University of California, Davis
Department of Economics
Microeconomics

Date: June 22, 2020
Time: 3 hours
Reading Time: 15 minutes

PRELIMINARY EXAMINATION FOR THE Ph.D. DEGREE

==

Answer <u>two</u> of the following three questions

QUESTION 1

It is common knowledge between Countries 1 and 2 that Country 1 plans to attack Country 2. The attack can occur at one of two locations, *C* and *D*. The success or failure of the attack depends on three factors: where Country 1's troops are amassed prior to the attack (near *C* or near *D*), where the attack occurs, and which location is defended. Let x denote the fraction of Country 1's troops amassed near *C*, and $(1-x)$ the fraction of Country 1's troops amassed near *D* ($0 \le x \le 1$). The game is played as follows: **simultaneously**, Country 1 chooses to attack either C or D, and Country 2 decides to defend either C or D. There are only two possible outcomes, from the point of view of Country 1: *Success* and *Failure*. Country 1 strictly prefers Success to Failure and Country 2 has the opposite ranking. The two countries have von Neumann-Morgenstern preferences over lotteries involving these two outcomes. The probability of a successful attack is determined as follows. Let z denote the fraction of Country 1's troops amassed near the location where Country 1 attacks (so $z = x$ if Country 1 attacks at C, and $z = 1-x$ if Country 1 attacks at D). Then the probability of a successful attack is z if Country 2 does not defend the location where Country 1 attacks, and $\frac{1}{2}z$ if Country 2 does defend the location where Country 1 attacks.

For parts (a)-(f) assume that the value of x is fixed and cannot be changed; furthermore, the value of x is common knowledge between the two countries.

(a) Write a strategic-form game that represents the situation described above.

(b) Is there a range of values of x for which Country 1 has a dominant strategy? If so, state the range and specify whether it is strict or weak dominance.

(c) Is there a range of values of x for which Country 2 has a dominant strategy? If so, state the range and specify whether it is strict or weak dominance.

(d) Are there ranges of values of x for which **pure** strategy Nash equilibria exist? If so, indicate the ranges and specify the equilibrium strategies.

(e) Are there ranges of values of x for which a **mixed** strategy Nash equilibrium exists? If so, indicate the ranges and specify the equilibrium strategies and corresponding payoffs.

(f) Draw a graph representing Country 1's payoff at the Nash equilibrium as a function of x.

(g) Now imagine that, instead of being fixed, the value of x is chosen by Country 1. Events occur in the following order:

1. Country 1 decides how many troops to amass near each location (that is, it chooses x).
2. Country 2 observes the deployment of Country 1's troops (that is, it observes x).
3. Simultaneously, Country 1 chooses to attack either *C* or *D*, and Country 2 decides to defend either *C* or *D*.

For each country define: **(g.1)** a pure strategy, **(g.2)** a behavioral strategy.

Question 2

a) Roger lives a simple life: For breakfast, he eats eggs with coffee, and for dinner he eats hot dogs with beer. In between he watches Fox News and earns his money with maintaining a couple of thousand twitter bots. Since he likes everything to be in order and simple, he puts his income into two pots: One with money for breakfast and one with money for dinner. Eggs and coffee are paid only from the breakfast pot; hot dogs and beer only from the dinner pot. That is,

$$\begin{aligned} p_1 x_1 + p_2 x_2 &\leq w_B \\ p_3 x_3 + p_4 x_4 &\leq w_D \end{aligned}$$

with $p_1, p_2, p_3, p_4, w_B, w_D > 0$, where subscripts $1, 2, 3, 4, B$, and D refer to eggs, coffee, hot dogs, beer, breakfast, and dinner, respectively. As usual, p_i is the price of one unit of commodity i, x_i is the quantity consumed of commodity i, and w_B and w_D is the amount of money in his breakfast or dinner pot, respectively.

His utility function is given by

$$u(x_1, x_2, x_3, x_4) = \left(x_1^e x_2^c + x_3^h x_4^b\right)^a$$

with $e, c, h, b, a > 0$.

aa) Given w_B and w_D, derive step-by-step Roger's Walrasian demand functions for eggs, coffee, hot dogs, and beer. Verify also second-order conditions.

ab) While watching Fox News, Roger heard about the government shifting money earmarked for fighting drugs to the construction of the border wall. He suddenly thought whether it would be better for him to move one dollar from his breakfast pot to the dinner pot. Find a condition on the primitives (i.e., parameters e, c, h, b, a, prices p_1, p_2, p_3, p_4, and budgets w_B and w_D) under which moving a dollar from his breakfast pot and putting it in the dinner pot is better for him.

ac) Suppose that the primitives are such that it is better for Roger to move a dollar from his breakfast pot to his dinner pot. Suppose further that both $e + c \geq 1$ and $h + b \geq 1$. Would it be better for Roger to skip breakfast altogether and just spend all the money on dinner?

b) Verify for the case of Cobb-Douglas utility functions on $\mathbb{R}^2_+$ that the Slutsky substitution matrix is negative semidefinite and symmetric.

Question 3

In this question you will argue that the set of competitive equilibrium prices of a competitive economy has essentially no structure other than *closedness*.

Consider a two-commodity world, let prices be normalized to the sphere

$$\mathcal{S} = \{p \in \mathbb{R}^2_{++} \mid \|p\| = 1\},$$

fix $\varepsilon > 0$, and denote

$$\mathcal{S}_\varepsilon = \{p \in \mathcal{S} \mid p_1 \geqslant \varepsilon \text{ and } p_2 \geqslant \varepsilon\}.$$

Fix an arbitrary set $E \subseteq \mathcal{S}_\varepsilon$ and suppose that it is closed. Define the function $Z : \mathcal{S} \to \mathbb{R}^2$ as follows:

(i) for commodity 1,

$$Z_1(p) = \min_{\hat{p}} \{\|\hat{p} - p\| : \hat{p} \in E\}; \tag{1}$$

(ii) and for commodity 2,

$$Z_2(p) = -\frac{p_1}{p_2} Z_1(p). \tag{2}$$

With this construction:

(a) Argue that Z is defined for all $p \in \mathcal{S}$.

(b) Argue that Z is continuous and satisfies Walras's law.[1]

(c) Argue that there exists an exchange economy

$$\{\mathcal{I}, (u^i, w^i)_{i \in \mathcal{I}}\}$$

where each $u^i : \mathbb{R}^2_+ \to \mathbb{R}$ is continuous, locally non-satiated and quasi-concave and such that for all $p \in \mathcal{S}_\varepsilon$,

$$\textstyle\sum_i [x^i(p) - w^i] = Z(p),$$

where

$$x^i(p) = \operatorname{argmax}_x \left\{u^i(x) : p \cdot x \leqslant p \cdot w^i\right\}.$$

(d) Conclude that every $p \in E$ is an equilibrium price vector for that economy.

(e) Use the analysis above to state a theorem to formalize the claim that "that the set of competitive equilibrium prices of a competitive economy has essentially no structure other than closedness".

(f) Suppose that instead of Eq. (1), we let $Z_1(p) = 1$ for all $p \in \mathcal{S}$, with Z_2 still defined by Eq. (2). Argue that the same conclusion of part (c) still applies, but explain why the fact that there is no p for which $Z(p) = 0$ is *not* a counter-example to the Arrow-Debreu existence theorem studied in class.

[1] That is, that for all $p \in \mathcal{S}$, $p \cdot Z(p) = 0$.