

University of California, Davis
Department of Economics

PRELIMINARY EXAMINATION FOR THE Ph.D. DEGREE

MACROECONOMICS

July 1, 2019

Directions: The exam consists of six questions. Questions 1,2 concern ECN 200D (Geromichalos), questions 3,4 concern ECN 200E (Cloyne), and questions 5,6 concern ECN 200F (Caramp). You only need to answer five out of the six questions. If you prefer (and have time), you can answer all six questions, and your grade will be based upon the best five scores. Feel free to impose additional structure on the problems below, but please state your assumptions clearly. You have 5 hours to complete the exam and an additional 20 minutes of reading time.

Question 1 (20 points)

Consider the standard neoclassical growth model in discrete time. There is a large number of identical households normalized to 1. Each household wants to maximize life-time discounted utility

$$U(\{c_t\}_{t=0}^{\infty}) = \sum_{t=0}^{\infty} \beta^t u(c_t), \quad \beta \in (0, 1).$$

Each household has an initial capital k_0 at time 0, and one unit of productive time in each period that can be devoted to work. Final output is produced using capital and labor, according to a CRS production function F . This technology is owned by firms (whose measure does not really matter because of the CRS assumption). Output can be consumed (c_t) or invested (i_t). Households own the capital (so they make the investment decision), and they rent it out to firms. Let $\delta \in (0, 1)$ denote the depreciation rate of capital. Households own the firms, i.e., they are claimants to the firms' profits, but these profits will be zero in equilibrium.

The function u is twice continuously differentiable and bounded, with $u'(c) > 0$, $u''(c) < 0$, $u'(0) = \infty$, and $u'(\infty) = 0$. Regarding the production technology, we will introduce the useful function $f(x) \equiv F(x, 1) + (1 - \delta)x$, $\forall x \in \mathbb{R}_+$. The function f is twice continuously differentiable with $f'(x) > 0$, $f''(x) < 0$, $f(0) = 0$, $f'(0) = \infty$, and $f'(\infty) = 1 - \delta$.

In this model the government taxes households' investment at the constant rate $\tau \in [0, 1]$. The government returns all the tax revenues, T , to the households in the form of lump-sum transfers. Throughout this question focus on recursive competitive equilibrium (RCE).

a) Write down the problem of the household recursively.¹ Carefully distinguish between aggregate and individual state variables. Then, define a RCE.

b) Write down the dynamic equation that the aggregate capital stock follows in this economy.²

c) Now focus on steady-states. Describe the steady-state equilibrium value of the aggregate capital stock in this economy, and denote it by $K^*(\tau)$.

d) Describe the value of K^* when $\tau = 0$ and when $\tau = 1$.

¹ Here firms face a static problem. I am not asking you to explicitly spell it out, but it will be critical for correctly defining a RCE.

² Hint: Obtain the Euler equation for the typical household and impose the RCE conditions. I recommend you express the equilibrium condition(s) in terms of the function f , rather than F . This will make life easier in the forthcoming parts.

e) In class, we studied the RCE steady state level of capital in an economy where the government taxed the income from renting capital (as opposed to investment, which is the case here). In that model, we saw that for $\tau = 1$ the equilibrium capital stock reached zero. Based on your answer to part (d), does this also happen here? Provide an intuitive explanation of why (or why not).

f) Focus on a Cobb-Douglas production function, i.e., let $F(K, N) = K^a N^{1-a}$, $a \in (0, 1)$. Provide a closed-form solution for $K^*(\tau)$.

g) Again using a Cobb-Douglas production function, calculate the government's total tax revenue, T , and plot it as a function of the tax rate τ (i.e., plot the Laffer curve). Which value of τ maximizes tax revenues?

Question 2 (20 points)

This question studies the co-existence of money and credit. Time is discrete with an infinite horizon. Each period consists of two subperiods. In the day, trade is bilateral and *partially* anonymous as in Kiyotaki and Wright (1991) (call this the KW market). At night trade takes place in a Walrasian or centralized market (call this the CM). There are two types of agents, buyers and sellers, and the measure of both is normalized to 1. The per period utility for buyers is $u(q) + U(X) - H$, and for sellers it is $-q + U(X) - H$, where q is the quantity of the day good produced by the seller and consumed by the buyer, X is consumption of the night good (the numeraire), and H is hours worked in the CM. In the CM, all agents can turn one unit of labor into a unit of good. The functions u, U satisfy the usual assumptions; I will only spell out the most crucial ones: There exists $X^* \in (0, \infty)$ such that $U'(X^*) = 1$, and we define the first-best quantity traded in the KW market as $q^* \equiv \{q : u'(q^*) = 1\}$.

Here we will assume that there are two types of sellers. Type-1 sellers, with measure $1 - \sigma$, never accept unsecured credit. Hence, in any meeting with this type of seller, the buyer must pay on the spot (*quid pro quo*) with money. In contrast, type-2 sellers, with measure σ , accept money but they also accept unsecured credit, in the form of a promise by the buyer to repay the seller in the forthcoming CM with numeraire good. However, there is a credit limit: the buyer can credibly promise to repay only an amount up to $C < q^*$.³

The rest is standard. Goods are non storable. There exists a storable and recognizable object, fiat money, that can serve as a medium of exchange. Money supply is controlled by a monetary authority, and we consider simple policies of the form $M_{t+1} = (1 + \mu)M_t$, $\mu > \beta - 1$. New money is introduced, or withdrawn if $\mu < 0$, via lump-sum transfers to buyers in the CM. Let ϕ denote the unit price of money (in terms of the numeraire). In KW meetings buyers have all the bargaining power.

a) Describe the CM value function of the typical buyer. (Be sure to correctly identify the state variables.) Show that this value function is linear in its argument(s).

b) Describe the CM value functions of the typical seller of type 1 and 2. As in part (a), show that these value functions are linear.

c) Let q_j denote the quantity of good and d_j the amount of money exchanged in a type- $j = 1, 2$ meeting in the KW market. Describe the bargaining solution in a typical

³ Here is a summary of the environment. All buyers meet a seller in the KW market. With probability $1 - \sigma$ that seller is of type-1; if that happens the buyer must use money. With probability σ the representative buyer meets a type-2 seller; if that happens, the buyer can use credit, but only up to the amount C , which, importantly, is less than q^* . Thus, money is useful even for buyers who end up meeting sellers who accept credit.

type- j meeting.⁴

At this point, I highlight an important detail of the model that will help you answer the remaining questions. As we have already discussed, here money is useful both for buyers who end up meeting type-1 sellers and for those who meet type-2 sellers, but, clearly, it is more useful for the former (because these buyers can only use money). Therefore, there exists an amount of money holdings, let's call it m_2^* , such that a buyer who carries m_2^* or more units of money can afford to purchase q^* if she meets a type-2 seller, but she *cannot* afford q^* if she meets a type-1 seller.

d) Describe the objective function of the representative buyer, $J(m')$, where the prime denotes next period's choices.⁵

e) Describe the equilibrium real balances, $z = \phi m = \phi M$, as a function of the nominal interest rate, i .⁶

f) Based on your work in part (e), describe the equilibrium values q_1, q_2 , i.e., the amount of day good exchanged in the two types of meetings.

Finally, define the welfare function of this economy as the measure of the various KW market meetings times the net surplus generated in each meeting, i.e.,

$$\mathcal{W} = \sigma[u(q_2) - q_2] + (1 - \sigma)[u(q_1) - q_1].$$

Also, assume that the KW utility function is quadratic, i.e., $u(q) = (1 + \gamma)q - \frac{q^2}{2}$. (This will allow you to find closed form solutions for all the equilibrium objects.)

g) Intuition suggests that welfare should be increasing in the credit limit, C , i.e., $\partial \mathcal{W} / \partial C > 0$. Is this intuition (always) correct in this model and why?

⁴ **Hint:** Feel free to analyze the bargaining problems in detail, but I think that with a little intuitive thinking you can answer this question in a couple of minutes. In particular, you should think how the amount of money exchanged in a type-2 meeting is affected by the exogenous credit limit C .

⁵ **Hint:** Here you should feel free to skip all the (formal) work that leads to the objective function, and simply guess the form of J . Notice that for the determination of J , it is important to distinguish between two cases: $m' \geq m_2^*$ and $m' < m_2^*$.

⁶ **Hint:** In the previous hint, I explained that the J function will behave differently around the point m_2^* . This property will also be reflected in the demand for real balances. More precisely, there will be a critical level of i , call it \bar{i} , such that for $i \leq \bar{i}$ buyers who meet type-2 sellers will be able to purchase q^* (but that will not be true for buyers who meet type-1 sellers). What is that critical level \bar{i} , and how is z affected by whether i is less or greater than \bar{i} ?

Question 3 (20 points)

Consider the following decentralized business cycle model. The representative household makes consumption (C) decisions to maximize lifetime expected utility:

$$E_0 \sum_{t=0}^{\infty} \beta^t (\ln C_t) \quad (1)$$

subject to their budget constraint:

$$C_t + I_t = w_t N_t + r_t^k K_t + \Pi_t \quad (2)$$

where w is the real wage, N is hours worked, K is capital, I is investment, r^k is the rental price of capital and Π are profits from firms. As usual, $0 < \beta < 1$. Assume that labor is supplied inelastically so $N_t = 1$. There are adjustment costs to capital. As a result, capital evolves according to the following capital production function:

$$K_{t+1} = K_t^\delta I_t^{1-\delta} \quad (3)$$

When $\delta = 0$, this becomes the simple model we saw in class with full depreciation (i.e. where $K_{t+1} = I_t$).

Competitive firms produce output, Y_t , using capital and labor. The production function is given by:

$$Y_t = Z_t K_t^\alpha (N_t)^{1-\alpha} \quad (4)$$

where Total Factor Productivity, Z_t , is stochastic and follows a Markov process. There is no trend growth.

a) Write down the household's problem in recursive form and write down the firm's maximization problem. Derive the household's first order conditions and the firm's optimal hiring rules. For households, use two constraints: the budget constraint and the capital production function. Denote the Lagrange multiplier on the budget constraint as λ_t and the one on the capital production constraint as $\lambda_t q_t$ (where q_t is the price of capital in terms of consumption goods).

b) Carefully define a recursive competitive equilibrium.

c) Show that the equilibrium conditions in parts (a) and (b) are equivalent to the following equilibrium conditions from the social planner's problem:⁷

$$q_t = \beta E_t \left[\frac{C_t}{C_{t+1}} \left(\alpha \frac{Y_{t+1}}{K_{t+1}} + \frac{\delta}{1 - \delta} \frac{I_{t+1}}{K_{t+1}} \right) \right]$$

$$q_t = \frac{1}{(1 - \delta)} \frac{I_t}{K_{t+1}}$$

$$Y_t = C_t + I_t$$

$$Y_t = Z_t K_t^\alpha$$

d) Solve this model using guess and verify. In particular, find the policy functions for K_{t+1} , I_t and C_t (**Hint:** start by guessing that investment and consumption are a constant share of output). Briefly discuss how, and why, the responses of consumption and investment to TFP shocks vary with δ .

e) Briefly explain how you would solve this model computationally using a linearization-based technique. Give one advantage of this method over Value Function Iteration.

⁷You do not need to set-up and solve the social planner's problem.

Question 4 (20 points)

This question considers a time-varying inflation target in the New Keynesian model. Consider the following set of linearized equilibrium conditions for the standard New Keynesian model, but where the policy rule includes a stochastic inflation target $\bar{\pi}_t$.

$$E_t \tilde{y}_{t+1} - \tilde{y}_t = (\hat{i}_t - E_t \hat{\pi}_{t+1}) \quad (5)$$

$$\hat{\pi}_t = \beta E_t(\hat{\pi}_{t+1}) + \kappa \tilde{y}_t \quad (6)$$

Monetary policy follows a rule:

$$\hat{i}_t = \hat{\pi}_t + \phi(\hat{\pi}_t - \bar{\pi}_t) \quad (7)$$

where $\bar{\pi}_t$ is the inflation target, which is stochastic and follows an AR(1) process

$$\bar{\pi}_t = \rho \bar{\pi}_{t-1} + e_t \quad (8)$$

e_t is i.i.d.

\tilde{y}_t is the output gap. In deviations from steady state: \hat{i}_t is the nominal interest rate and $\hat{\pi}_t$ is inflation. Although this model is written in terms of a time-varying inflation target, everything has still been linearized around the usual New Keynesian steady state with zero inflation. κ is a function of model parameters, including the degree of price stickiness.⁸ Assume that $\phi > 0$ and $0 < \beta < 1$. $0 \leq \rho \leq 1$. The elasticity of intertemporal substitution is 1.

a) Using the method of undetermined coefficients, find the response of the output gap and inflation to an exogenous increase in the inflation target $\bar{\pi}_t$ when prices are sticky and monetary policy follows the policy rule above. To do this, guess that the solution for each variable is a linear function of $\bar{\pi}_t$.

b) What is the solution for the path of the nominal interest rate following an increase in the inflation target? (**Hint:** one way to find this is by using the policy rule together with the solution you found in part (a)).

c) First assume $\rho = 0$. Discuss how, and why, an increase in the inflation target affects nominal interest rates, the output gap and inflation. How would your results change if $\rho = 1$?

⁸ $\kappa = (1 + \psi) \frac{(1-\theta)(1-\beta\theta)}{\theta}$ where θ is the probability that a firm cannot adjust its price and ψ is the inverse Frisch elasticity of labor supply.

d) Now suppose that prices are flexible. Under flexible prices, the real interest rate is exogenous and constant. This model can be written as:

$$\hat{i}_t = E_t \hat{\pi}_{t+1} \quad (9)$$

$$\hat{i}_t = \hat{\pi}_t + \phi(\hat{\pi}_t - \bar{\pi}_t) \quad (10)$$

Show that $\phi > 0$ is required to ensure a unique stable solution for inflation in this flexible price model and find the stable solution for inflation.

e) When $\rho = 1$, how does the result for inflation in part (d) compare to part (c) under sticky prices. Briefly discuss.

Question 5 (20 points)

This problem analyzes the welfare effects of a “capital injection” in a model with financial frictions. There are two periods, 0 and 1. There are two types of agents, consumers and entrepreneurs. Both types have a linear utility function, $c_0 + c_1$. Consumers have a large endowment of the consumption good in each period, and a unit endowment of labor in period 1 which they sell inelastically in a competitive labor market at the price w_1 . Entrepreneurs have an endowment of the consumption good n_0 in period 0, and no initial capital. In period 0 they can borrow b_1 and invest k_1 . In period 1 they hire workers at the wage w_1 and produce the consumption good according to the Cobb-Douglas production function

$$y_1 = k_1^\alpha l_1^{1-\alpha}$$

Assume that k_1 completely depreciates after use in production.

Moreover, the entrepreneurs face a borrowing constraint

$$b_1 \leq \lambda(y_1 - w_1 l_1),$$

where $\lambda \in (0, 1)$ is a given scalar. Assume that the consumers endowment is large enough so that the gross interest rate is always 1 in equilibrium.

a) State the entrepreneurs’ problem. Take the first order condition with respect to l_1 . Show that the entrepreneurs always choose l_1 to maximize profits in period 1, that is, they choose l_1 to solve

$$w_1 = (1 - \alpha)k_1^\alpha l_1^{-\alpha}$$

Show that this implies that profits are a linear function of the capital stock k_1 , that is,

$$y_1 - w_1 l_1 = R(w_1)k_1$$

for some function $R(\cdot)$ that depends only on w_1 and parameters.

b) Using the result from a), restate the entrepreneurs’ problem as a simpler linear problem, replacing $y_1 - w_1 l_1$ by $R(w_1)k_1$. Characterize the solution to the entrepreneurs’ problem with first order conditions.

c) Show that if $\lambda R(w_1) < 1 \leq R(w_1)$ the entrepreneurs’ investment in capital is positive and finite. What happens if $\lambda R(w_1) \geq 1$? What if $R(w_1) < 1$? **Hint:** Combine the first order conditions with respect to capital and bond.

d) Show that if the entrepreneurs are constrained, their choice of capital is

$$k_1 = \frac{n_0}{1 - \lambda R(w_1)}$$

e) Show that when entrepreneurs are constrained, the equilibrium level of capital solves

$$k_1 = \frac{n_0}{1 - \lambda \frac{\alpha}{k_1^{1-\alpha}}}$$

Show that k_1 and the entrepreneurs' profits are increasing in n_0 . **Hint:** Remember that we limit attention to the case in which $\lambda R(w_1) < 1 \leq R(w_1)$.

f) Assume that the entrepreneurs are constrained. Suppose the government taxes a lump-sum τ to consumers in period 0, and transfers the receipts from the tax to the entrepreneurs. Derive an expression for the utility of consumers and entrepreneurs as a function of τ .

g) Show that if n_0 is sufficiently small, a small positive tax increases the utility of both consumers and entrepreneurs.

Question 6 (20 points)

Consider an economy consisting of a large and equal number (each of mass 1) of two types of infinitely lived agents. ‘Odd’ agents receive the endowment stream $\{y_t^o\}_{t=0}^\infty$, while ‘even’ agents receive the endowment stream $\{y_t^e\}_{t=0}^\infty$. The endowment streams are given by:

$$y_t^o = \begin{cases} 1 & \text{if } t \text{ is odd} \\ 0 & \text{if } t \text{ is even} \end{cases}$$

and

$$y_t^e = \begin{cases} 0 & \text{if } t \text{ is odd} \\ 1 & \text{if } t \text{ is even} \end{cases}$$

Agents of type i want to maximize:

$$\sum_{t=0}^{\infty} \beta^t \ln c_t^i, \quad i = \{\text{odd}, \text{even}\} \quad \beta \in (0, 1)$$

where c_t^i is the time t consumption of an agent of type i .

a) Suppose agents have access to a market for an asset, denoted by a_{t+1} , which pays a rate of return $1 + r_t$. Assume that agents face no borrowing constraint except for the natural borrowing limit. Define a competitive equilibrium for this economy.

b) Assume that initial asset holdings are zero for all agents. Compute all the equilibria of the economy. That is, find all the prices and quantities (including asset holdings) consistent with your definition of equilibrium. Are the equilibria Pareto optimal? Conclude that Pareto optimal allocations require a constant consumption path for each type of agent. **Hint:** It might be useful to compute the agents’ intertemporal budget constraint.

For the rest of the exercise, assume that all borrowing and lending is prohibited. At time $t = 0$, all odd agents are endowed with M units of an irreproducible and useless asset (that is, it pays zero dividend and does not depreciate), and all even agents are endowed with 0 units of the asset. Agents cannot issue the asset, so holdings have to be weakly positive. Let p_t be the price of a unit of the asset at time t .

c) Define and compute a steady state with a positive and constant price for the useless asset. Argue that this equilibrium is not Pareto optimal. **Hint:** Guess and verify that agents’ consumption takes only two values, \bar{c} and \underline{c} , with $\bar{c} \geq \underline{c}$.

Let’s reinterpret the useless asset as money. Money is a durable asset that does not pay dividends. The government can choose the supply of money according to

$$M_t = \mu M_{t-1}$$

where $\mu > 0$ and M_t is the money supply in period t . At time t the government makes a transfer of $(\mu - 1)M_{t-1}$ units of money (which can be negative) to the agents, on terms to be described in more detail below. We will look for stationary equilibria in which the price of money grows at a constant rate: $\frac{p_{t+1}}{p_t} = 1 + \gamma$ where $\gamma > -1$ (that is, the growth rate of the price of money is allowed to be negative).

d) Assume that the government hands out the new currency (or withdraws old currency if $\mu < 1$) proportionally to initial holdings of money. That is, if an agent holds m_{t-1} units of currency from $t - 1$ to t , the government augments her holdings by $(\mu - 1)m_{t-1}$ for a total of μm_{t-1} units at the beginning of period t . Compute a stationary equilibrium with a positive price for money that grows (or decreases) at the constant rate γ . Is there any μ and γ such that the equilibrium is Pareto optimal? Explain. **Hint:** Use your findings from b).

e) Assume that the transfers are lump sum. That is, each agent, regardless of type, receives $(\mu - 1)M_{t-1}/2$ units of money from the government at the beginning of t . Compute a stationary equilibrium with a positive price for money that grows at the constant rate γ . Is there any μ and γ such that the equilibrium is Pareto optimal? Explain.