_{i,0} = 1$) and the optimal reset price to the negative territory. On the margin, this creates an incentive for firms whose idiosyncratic productivity is just to the left of baseline bands to adjust; simultaneously, this creates an incentive for firms just to the right of the baseline bands to stop adjusting. As a result, the mass of non-adjusters moves towards firms with higher idiosyncratic productivity, corresponding to a rightward shift in the midpoint of the bands.

We can now combine (41) and (42) to express first order movements in terms of sectoral shocks and monetary policy response:

Proposition 2 (Sectoral prices)**.** *Near the baseline equilibrium, the first-order change in the sectoral price is given by:*

$$d\log P_i = d\log \mathcal{Q}_i = -\sum_k \Psi_{ik} d\log A_k + d\log M \tag{43}$$

where Ψ_{ik} is the (i,k) entry of the cost-based Leontief inverse:

$$\Psi \equiv (I - \overline{\Omega})^{-1} \tag{44}$$

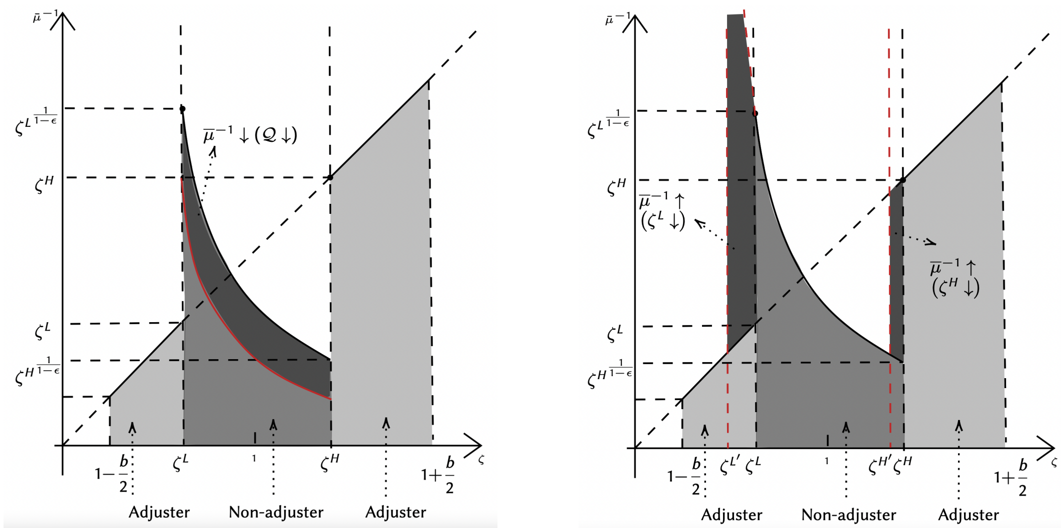
where $[\overline{\Omega}]_{i,j} = \overline{\omega}_{ij}$.

Proposition 2 estbalishes a key result. Up to first order, sectoral prices move one-for-one with the sectoral marginal cost, just as they would in the *flexible-price* equilibrium. This first-order sectoral price flexibility property holds even though the width of the adjustment bands does not shrink to zero following sectoral/aggregate shocks, so that there is still a non-zero mass of firms that choose to not adjust. Key here is the *selection* effect, which changes the composition of adjusters in a non-random way. In particular, following a shock which increases the sectoral marginal cost, the mass of adjusters shifts towards firms with lower idiosyncratic productivity, which delivers a further upward push to the sectoral price. In fact, the push is just strong enough to make the sectoral price move one-for-one with the sectoral marginal cost.

Crucially, even though sectoral prices behave like in the flexible-price equilibrium up to first order, other important aspects of the economy behave very differently in the frictionless environment. In the next subsection, we show that when it comes to misallocation, the model features a first-order deviation from the flexible-price benchmark.

Figure 2: First-order changes in within-sector misallocation

(a) Price effect **(b)** Bands effect



Notes: the figure graphically illustrates the two effects that drive first-order changes in the sectoral sales-weighted harmonic average markup.

4.2 Sectoral misallocation

In this subsection, we characterize first-order changes in sectoral sales-weighted harmonic average markups, corresponding to changes in misallocation within a specific sector. In the next section, we use these results to derive changes in aggregate misallocation.

The proposition below formally derives the first-order change in within-sector misallocation near the baseline:

Proposition 3 (Within misallocation)**.** *Near the baseline, the first-order change in the inverse harmonic average markup of any sector i is given by:*

$$d\mu_i^{-1} = \underbrace{\frac{\varphi_i^P}{b_i} d\log P_i}_{\text{Price effect}} - \underbrace{\left[\frac{\varphi_i^H}{b_i} d\zeta_i^H + \frac{\varphi_i^L}{b_i} d\zeta_i^L\right]}_{\text{Bands effect}}$$

where φ_i^P, φ_i^H and φ_i^L are given by:

$$\varphi_i^P \equiv \epsilon(\overline{\mu}_i^{-1} - 1)b_i + \overline{\zeta}_i^H - \overline{\zeta}_i^L > 0, \qquad \varphi_i^H \equiv 1 - (\overline{\zeta}_i^H)^{\frac{1}{1-\epsilon}} > 0, \qquad \varphi_i^L \equiv (\overline{\zeta}_i^L)^{\frac{1}{1-\epsilon}} - 1 > 0.$$

One can see that first-order changes in within-sector misallocation can be decomposed into the *price effect* and the *bands effect*.

The price effect signifies that, *ceteris paribus*, a first-order fall in the sectoral price leads to first-order fall in that sector's misallocation. Recall that firms that choose not to adjust their price charge an inefficient markup of $[\mu_i(j)|\chi_i(j) = 0] = \zeta_i(j)^{\frac{1}{\epsilon-1}} \mathcal{Q}_i^{-1}$. By Proposition 2, a first order fall in P_i delivers a one-for-one first-order fall in $\mathcal{Q}_i$. As a result, markups of all non-adjusting firms rise, which lowers the resources allocated to the non-adjusting firms and hence the misallocation within sector i. Panel (a) of Figure 2 makes the same point graphically: a fall in $\mathcal{Q}$ shifts the intermediate segment of the misallocation curve downward, lowering the total area under the three segments.

The bands effect, on the other hand, states that, *ceteris paribus*, a first order decrease in the adjustment bands ζ_i^L and ζ_i^H leads to an increase in misallocation in sector i. A leftward shift in the bands implies that the mass of non-adjusters shifts towards firms with lower idiosyncratic productivity. Therefore, the new marginal non-adjusters have lower markups, which increases the resources allocated to them and, hence, the misallocation within the sector. Panel (b) of Figure 2 makes the same point graphically.

Note that by Propositions 1 and 2, shocks that lead to a fall in $\mathcal{Q}_i$ simultaneously lead to a fall in the sectoral price and a leftward shift in the sectoral adjustment bands. Therefore, the cyclical behaviour of within-sector misallocation near the baseline can be ambiguous. In the next section, we show that the ambiguity is resolved in the limit of small menu costs. However, before that, we characterize the cyclical behaviour of macroeconomic aggregates near the baseline.

5 Aggregation

5.1 Aggregate GDP

First, we characterize the first-order behaviour of aggregate GDP:

Proposition 4 (Aggregate GDP)**.** *Near the baseline, the first-order change in aggregate GDP is given by:*

$$d \log C = \sum_{k} \tilde{\lambda}_k d \log A_k$$

where $\tilde{\lambda}_k$ is the sales share (Domar weight) in flexible-price equilibrium:

$$\tilde{\boldsymbol{\lambda}} = (I - \overline{\Omega}^T)^{-1} \overline{\boldsymbol{\varpi}}^{\boldsymbol{C}} = \Psi^T \overline{\boldsymbol{\varpi}}^{\boldsymbol{C}}.$$

Up to the first order, aggregate GDP behaves as in the flexible-price model. In particular, sector-specific productivity shocks are aggregated with the flexible-price (cost-based) Domar weights, consistent with the classical result of Hulten (1978). At the same time, changes in money supply have no first-order effect on aggregate GDP. The last result is reminiscent of the quasi-neutrality of Golosov and Lucas Jr (2007).

As a direct consequence of this result, we have that regardless of the underlying monetary rule, there is no "output gap" up to first order. Nonetheless, despite the approximate neutrality of aggregate GDP, the economy features misallocation as the presence of menu costs induces some firms to be too large and others to be too small. This cross-sectional inefficiency manifests in an inefficient aggregate labor supply.

5.2 Aggregate Employment and Labor Share

We now turn to studying the aggregate supply side of the economy. In particular, we derive first-order movements in aggregate labor supply.

Note that under the log-linear preferences we use, the equilibrium aggregate labor supply is exactly equal to the equilibrium labor share. In particular, letting $\Theta \equiv \frac{WL}{P^C C}$ be the aggregate share of value added, the households' optimality condition $P^C C = W$ implies that in equilibrium $\Theta = L$. We are, therefore, going to be using Θ and L interchangeably.

From the households' budget constraint, we can write the equilibrium aggregate labor supply/labor share as:

$$L = \Theta = 1 + \underbrace{\sum_i \lambda_i (\mu_i^{-1} - 1)}_{\text{Misallocation}} + \underbrace{\sum_i \upsilon_i \left(1 - \frac{\zeta_i^H - \zeta_i^L}{b_i}\right)}_{\text{Menu cost payment}}. \tag{45}$$

As a special case, the flexible-price economy with ($\upsilon_i = 0$ and $\mu_i^{-1} = 1$) features an aggregate labor share equal to one. Price rigidities in the form of menu costs create additional (inefficient) labor supply in equilibrium. First, due to the fact that menu costs are paid with labor. Second, additional labor supply is needed to compensate for within-sector misallocation, which further propagates through input-output linkages as governed by the Domar weights λ_i.

We can now use (45) to derive the first-order change in aggregate employment and the labor share:

$$dL = d\Theta = \underbrace{\sum_i d\lambda_i(\mu^{-1} - 1)}_{\text{①:pre-existing distortions}} \underbrace{-\sum_i \upsilon_i d\left[\frac{\zeta_i^H - \zeta_i^L}{b_i}\right]}_{\text{②: menu cost payment}}$$

$$+ \underbrace{\sum_i \lambda_i \frac{\varphi_i^P}{b_i} d\log P_i}_{\text{③: aggregate price effect}} \underbrace{-\sum_i \lambda_i \left[\frac{\varphi_i^H}{b_i} d\zeta_i^H + \frac{\varphi_i^L}{b_i} d\zeta_i^L\right]}_{\text{④: aggregate bands effect}}. \tag{46}$$

Let us now analyze the four effects that pin down the first-order changes in aggregate employment (labor share). First, there is the effect of *pre-existing distortions*: all else equal, if sectoral/aggregate shocks make a sector i larger, the importance of pre-existing misallocation in sector i increases. As a consequence, additional labor needs to be supplied to make up for misallocation within that sector. Second, there is the *menu cost payment effect*: if sectoral/aggregate shocks increase the fraction of adjusters in a specific sector, then there will be more aggregate labor required to pay the menu costs of adjusters. Third, there is the *aggregate price effect*. Recall from Proposition 3 that a cyclical rise in the sectoral price increases misallocation within that sector. The aggregate price effect scales each sector-specific price effect by the sales share of that sector. This is because more labor is required to compensate for the inefficient allocation of resources within a sector that is a major supplier either to other sectors or to households. Fourth, there is the *aggregate bands effect*. Once again, from Proposition 3, we know that a cyclical leftward shift in the adjustment bands increases within-sector misallocation. For the same reason as with the aggregate price effect, the aggregate bands effect scales each sector-specific bands effect by the sectoral sales share.

Note that, in general, for a given sectoral/aggregate shock, the four effects do not move in the same direction, resulting in an ambiguous cyclicality of aggregate labor. In particular, we know from Propositions 1 and 2 that productivity shocks that lead to a fall in sectoral marginal costs lead to a reduction in all sectoral prices and leftward shifts in all sectoral bands, implying that the aggregate price effect and the aggregate bands effect move in opposite directions.

However, the ambiguity in aggregate labor cyclicality can be resolved in a specific setting we now consider. First, suppose monetary policy amounts to stabilizing aggregate nominal GDP at one ($P^C C = 1$), implying that money supply is fixed up to first order ($d\log M = 0$). Second, consider perturbations of each of the four effects in the *normalized* menu costs

$\{\rho_i\}_{i=1}^N$. In particular, we assume the normalized costs to be small, in the sense that ρ_i is second order and $\sqrt{\rho_i}$ is first order in the perturbations. The next proposition establishes the first-order response of each of the four components, as well as aggregate labor, in response to a sectoral productivity shock, up to an error that is $\mathcal{O}\left(\sum_i \rho_i^2\right)$:

Proposition 5 (Aggregate labor)**.** *Suppose that monetary policy amounts to fully stabilizing aggregate nominal demand ($P^C C = 1$). Then the four components of the response of aggregate labor supply (labor share) to a productivity shock in a sector k can be approximated as:*

1: $\sum_i \frac{d\lambda}{d\log A_k}(\overline{\mu}_i^{-1} - 1) \approx 0$

2: $-\sum_i \frac{\overline{\lambda}_i}{b_i}\rho_i d\left[\frac{\zeta_i^H - \zeta_i^L}{d\log A_k}\right] \approx -\sum_i \left[\frac{(\epsilon-1)\sqrt{2\epsilon}}{b_i}\tilde{\lambda}_i \Psi_{ik}\right] \rho_i^{1.5}$

3: $\sum_i \overline{\lambda}_i \frac{\varphi_i^P}{b_i}\frac{d\log P_i}{d\log A_k} \approx -\sum_i \left[\frac{2\varepsilon}{b_i}\tilde{\lambda}_i \Psi_{ik}\right]\rho_i^{0.5} - \sum_i \left[\frac{\epsilon^2\varepsilon^3}{3(\epsilon-1)^2 b_i}\tilde{\lambda}_i \Psi_{ik}\right]\rho_i^{1.5}$

4: $-\sum_i \overline{\lambda}_i \left[\frac{\varphi_i^H}{b_i}\frac{d\zeta_i^H}{d\log A_k} + \frac{\varphi_i^L}{b_i}\frac{d\zeta_i^L}{d\log A_k}\right] \approx \sum_i \left[\frac{2\varepsilon}{b_i}\tilde{\lambda}_i \Psi_{ik}\right]\rho_i^{0.5} + \sum_i \left[\frac{\epsilon\varepsilon^2\{(2\epsilon-1)\varepsilon + 3(\epsilon-1)\}}{3(\epsilon-1)^2 b_i}\tilde{\lambda}_i \Psi_{ik}\right]\rho_i^{1.5}$

Combining them:

$$\frac{dL}{d\log A_k} = \frac{d\Theta}{d\log A_k} = \frac{(9\epsilon-4)\varepsilon}{6}\sum_i \left[\frac{\tilde{\lambda}_i \Psi_{ik}}{b_i}\right]\rho_i^{1.5} + \mathcal{O}\left(\sum_i \rho_i^2\right). \tag{47}$$

Let us now analyze the key properties established above. First, the approximation allows us to clearly assess the relative magnitudes and directions of the four effects driving first-order changes in aggregate labor. The pre-existing distortions effect is the smallest one in terms of magnitudes; in fact, it is zero up to $\mathcal{O}\left(\sum_i \rho_i^2\right)$. The menu cost payment effect lowers aggregate labor after any sector-specific positive productivity shock since the latter decreases the fraction of adjusting firms in every sector. As for the aggregate price effect, it similarly has a negative effect on aggregate labor after a positive productivity shock, since the latter lowers sectoral prices, which in turn reduces misallocation in every sector, diminishing the need for extra labor to compensate for it. Moreover, the aggregate price effect is two orders larger than the menu cost payment effect. Finally, the aggregate bands effect is of the same order as the aggregate price effect, although it has a different direction following a productivity shock. In particular, any sector-specific productivity improvement shifts all sectoral adjustment bands leftwards, thus increasing within-sector misallocations and, therefore, making the aggregate labor supply rise to compensate for the extra inefficient allocation of resources across firms.

Second, the proposition above gives an unambiguous prediction regarding the direction of the first-order change in aggregate labor following any productivity shock. Specifically, following any sector-specific productivity improvement, the aggregate labor supply *increases*. This is because the aggregate bands effect dominates over the combination of the aggregate price effect and the menu cost payment effect. However, despite the fact that the aggregate bands effect is $\mathcal{O}\left(\sum_i \rho_i^{0.5}\right)$, the cyclical movements in the aggregate labor are only of the order $\mathcal{O}\left(\sum_i \rho_i^{1.5}\right)$. This is because the two components of the aggregate bands and price effects that are $\mathcal{O}\left(\sum_i \rho_i^{0.5}\right)$ exactly cancel out.

The first-order changes in aggregate labor can also be approximated in terms of sufficient statistics observable in the data:

Corollary 1 (Sufficient Statistic)**.** *Suppose that monetary policy amounts to fully stabilizing aggregate nominal demand ($P^C C = 1$). Denote by* $\alpha_i \equiv \frac{\bar{\zeta}_i^H - \bar{\zeta}_i^L}{b_i}$ *the sector-specific frequencies of non-adjustment, and by* $\sigma_i^2 \equiv Var(\zeta_i)$ *the sector-specific variances of idiosyncratic shocks. Then the first-order change in the aggregate employment (labor share) can be approximated as:*

$$\frac{dL}{d\log A_k} = \frac{d\Theta}{d\log A_k} \approx \quad \xi \times \mathbb{E}_{\tilde{\boldsymbol{\lambda}}}[\boldsymbol{\sigma^2}\boldsymbol{\alpha^3}] \times \mathcal{S}_k \quad + \quad \xi \times \ell \times Cov_{\tilde{\boldsymbol{\lambda}}}[\boldsymbol{\sigma^2}\boldsymbol{\alpha^3}, \Psi_{(k)}] \tag{48}$$

where $\xi \equiv \frac{(9\epsilon-4)\epsilon}{8(\epsilon-1)^2}$, $\quad \ell \equiv \sum_i \tilde{\lambda}_i$, $\quad \boldsymbol{\sigma^2}\boldsymbol{\alpha^3} \equiv [\sigma_1^2\alpha_1^3, ..., \sigma_N^2\alpha_N^3]^T, \Psi_{(k)}$ *is the k'th column of the Leontief inverse and*

$$\mathcal{S} \equiv \Psi^T \tilde{\boldsymbol{\lambda}} = (\Psi^T)^2 \overline{\boldsymbol{\omega}}_{\boldsymbol{c}} \tag{49}$$

*is the **supplier-of-suppliers (SS) centrality**.*

Let us discuss the role played by each component in the sufficient statistic formula. First, aggregate labor increases by more following a positive productivity shock to sectors that have a high *supplier-of-suppliers (SS)* centrality, denoted by $\mathcal{S}_k$. The novel SS-centrality vector is given by the product of the *square* of the (transpose) of the Leontief inverse and the vector of final consumption shares. Intuitively, a sector has a high SS-centrality if it acts as a major supplier to sectors (represented by the first Ψ^T) that in turn act as a major supplier either to other sectors (represented by the second Ψ^T), or to households (represented by the final consumption share vector $\overline{\boldsymbol{\omega}}_{\boldsymbol{c}}$). Second, productivity shocks to any sector have a higher effect on aggregate labor if the economy has a higher (sales share weighted) average of variances of idiosyncratic shock times the cubed frequencies of non-adjustement ($\mathbb{E}_{\tilde{\boldsymbol{\lambda}}}[\boldsymbol{\sigma}^{\mathbf{2}}\boldsymbol{\alpha}^{\mathbf{3}}]$). Third, shocks to a sector have a large effect on aggregate employment if its customers (direct or indirect) have either a larger variance of idiosyncratic shocks or cubed frequency of non-adjustment, as represented by the covariance between the column of Leontief inverse corresponding to the shocked sector and the vector of products of variances of idiosyncratic shocks and cubed frequencies of non-adjustment.

5.3 Aggregate measured TFP and Welfare

We can now derive the first-order movements in aggregate measured TFP, given by $TFP \equiv \frac{C}{L}$ and welfare. In response to a productivity shock to any sector i, the TFP response is given by:

$$\frac{d\log TFP}{d\log A_i} = \frac{d\log C}{d\log A_i} - \frac{d\log L}{d\log A_i} = \tilde{\lambda}_i - \frac{1}{L}\frac{dL}{d\log A_i},$$

where $\frac{dL}{d\log A_i}$ is given in Proposition 5.

Note that from Proposition 5 it follows that in the limit of fully flexible prices ($\rho_i = 0, \forall i$), the first order change in aggregate labor is zero: $\frac{dL^{flex}}{dlogA_i} = 0, \forall i$. Letting $TFP^{gap} \equiv \frac{TFP}{TFP^{flex}}$ be the TFP gap, we can write its first-order change near the baseline as:

$$\frac{d\log TFP^{gap}}{d\log A_i} = -\frac{1}{\overline{\overline{L}}}\frac{dL}{d\log A_i}. \tag{50}$$

It, therefore, follows that all first-order losses in aggregate TFP due to menu costs are captured by the first-order movements in aggregate labor.

We can also characterize *welfare* losses coming from sticky prices driven by the presence of menu costs. More specifically, we can distinguish two elements of these welfare costs: i) the steady-state distortion that is present even without sectoral shocks and ii) the cost of sectoral shocks. The next proposition characterizes both types of costs:

Proposition 6 (Welfare Costs)**.** *Denote δ^{ss} the drop in consumption required to make the household indifferent between living in a flexible-price frictionless economy and the economy* *with menu costs, absent sectoral shocks. Then*

$$\delta^{ss} = 1 - \exp\{1 - \overline{\Theta}\} \tag{51}$$

Where $\overline{\Theta}$ *is given by equation (45) evaluated in the baseline equilibrium.*

Denote δ_i^{bc} *the consumption-equivalent welfare loss associated with the presence of sectoral shock of magnitude* $\Delta \log A_i$ *and menu costs, then*

$$\delta_i^{bc} = 1 - \exp\left\{-\frac{dL}{d\log A_i}\Delta \log A_k\right\}. \tag{52}$$

Notice that the first-order change in aggregate labor is a sufficient statistic for the consumption-equivalent welfare loss due to menu costs. In fact, one can use Corollary 1 to express the welfare loss in terms of observable statistics. In particular:

$$\delta_i^{bc} \approx \left\{\xi \mathbb{E}_{\tilde{\boldsymbol{\lambda}}}[\boldsymbol{\sigma}^{\mathbf{2}}\boldsymbol{\alpha}^{\mathbf{3}}]\mathcal{S}_i + \xi \ell Cov_{\tilde{\boldsymbol{\lambda}}}[\boldsymbol{\sigma}^{\mathbf{2}}\boldsymbol{\alpha}^{\mathbf{3}}, \Psi_{(i)}]\right\} \times \Delta \log A_i. \tag{53}$$

5.4 Properties of SS-centrality $\mathcal{S}$

The sufficient statistic $\mathcal{S}$ provides an easily measurable object to quantify the effect of menu costs on the labor share and, consequently, on TFP. Later in the paper, we study its empirical distribution. For now, to better understand its properties, the following three remarks discuss how it relates to common measures of network importance and its distribution.

Remark 1. *The sufficient statistic* $\mathcal{S}$ *is related to the notion of Upstreamness introduced by Antràs et al. (2012)* $U = \hat{Y}^{-1}(\Psi^T)^2\omega_c$*, where* $\hat{Y}^{-1} = diag(\{y_i^{-1}\}_i)$ *and* y_i *is sector* i *output. Formally,* $\mathcal{S} = U \cdot Y$*.*

Remark 2. *The sufficient statistic* $\mathcal{S}$ *is closely related to the degree distribution. Denote the vector containing the* n^{th} *outdegree as* $D_n = \bar{\Omega}^n B$*. Then the* $\mathcal{S} = \Lambda + \sum_{n=0} nD_n = \sum_{n=0}(n+1)D_n$*.*

Remark 1 shows that the supplier-of-supplier centrality is simply the product of upstreamness and sectoral output. Upstreamness measures the average number of steps of production between a sector and consumption across all paths in the graph. Our sufficient statistic is, therefore, higher when sectors are large and upstream in the production network.

Remark 2 highlights how we can view supplier-of-supplier centrality as a weighted sum of outdegrees, where we give increasingly larger weights to higher-order degrees. Recall that the Domar weight is defined as the sum of all outdegrees of an industry. The fact that TFP and welfare losses are governed by $\mathcal{S}$ rather than the Domar weight Λ highlights again the novel role of higher-order connections as the source of propagation of pricing frictions and productivity shocks.

Remark 3. *Suppose that the distribution of degree D_n follows a Power Law with a tail parameter χ_n, Then, the distribution of $\mathcal{S}$ is governed by a tail parameter $\chi_{\mathcal{S}} = \min\{\chi_1, \ldots, \chi_n\}$.*

Finally, Remark 3 shows that if any of the degree distributions are fat-tailed, then so is the distribution of $\mathcal{S}$. As $\mathcal{S}$ governs the size of welfare costs associated with sectoral shocks, we can potentially have large welfare losses even when menu costs are very small.

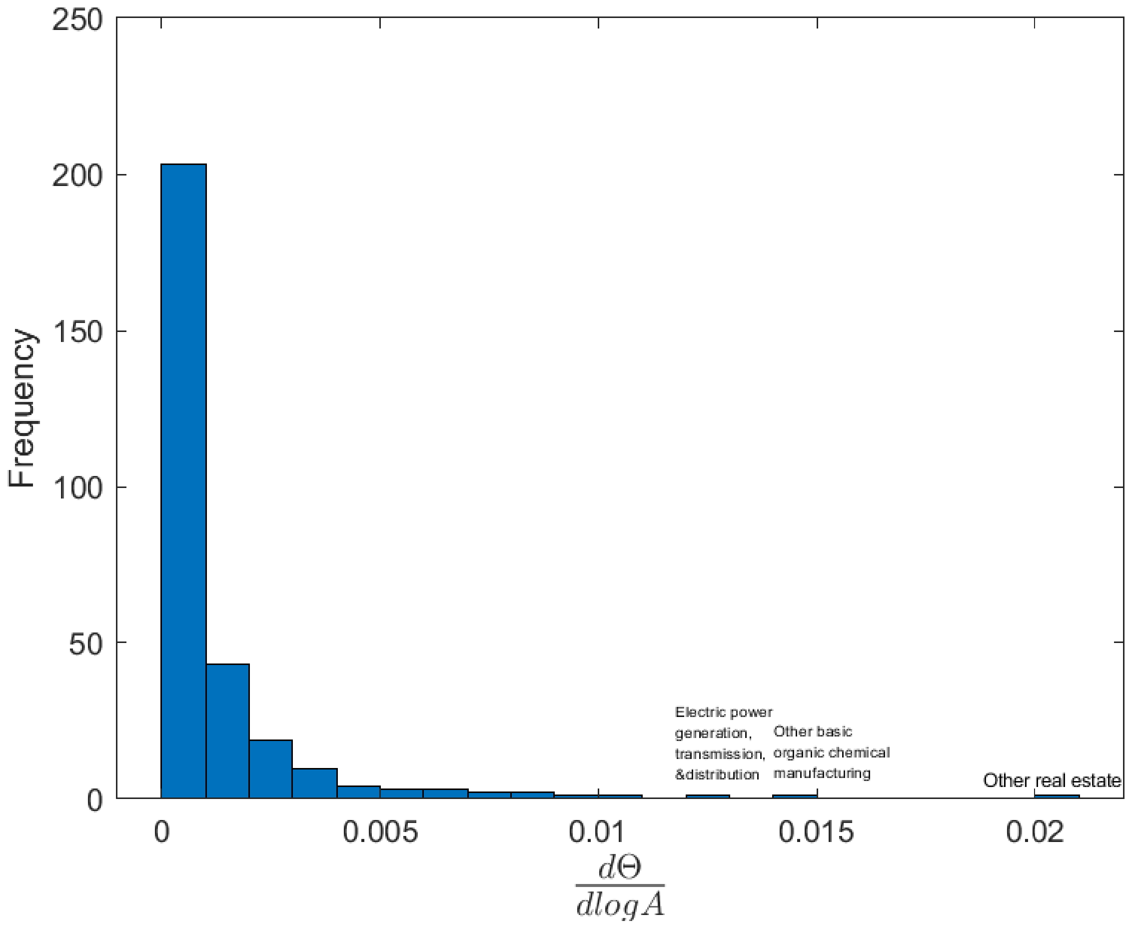
6 Quantitative Evaluation

In this section, we quantitatively explore the properties of our economy. We start by calibrating the model to the US input-output data.

Calibration. We use the 2017 BEA Input-Output Table. We exclude government purchases and redefine the table consistently. Next, we set $\upsilon_k/\lambda_k = .015$, where λ_k is the measured Domar weight so that menu costs represent 0.1% of industry revenues. We set $\epsilon_k = 10$, $\forall k$. We take the sector-specific expenditure share in consumption directly from the I-O Table, where we define consumption as personal and government use. These parameters, jointly with the sector-specific support of the shock distribution b_k, determine the frequency of price resetting for each sector. We use the data on the frequency of price adjustment from Pasten et al. (2020) to back out a vector of b_k. Our data includes 324 sectors for which all these parameters can be estimated. Formally, we use $b_i = \frac{2\sqrt{2\frac{(\epsilon-1)^2}{\epsilon}}\sqrt{\upsilon_i/\lambda_i}}{\alpha_i}$ to back out the vector b_i based on the empirical frequency of non-adjustment a_i. We exclude sectors for which the implied b is larger than 2 as well as sectors with a zero consumption share.[1] After the calibration, we work with 295 sectors.

[1]The exclusion of sectors with no consumption share is motivated by our comparisons with models without networks. In these counterfactuals, a sector's size is only driven by its importance as a supplier to the household; hence, in an economy without a network, sectors with $\omega^c = 0$ would not exist. To maintain a consistent set of sectors in all our experiments, we exclude them from the analysis.

Figure 3: Distribution of $d\Theta/d\log A$



6.1 Results

We provide two sets of quantitative results. First, we describe how our economy behaves in terms of labor share and TFP in response to sectoral shocks. Next, we study the welfare costs associated with sectoral fluctuations in the presence of menu costs and input-output networks.

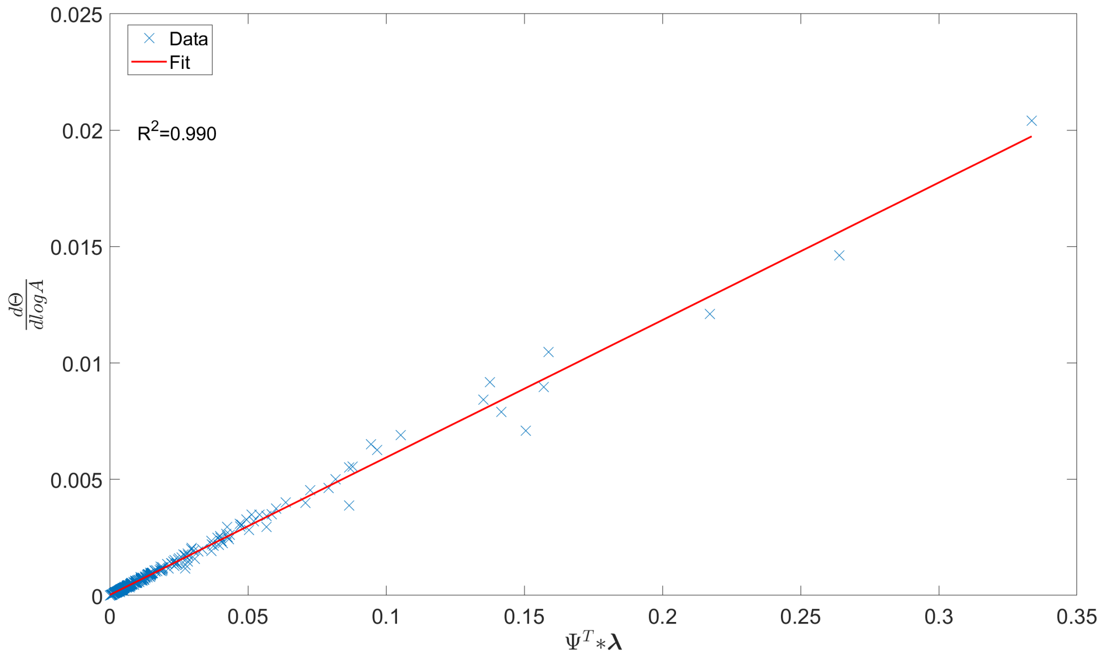
Total Factor Productivity In our economy, changes in TFP in response to a sectoral shock are given by

$$\frac{d\log TFP}{d\log A_i} = \tilde{\lambda}_i - \frac{d\log\Theta}{d\log A_i}.$$

Recall that an economy without menu costs would only have the first term. To summarize the importance of menu costs, Figure 3 shows the distribution of $d\Theta/d\log A$.

The first observation is that the contribution of menu costs to the impact of sectoral

Figure 4: $d\Theta/d\log A$ and SS-centrality $\mathcal{S}$

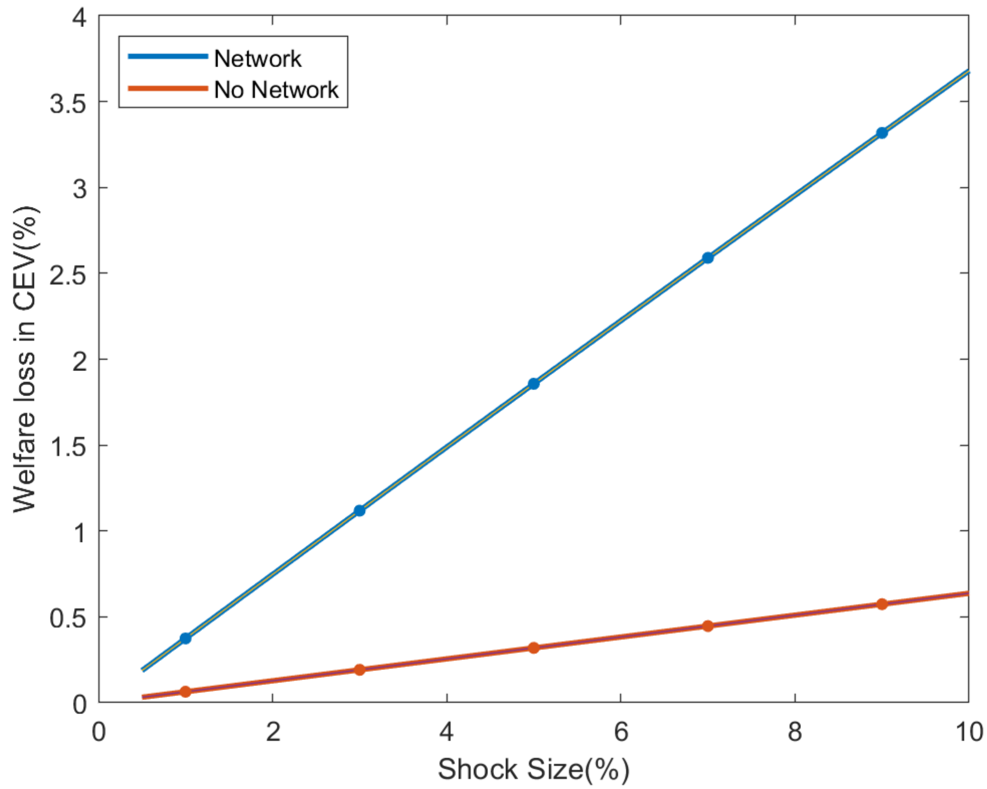


shocks on TFP is itself fat-tailed. This should not be surprising since changes in the labor share Θ are governed by the sufficient statistic $\mathcal{S}$, which itself is the product of two fat-tailed objects. Quantitatively, a 1% increase in sectoral TFP can generate up to 2 percentage points increases in the labor share Θ.

Next, we use the result in Corollary 1. Figure 4 plots the relation between $d\Theta/d\log A$ and our sufficient statistic $\mathcal{S}$. Corollary 1 tells us that the slope is governed by the elasticity ϵ, the frequency of non-adjustment α, and the variance of idiosyncratic shocks σ^2. The intercept is given by the covariance between these and the Leontief element of each sector. Importantly, our calibrated model suggests that the covariance term is negligible as the intercept is very close to zero. As a consequence, our sufficient statistic provides a very good approximation of the distortions associated with sectoral fluctuations.

Welfare We conclude by analyzing the welfare costs of sectoral productivity shocks in our quantitative economy. First, in Figure 5, we plot the consumption equivalent welfare loss associated with sectoral productivity shocks up to 10%. In an economy without a network structure, these shocks induce welfare losses up to approximately 1% CEV. When we apply the same sectoral fluctuations to the network economy, we note that these welfare losses are

Figure 5: Welfare losses following sector-specific shocks



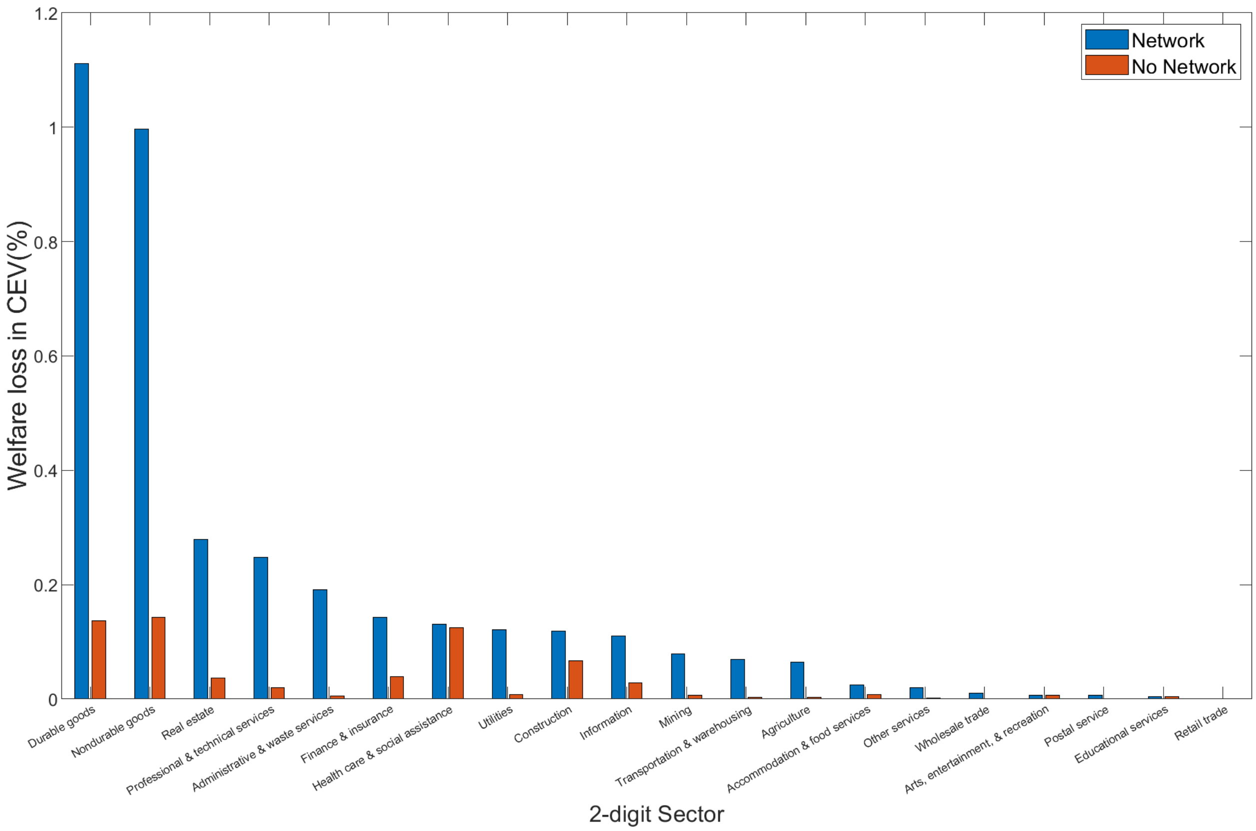
up to 3 times larger.

Finally, we study the sectoral heterogeneity in inducing welfare losses. In Figure 6, we plot the distribution of the welfare losses associated with a 1% change in sectoral productivity for each 2-digit sector in our economy. First, note that the distribution of welfare costs is significantly more heterogeneous in an economy with a network structure, compared to one without it. This is largely driven by a few sectors (durable and non-durable goods), whose contribution to welfare losses increases approximately 6 fold. On the contrary, sectors like health services, which are typically a large component of consumer expenditure but relatively unimportant as suppliers of other industries, do not increase their contribution to welfare losses.

7 Conclusion

We develop an analytically-tractable multi-sector model with a fully general input-output structure and pricing decisions subject to small menu costs. We provide a novel analytic

Figure 6: Distribution of $\mathcal{S}$ by 2-digit sector



aggregation result, which links first-order changes in macroeconomic variables such as GDP, total factor productivity and welfare, to microeconomic shocks, the input-output topology, as well as sector-specific average frequencies and sizes of price adjustment. Crucially, we show that relative to the flexible-price efficient benchmark, input-output linkages amplify the productivity and welfare losses associated with menu costs by an order given by a novel centrality measure, which captures a sector's importance as a *supplier of important suppliers*. This generates a powerful amplification of productivity and welfare losses, since input-linkages create two rounds of misallocation: first, within sectors due to the effect on the location adjustment bands; second, across sectors due to the inefficient reallocation of resources towards the key supplier sectors.

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