

## MACROECONOMICS PRELIM, AUGUST 2020

### ANSWER KEY FOR QUESTION 1

#### Question 1

a) The fact that here  $e_t$  is not constant does not affect the definition of equilibrium (the endowment process is still deterministic). Hence, the definitions of ADE and SME are just like in the lecture notes (in the deterministic version of the model).

b) To characterize ADE prices, we need to set up the ADE problem of the typical agent of island  $i$ . Let  $L_i$  denote that agent's Lagrangian function. Then,

$$L_i = \sum_{t=0}^{\infty} \beta^t \frac{(c_t^i)^\sigma}{1 - \sigma} - \lambda [p_t(e_t^i - c_t^i)],$$

for all  $i$ . Taking FOCs with respect to  $c_t^i$  and  $c_{t+1}^i$  yields:

$$\begin{aligned} \beta^t u'(c_t^i) &= \lambda p_t, \\ \beta^{t+1} u'(c_{t+1}^i) &= \lambda p_{t+1}. \end{aligned}$$

Dividing the two FOCs by parts, yields:

$$\frac{p_{t+1}}{p_t} = \beta \frac{u'(c_{t+1}^i)}{u'(c_t^i)} = \beta \left( \frac{c_t^i}{c_{t+1}^i} \right)^\sigma. \quad (1)$$

Since the last equation holds for  $i = 1, 2$ , we can conclude that

$$\frac{c_t^1}{c_{t+1}^1} = \frac{c_t^2}{c_{t+1}^2} \Rightarrow \frac{c_{t+1}^2}{c_{t+1}^1} = \frac{c_t^2}{c_t^1} \equiv c. \quad (2)$$

The last equation simply states that the ratio of consumption between agents 2 and 1 will be constant in all periods, and we define that ratio as  $c$ . Combining (2) with the feasibility constraint,  $c_t^1 + c_t^2 = e_t$ , implies

$$c_t^1 = \frac{e_t}{1 + c}, \quad (3)$$

$$c_t^2 = \frac{c e_t}{1 + c}. \quad (4)$$

In words, agent 1's consumption is always equal to the fraction  $1/(1 + c)$  of the total endowment, where  $c$  is an unknown to be determined (an analogous statement applies for agent 2, but for that agent the fraction is  $c/(1 + c)$ ). Before we move on to characterizing  $c$ , we need to specify prices. To that end, use either (3) or (4) in (1) to obtain

$$\frac{p_{t+1}}{p_t} = \beta \left( \frac{e_t}{e_{t+1}} \right)^\sigma.$$

Normalizing  $p_0 = 1$  allows us to solve this simple first-order difference equation and obtain the final formula for the ADE prices, which is given by:

$$p_t = \beta^t \left( \frac{e_0}{e_t} \right)^\sigma. \quad (5)$$

c) In this part I give you a very important hint. We have seen numerous times in class that in the simple endowment economy the ADE and SME consumption allocations coincide. Since here I am only asking for the consumption allocation of the SME (not the bond holdings), and since we have already done all this work characterizing the ADE, it would be a waste of time to use any other method: so you should just finish the characterization of the ADE and explain that the consumption allocation we find for the ADE is also the SME consumption allocation.<sup>1</sup>

Given this discussion, all we need is to find the value of the constant  $c$  defined in part b. If we know  $c$ , then equations (3) and (4) fully characterize equilibrium consumption. To find  $c$ , we need to use the agent's budget constraint. Without loss of generality, take agent 1. That agent's BC is:

$$\sum_{t=0}^{\infty} p_t c_t^1 = \sum_{t=0}^{\infty} p_t e_t^1,$$

which, using (5), leads to

$$\frac{1}{1+c} \sum_{t=0}^{\infty} \beta^t \left( \frac{e_0}{e_t} \right)^\sigma e_t = \sum_{t=0}^{\infty} \beta^t \left( \frac{e_0}{e_t} \right)^\sigma e_t^1.$$

Replacing for  $e_t^1$  and after some algebra, we conclude that:

$$\left( \frac{1}{1+c} - a \right) \sum_{t=0}^{\infty} \beta^{2t} e_{2t}^{1-\sigma} = \left( 1 - a - \frac{1}{1+c} \right) \sum_{t=0}^{\infty} \beta^{2t+1} e_{2t+1}^{1-\sigma}. \quad (6)$$

This is one equation in one unknown,  $c$ . Without a specific functional form for  $e_t$  we cannot characterize equilibrium more precisely, but we already know a lot: we know that agent 1 will always consume a constant fraction equal to  $1/(1+c)$  of the aggregate endowment, where  $c$  solves equation (6) above.

d) With the parametric specification  $e_t = \gamma^t e$  the equilibrium characterization can be much sharper, in fact, we can find closed form solutions. Plug  $e_t = \gamma^t e$  into (6), and after some algebra we find that:

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<sup>1</sup> Using another method is not wrong (and you would still get all the points if you did it correctly), but, like I said, it is a waste of time.

$$\frac{1}{1+c} = \frac{a + (1-a)\beta\gamma^{1-\sigma}}{1 + \beta\gamma^{1-\sigma}}.$$

We conclude that the equilibrium (ADE or SME, they are the same) consumption allocation is given by

$$c_t^1 = \frac{a + (1-a)\beta\gamma^{1-\sigma}}{1 + \beta\gamma^{1-\sigma}} \gamma^t e, \quad (7)$$

$$c_t^2 = \frac{1 - a(1 - \beta\gamma^{1-\sigma})}{1 + \beta\gamma^{1-\sigma}} \gamma^t e. \quad (8)$$

e) When the endowment does not grow over time, i.e.,  $\gamma = 1$ , neither does the agents' consumption. So we have  $c_t^i = c^i$ , and, more specifically,

$$c^1 = \frac{a + (1-a)\beta}{1 + \beta} e,$$

$$c^2 = \frac{1 - a(1 - \beta)}{1 + \beta} e,$$

for all  $t$ . If, moreover, we have  $a = 1$ , these formulas reduce to

$$c^1 = \frac{1}{1 + \beta} e,$$

$$c^2 = \frac{\beta}{1 + \beta} e.$$

This should not be surprising; it is just the result we saw in the lecture, since there we had assumed that agent 1 takes all the endowment in even periods, and agent 2 takes all the endowment in odd periods, which is just another way of saying that  $a = 1$ .

f) Take any period  $t$ . From (7) and (8), in that period we will have  $c_t^1 > c_t^2$ , if and only if

$$\frac{a + (1-a)\beta\gamma^{1-\sigma}}{1 + \beta\gamma^{1-\sigma}} \gamma^t e > \frac{1 - a(1 - \beta\gamma^{1-\sigma})}{1 + \beta\gamma^{1-\sigma}} \gamma^t e.$$

After some algebra and terms canceling out, this simplifies to the intuitive condition:

$$a > \frac{1}{2}.$$

g) First, let's calculate ADE prices for  $e_t = \gamma^t e$ . Equation (5) implies that

$$p_t = \beta^t \left( \frac{e_0}{e_t} \right)^\sigma = \beta^t \left( \frac{e}{\gamma^t e} \right)^\sigma = (\beta\gamma^{-\sigma})^t.$$

Thus, the price sequence will be increasing if and only if

$$\gamma < \beta^{\frac{1}{\sigma}}.$$

This is intuitive. Prices tend to increase in  $t$  when  $\beta$  is large (agents are patient) and  $\gamma$  is small (supply falls in the future, the fruit is scarce, prices go up). The condition derived above says that for  $p_t$  to be increasing in  $t$ , we need a combination of a low  $\gamma$  and a high  $\beta$ . Interestingly, the curvature of the agent's utility function also plays a role here. If we had assumed logarithmic utility, the condition would reduce simply to

$$\gamma < \beta.$$

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**PRELIMINARY EXAMINATION FOR THE Ph.D. DEGREE**

**MACROECONOMICS: 200E Question Answer Key**

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**Question 2 (50 points)**

This question considers the macroeconomic effects of a collapse in consumer demand in the New Keynesian model.

There are a continuum of identical households. The representative household makes consumption ( $C$ ) and labor supply ( $N$ ) decisions to maximize lifetime expected utility:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left( \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\psi}}{1+\psi} \right) Z_t \quad (1)$$

subject to their budget constraint:

$$C_t + B_t = w_t N_t + (1 + i_{t-1}) \frac{P_{t-1}}{P_t} B_{t-1} + D_t \quad (2)$$

where  $w_t$  is the real wage,  $N_t$  is hours worked,  $B_t$  are real bond holdings at the end of period  $t$ ,  $i_{t-1}$  is the nominal interest rate paid between  $t-1$  and  $t$ ,  $P_t$  is the price of the final consumption good and  $D_t$  are real profits from firms that are distributed lump sum. As usual,  $0 < \beta < 1$ ,  $\psi > 0$  and  $\sigma > 0$ .  $Z_t$  is a household preference shock, which is a way of generating variations in consumer demand.

The production side of the model is the standard New Keynesian environment. Monopolistically competitive intermediate goods firms produce an intermediate good using labor. Intermediate goods firms face a probability that they cannot adjust their price each period (the Calvo pricing mechanism). Intermediate goods are aggregated into a final (homogenous) consumption good by final goods firms. The production side of the economy, when aggregated and linearized, can be described by the following set of linearized equilibrium conditions (the production function, the optimal hiring condition for labor and the dynamic evolution of prices):

$$\hat{y}_t = \hat{n}_t \quad (3)$$

$$\hat{w}_t = \hat{m}c_t \quad (4)$$

$$\hat{\pi}_t = \beta E_t(\hat{\pi}_{t+1}) + \lambda \hat{m}c_t \quad (5)$$

The resource constraint is:  $\hat{y}_t = \hat{c}_t \quad (6)$

Monetary policy follows a simple Taylor Rule:  $\hat{i}_t = \phi_\pi \hat{\pi}_t \quad (7)$

The (linearized) preference shock follows an AR(1) process:  $\hat{z}_t = \rho \hat{z}_{t-1} + e_t \quad (8)$

$e_t$  is i.i.d. In percentage deviations from steady state:  $\hat{m}c_t$  is real marginal cost,  $\hat{c}_t$  is consumption,  $\hat{w}_t$  is the real wage,  $\hat{n}_t$  is hours worked,  $\hat{y}_t$  is output. In deviations from

steady state:  $\hat{i}_t$  is the nominal interest rate,  $\hat{\pi}_t$  is inflation.  $\lambda$  is a function of model parameters, including the degree of price stickiness.<sup>1</sup> Assume that  $\phi_\pi > 1$ ,  $0 < \rho < 1$ .

a) First consider the representative household's problem. Write down the household's problem in *recursive form* and derive the household's first order conditions.

**Answer**

$$V(B_{t-1}, Z_t, \mathcal{B}_{t-1}) = \max_{C_t, B_t, N_t} \left\{ \left( \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\psi}}{1+\psi} \right) Z_t + \beta E_t V(B_t, Z_{t+1}, \mathcal{B}_t) \right\} \quad (9)$$

subject to

$$C_t + B_t = w_t N_t + (1 + i_{t-1}) \frac{P_{t-1}}{P_t} B_{t-1} + D_t \quad (10)$$

where  $\mathcal{B}$  are aggregate bond holdings. Setting this up as a Lagrangian (using  $\lambda_t$  as the multiplier), we can derive the first order conditions:

$$C_t^{-\sigma} Z_t = \lambda_t \quad (11)$$

$$N_t^\psi Z_t = \lambda_t w_t \quad (12)$$

$$\lambda_t = \beta E_t \frac{\partial V}{\partial B_t} \quad (13)$$

Using the envelope condition (take partial derivative of the value function today wrt  $B_{t-1}$  and shift forward one period):

$$\lambda_t = \beta E_t \lambda_{t+1} (1 + i_t) \frac{P_t}{P_{t+1}} \quad (14)$$

b) Show that the linearized first order condition for labor supply from part (a) is:

$$\hat{w}_t = \sigma \hat{c}_t + \psi \hat{n}_t \quad (15)$$

and show that under flexible prices demand shocks have no effect on real GDP.

**Hints:** you will need to use equations (3), (4), (6) and (17).

**Answer**

After combining equations 11 and 12 the easiest way to linearize this is to take logs and then subtract the same expression evaluated at the steady state:

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<sup>1</sup>  $\lambda = \frac{(1-\theta)(1-\beta\theta)}{\theta}$  where  $\theta$  is the probability that a firm cannot adjust its price.

$$\ln w_t - \ln w = \psi(\ln N_t - \ln N) + \sigma(\ln C_t - \ln C) \quad (16)$$

Using hat notation to denote percentage deviations from steady state yields the equation in the question:

$$\hat{w}_t = \sigma \hat{c}_t + \psi \hat{n}_t \quad (17)$$

Next, note that

$$\hat{w}_t = \hat{m}c_t \quad (18)$$

Furthermore, under flexible prices,  $\hat{m}c_t = 0$ . Because all firms are free to set the same price, there is no markup dispersion and  $\hat{m}c_t = 0$ . Also making use of the production function and the resource constraint yields:

$$0 = \sigma \hat{y}_t + \psi \hat{y}_t \quad (19)$$

Solving for  $\hat{y}_t$  and putting a superscript  $n$  to denote the level of real GDP under the assumption of flexible prices yields:

$$\hat{y}_t^n = 0 \quad (20)$$

The flexible price outcome for real GDP is not affected by demand shocks  $Z$ . This is because increases in demand raise prices and real interest rates but leave real GDP unchanged. Since the natural rate of output is always equal to its steady state level, it also follows that this is unaffected by monetary policy.

c) This model can be reduced to three equations:

$$E_t \tilde{y}_{t+1} - \tilde{y}_t = \frac{1}{\sigma} (\hat{i}_t - E_t \hat{\pi}_{t+1}) + \frac{1}{\sigma} E_t (\hat{z}_{t+1} - \hat{z}_t) \quad (21)$$

$$\hat{\pi}_t = \beta E_t (\hat{\pi}_{t+1}) + \kappa \tilde{y}_t \quad (22)$$

$$\hat{i}_t = \phi_\pi \hat{\pi}_t \quad (23)$$

where  $\hat{z}_t$  follows the process in equation (8) and  $\kappa = (\sigma + \psi)\lambda$ .

Using the method of undetermined coefficients, find the response of the output gap and inflation to a collapse in consumer demand when prices are sticky and monetary policy follows the Taylor Rule above. To do this, guess that the solution for each variable is a linear function of the shock  $\hat{z}_t$ .

**Answer:**

Let's guess:

$$\hat{\pi}_t = \Lambda_\pi \hat{z}_t$$



$$\tilde{y}_t = \Lambda_y \hat{z}_t$$

Substitute the guesses into the NKPC and make use of the stochastic process to substitute for the  $E_t \hat{z}_{t+1}$  terms. Solving for  $\Lambda_y$  in terms of  $\Lambda_\pi$  yields:

$$\Lambda_y = \frac{\Lambda_\pi(1 - \beta\rho)}{\kappa}$$

Next, substitute the guesses and the monetary policy rule into the dynamic IS curve, and make use of the stochastic process to substitute for the  $E_t \hat{z}_{t+1}$  terms. Also make use of the expression for  $\Lambda_y$  that we just derived. Solve for  $\Lambda_\pi$ :

$$\Lambda_\pi = \kappa \left( \sigma(1 - \beta\rho) + \frac{\kappa(\phi_\pi - \rho)}{1 - \rho} \right)^{-1} > 0$$

Combining this with the solution for  $\Lambda_y$  we found above:

$$\Lambda_y = (1 - \beta\rho) \left( \sigma(1 - \beta\rho) + \frac{\kappa(\phi_\pi - \rho)}{1 - \rho} \right)^{-1} > 0$$

If  $\hat{z}_t$  falls both output and inflation fall.

d) Discuss how, and why, a fall in demand affects the natural rate of output, the output gap and inflation in this model. Briefly comment on how a decrease in  $\hat{z}_t$  relates to typical recessions we see in the data.

**Answer:**

A negative shock to consumer preferences is like a shock to demand. It leads to a fall in output and inflation. Why? As consumer demand falls some firms cannot adjust their price (sticky prices come from the Calvo pricing mechanism). Some firms cut price and some firms cut output. As a result both prices and output fall. If prices were not sticky there would be a fast adjustment of prices by all firms and output would remain unchanged. This was shown in part (b) where the natural rate of output was unaffected by the demand shock. This accords with common views of recessions — a collapse in demand that leads to a fall in output and inflation.

The  $\Lambda$  terms also make sense. A higher  $\kappa$  — more flexible prices — raises the effect on inflation and lowers the effect on output. As  $\phi_\pi$  rises the response of output and inflation gets smaller. This makes sense because  $\phi_\pi$  is the coefficient in the monetary policy rule, a higher coefficient implies more aggressive policy.

e) Instead of following the Taylor Rule above, policy is now set optimally. Derive the optimal monetary policy rule under *discretionary policy*. (**Hint:** As in class, assume

that the loss function has quadratic terms for the output gap and inflation, with a relative weight  $\vartheta$  on the output gap. For simplicity, assume the steady state is efficient). Using this rule and your knowledge of the model, what is the optimal path for the output gap and inflation in response to a preference shock? Would your answer change if the central bank followed an optimal policy rule under commitment? Explain. (You do not need to derive anything for these last two discussion questions).

**Answer:**

Optimal policy under discretion means the policy maker resets policy choices each period. We are told to assume an efficient steady state, so  $\tilde{y}$  appears in the loss function. The policymaker therefore solves a static problem:

$$\min_{\tilde{y}, \pi} \frac{1}{2} (\hat{\pi}_t + \vartheta \tilde{y}_t)$$

subject to the Phillips Curve. If we denote the Lagrange multiplier on the constraint as  $\xi_t$ , the first order conditions for inflation and the output gap are:

$$\hat{\pi}_t + \xi_t = 0$$

$$\vartheta \tilde{y}_t - \kappa \xi_t = 0$$

Combining these equations leads to a targeting rule that keeps the output gap proportional to inflation:

$$\hat{\pi}_t = -\frac{\vartheta}{\kappa} \tilde{y}_t$$

There are no trade-off shocks in the Phillips Curve so it is possible to close the output gap and the inflation gap at the same time. 0 is a solution to this policy rule, it minimizes the loss function and is consistent with the IS and PC equations. Optimal policy therefore completely offset the recession in parts (c) and (d).

Commitment policy is able to shape expectations of future inflation. When there is a trade-off, making promises about the future can improve outcomes. Because the Phillips Curve and the IS curve are forward looking, expectations of the future affect outcomes today. In this model, however, there is no trade off. Discretionary policy is able completely stabilize the output gap and inflation. There is no additional benefit to commitment policy in this model (as long as there is an efficient steady state and no cost push shocks).

# Prelim 2020 - Retake: Answer Key

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## Question 3

1. The problem of a farmer is

$$\begin{aligned} \max E \left[ \sum_{t=0}^{\infty} -\beta^t \frac{\exp(-\gamma c_t)}{\gamma} \right] \\ \text{s.t. } c_t + k_{t+1} + a_{t+1} \leq y_t + (1 + r^K - \delta)k_t + (1 + r)a_t \\ \lim_{t \rightarrow \infty} \frac{a_t}{(1 + r)^t} \geq 0 \end{aligned}$$

Since farmers can produce capital at a cost of 1, the equilibrium price cannot be higher than 1. If the price was lower than 1, no farmer would produce capital and eventually the supply of capital would go to zero. Hence, the price of capital must be equal to 1.

Let  $\lambda_t$  be the Lagrange multiplier associated to the budget constraint. The FOCs are

$$\begin{aligned} (c_t) : \quad & \beta^t \exp(-\gamma c_t) = \lambda_t \\ (k_{t+1}) : \quad & E_t[(1 + r^K - \delta)\lambda_{t+1}] \leq \lambda_t \\ (a_{t+1}) : \quad & E_t[(1 + r)\lambda_{t+1}] = \lambda_t \end{aligned}$$

Since some farmers have to hold the capital in equilibrium, they need to be indifferent between holding capital or bonds. Hence

$$r = r^K - \delta.$$

2. The farmers' intertemporal Euler equation is

$$\exp(-\gamma c_t) = \beta(1 + r)E_t[\exp(-\gamma c_{t+1})]$$

or

$$1 = \beta(1+r)E_t[\exp(-\gamma(c_{t+1} - c_t))]$$

Let's use the guess. We have

$$\begin{aligned} c_{t+1} - c_t &= \underline{c} + r \left( \underbrace{a_{t+1} + k_{t+1}}_{y_t + (1+r)(k_t + a_t) - c_t} \right) + \frac{r}{1+r} y_{t+1} - c_t \\ &= \underline{c} + r(y_t + (1+r)(k_t + a_t) - c_t) + \frac{r}{1+r} y_{t+1} - c_t \\ &= \underline{c} + r(y_t + (1+r)(k_t + a_t)) + \frac{r}{1+r} y_{t+1} - (1+r)c_t \\ &= \underline{c} + r(y_t + (1+r)(k_t + a_t)) + \frac{r}{1+r} y_{t+1} - (1+r) \left( \underline{c} + r(a_t + k_t) + \frac{r}{1+r} y_t \right) \\ &= -r\underline{c} + \frac{r}{1+r} y_{t+1} \end{aligned}$$

Hence, the FOC is

$$1 = \beta(1+r)E_t[\exp(\gamma r \underline{c} - \frac{\gamma r}{1+r} y_{t+1})]$$

or

$$1 = \beta(1+r) \exp \left( \gamma r \underline{c} - \frac{\gamma r}{1+r} \bar{y} + \frac{1}{2} \left( \frac{\gamma r}{1+r} \right)^2 \sigma^2 \right)$$

After some algebra, we get

$$\underline{c} = -\frac{1}{\gamma r} \log[\beta(1+r)] + \frac{\bar{y}}{1+r} - \frac{1}{2} \frac{\gamma r}{(1+r)^2} \sigma^2$$

3. We have

$$C^F = \int c_f df = \underline{c} + r \left( \underbrace{\int a_f df}_{=0} + \underbrace{\int k_f df}_{=K^F} \right) + \frac{r}{1+r} \bar{y} = \underline{c} + rK^F + \frac{r}{1+r} \bar{y}.$$

Integrating the budget constraint, we get

$$C^F + K^F = \bar{y} + (1+r)K^F$$

For both expressions to hold, we need

$$\underline{c} + rK^F + \frac{r}{1+r}\bar{y} = \bar{y} + rK^F$$

or

$$\frac{\bar{y}}{1+r} = \underline{c} = -\frac{1}{\gamma r} \log[\beta(1+r)] + \frac{\bar{y}}{1+r} - \frac{1}{2} \frac{\gamma r}{(1+r)^2} \sigma^2$$

and hence

$$\log[\beta(1+r)] = -\frac{1}{2} \left( \frac{\gamma r}{1+r} \right)^2 \sigma^2 < 0$$

which implies

$$\beta(1+r) < 1$$

4. The problem of a worker is

$$\max E_t \left[ \sum_{t=0}^{\infty} \beta^t u(c_t) \right] \text{ s.t. } c_t + k_{t+1} \leq w_t + (1+r)k_t$$

5. Let  $\lambda_t$  be the Lagrange multiplier associated to the budget constraint. The FOCs of the worker's problem are

$$\begin{aligned} (c_t) : \quad & \beta^t u'(c_t) = \lambda_t \\ (k_{t+1}) : \quad & E_t[(1+r)\lambda_{t+1}] \leq \lambda_t \end{aligned}$$

If  $c_t$  constraint, then  $\lambda_t = \beta^t$ , and the intertemporal Euler equation simplifies to

$$\beta(1+r) \leq 1$$

Since  $\beta(1+r) < 1$  in equilibrium,  $k^W = 0$ .

Hence, putting together the problem of the farmers and workers, we can conclude that there exists an equilibrium with constant aggregate farmers consumption and capital holdings, constant individual workers' consumption, workers hold zero capital, and the interest rate and wage are constant.

6. From question 3., one can see that if  $\sigma^2$  increases,  $r$  decreases. Moreover,

$$\frac{\partial \underline{c}}{\partial r} = -\frac{\bar{y}}{(1+r)^2} < 0$$

(since  $\underline{c} = \frac{\bar{y}}{1+r}$ ), hence  $c(a, k, y)$  increases for all  $(a, k, y)$  and therefore  $C^F(K^F)$  increases for all  $K^F$ . From the budget constraint

$$C^F = \bar{y} + rK^F$$

Since  $r$  decreases with the increase in  $\sigma^2$ , it must be true that  $K^F$  increases. But if  $K^F$  increases, the wage  $w$  increases, and hence the welfare of workers increases.