

University of California, Davis  
Department of Economics

PRELIMINARY EXAMINATION FOR THE Ph.D. DEGREE

MACROECONOMICS

August 17, 2020

**Directions:** The exam consists of three questions. Question 1 concerns ECN 200D (Geromichalos), question 2 concerns ECN 200E (Cloyne), and question 3 concerns ECN 200F (Caramp). You only need to answer two out of the three questions. If you prefer (and have time), you can answer all three questions and your grade will be based upon the best two scores. Feel free to impose additional structure on the problems below, but please state your assumptions clearly. You have 3 hours to complete the exam and an additional 15 minutes of reading time.

### Question 1 (50 points)

Consider a pure exchange economy with 2 islands. Each island consists of infinitely-lived identical agents whose measure is normalized to one. There is a single consumption good, or fruit, that is non-storable. The representative agent of island  $i$  values different consumption streams according to

$$u_i(c^i) = \sum_{t=0}^{\infty} \beta^t \frac{(c_t^i)^\sigma}{1 - \sigma}, \forall i,$$

with  $\sigma > 0$ . The total endowment in this economy (i.e., in both islands) in period  $t$  is given by a deterministic sequence  $\{e_t\}_{t=0}^{\infty}$ , with  $e_t > 0$  for all  $t$ . However, due to the location of the two islands, weather conditions are different affecting the fruit yield on each island. Letting each period  $t$  denote a season, we assume that in even periods the fruit yield on island 1 is given by  $e_t^1 = a e_t$ ,  $a \in [0, 1]$ , and in odd periods it is given by  $e_t^1 = (1 - a) e_t$ . Of course, by definition, the endowment of fruit on island 2 must be given by  $e_t^2 = e_t - e_t^1$ , in all periods.

a) For this economy define an Arrow-Debreu equilibrium (ADE) and a Sequential Markets equilibrium (SME).

b) Fully characterize (i.e., find a closed-form solution for) the ADE prices.

c) Using any method you like, characterize the SME consumption allocation in as much detail as you can.<sup>1</sup>

In the remaining questions, we will assume that the total endowment in the economy (in both islands) follows the process  $e_t = \gamma^t e$ , with  $\gamma > 0$ .

d) Given this new parametric specification, provide a closed-form solution for the SME consumption allocation.

e) What happens to the consumption allocation you calculated in part (d) when  $\gamma = 1$  (no growth in the endowment)? What if ( $\gamma = 1$  and)  $a = 1$ ?

f) Back to the model with general  $a, \gamma$ ; can you specify parameter values for which agents on island 1 consume more than agents on island 2 in a typical period  $t$ ?

g) For what parameter values do the ADE prices you calculated in part (b) increase as a function of  $t$ ? Provide some intuition for your result.

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<sup>1</sup> **Hint:** Notice that I am asking you to find *only* the equilibrium consumption allocation. This has important consequences for which method is the most convenient to use here.

**Question 2 (50 points)**

This question considers the macroeconomic effects of a collapse in consumer demand in the New Keynesian model.

There are a continuum of identical households. The representative household makes consumption ( $C$ ) and labor supply ( $N$ ) decisions to maximize lifetime expected utility:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left( \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\psi}}{1+\psi} \right) Z_t \quad (1)$$

subject to their budget constraint:

$$C_t + B_t = w_t N_t + (1 + i_{t-1}) \frac{P_{t-1}}{P_t} B_{t-1} + D_t \quad (2)$$

where  $w_t$  is the real wage,  $N_t$  is hours worked,  $B_t$  are real bond holdings at the end of period  $t$ ,  $i_{t-1}$  is the nominal interest rate paid between  $t-1$  and  $t$ ,  $P_t$  is the price of the final consumption good and  $D_t$  are real profits from firms that are distributed lump sum. As usual,  $0 < \beta < 1$ ,  $\psi > 0$  and  $\sigma > 0$ .  $Z_t$  is a household preference shock, which is a way of generating shocks to demand.

The production side of the model is the standard New Keynesian environment. Monopolistically competitive intermediate goods firms produce an intermediate good using labor. Intermediate goods firms face a probability that they cannot adjust their price each period (the Calvo pricing mechanism). Intermediate goods are aggregated into a final (homogeneous) consumption good by final goods firms. The production side of the economy, when aggregated and linearized, can be described by the following set of linearized equilibrium conditions (the production function, the optimal hiring condition for labor and the dynamic evolution of prices):

$$\hat{y}_t = \hat{n}_t \quad (3)$$

$$\hat{w}_t = \hat{m}c_t \quad (4)$$

$$\hat{\pi}_t = \beta E_t(\hat{\pi}_{t+1}) + \lambda \hat{m}c_t \quad (5)$$

The resource constraint is:  $\hat{y}_t = \hat{c}_t \quad (6)$

Monetary policy follows a simple Taylor Rule:  $\hat{i}_t = \phi_\pi \hat{\pi}_t \quad (7)$

The (linearized) preference shock follows an AR(1) process:  $\hat{z}_t = \rho \hat{z}_{t-1} + e_t \quad (8)$

$e_t$  is i.i.d. In percentage deviations from steady state:  $\hat{m}c_t$  is real marginal cost,  $\hat{c}_t$  is consumption,  $\hat{w}_t$  is the real wage,  $\hat{n}_t$  is hours worked,  $\hat{y}_t$  is output. In deviations from

steady state:  $\hat{i}_t$  is the nominal interest rate,  $\hat{\pi}_t$  is inflation.  $\lambda$  is a function of model parameters, including the degree of price stickiness.<sup>2</sup> Assume that  $\phi_\pi > 1$ ,  $0 < \rho < 1$ .

a) First consider the representative household's problem. Write down the household's problem in *recursive form* and derive the household's first order conditions.

b) Show that the linearized first order condition for labor supply from part (a) is:

$$\hat{w}_t = \sigma \hat{c}_t + \psi \hat{n}_t \quad (9)$$

and show that under flexible prices demand shocks have no effect on real GDP.

**Hints:** you will need to use equations (3), (4), (6) and (9).

c) This model can be reduced to three equations:

$$E_t \tilde{y}_{t+1} - \tilde{y}_t = \frac{1}{\sigma} (\hat{i}_t - E_t \hat{\pi}_{t+1}) + \frac{1}{\sigma} E_t (\hat{z}_{t+1} - \hat{z}_t) \quad (10)$$

$$\hat{\pi}_t = \beta E_t (\hat{\pi}_{t+1}) + \kappa \tilde{y}_t \quad (11)$$

$$\hat{i}_t = \phi_\pi \hat{\pi}_t \quad (12)$$

where  $\hat{z}_t$  follows the process in equation (8) and  $\kappa = (\sigma + \psi)\lambda$ .  $\tilde{y}_t = \hat{y}_t - \hat{y}_t^n$  is the output gap and  $\hat{y}_t^n$  is the natural rate of output.

Using the method of undetermined coefficients, find the response of the output gap and inflation to a collapse in consumer demand when prices are sticky and monetary policy follows the Taylor Rule above. To do this, guess that the solution for each variable is a linear function of the shock  $\hat{z}_t$ .

d) Discuss how, and why, a fall in demand affects the natural rate of output, the output gap and inflation in this model. Briefly comment on how a decrease in  $\hat{z}_t$  relates to typical recessions we see in the data.

e) Instead of following the Taylor Rule above, policy is now set optimally. Derive the optimal monetary policy rule under *discretionary policy*. (**Hint:** As in class, assume that the loss function has quadratic terms for the output gap and inflation, with a relative weight  $\vartheta$  on the output gap. For simplicity, assume the steady state is efficient). Using this rule and your knowledge of the model, what is the optimal path for the output gap and inflation in response to a demand shock? Would your answer change if the central bank followed an optimal policy rule under commitment? Explain. (You do not need to derive anything for these last two discussion questions).

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<sup>2</sup> $\lambda = \frac{(1-\theta)(1-\beta\theta)}{\theta}$  where  $\theta$  is the probability that a firm cannot adjust its price.

### Question 3 (50 points)

Consider the following economy. Time is discrete and runs forever,  $t = 0, 1, 2, \dots$ . The economy is populated by two types of agents (a measure one of each): farmers and workers. Farmers own a piece of land that pays a stochastic income  $y_t$  every period. We assume that  $y_t$  is i.i.d. across farmers and time, and that  $y_t \sim N(\bar{y}, \sigma^2)$ . Farmers use all their time working their land. Their preferences are given by

$$u(c) = -\frac{\exp(-\gamma c)}{\gamma}$$

for some  $\gamma > 0$ . For simplicity, we assume that consumption of farmers can be negative, that is  $c \in \mathbb{R}$ . Moreover, farmers can save (or borrow) in a non-state contingent and non-defaultable asset  $a$ , which has a rate of return  $r$ . Farmers face the following “No-Ponzi” condition on assets

$$\lim_{t \rightarrow \infty} \frac{a_t}{(1+r)^t} \geq 0.$$

Moreover, farmers can produce and hold capital,  $k$ , which is rented to the representative firm in competitive markets (1 unit of final consumption good produces 1 unit of capital). Let  $r^K$  be the rental rate of capital and  $\delta$  the depreciation rate.

Unlike farmers, workers don't own land, and they use their available time to work in the representative firm. Assume each worker is endowed with one unit of time. They **cannot** trade the asset  $a$ , but **they can produce and hold capital**,  $k$  (with the same technology as farmers). Their per-period utility is given by  $u(c)$ , with  $u'(c) > 0$ ,  $u''(c) < 0$ ,  $\lim_{c \rightarrow 0} u'(c) = \infty$ ,  $\lim_{c \rightarrow \infty} u'(c) = 0$ .

Finally, there is a representative firm that combines capital and labor to produce final good according to  $f(K, L)$ , where  $K$  is the capital they operate, and  $L$  the hours/workers they hire. Assume  $f(\cdot)$  satisfies the standard Inada conditions.

Thus, the only financial market available in this economy is the market for assets  $a$ , where only farmers can trade. Moreover, there is a market for  $k$ , where all agents can trade.

Both type of agents have the same discount factor  $\beta \in (0, 1)$ .

a) Given a constant path for  $r$  and  $r^K$ , state the problem of a farmer. Argue that the equilibrium price of capital is equal to 1. Characterize the farmer's problem with the necessary FOCs. Show that in any equilibrium with positive capital it must hold that  $r = r^K - \delta$ .

b) Let  $c^F(a, k, y)$  denote the consumption policy function of the farmers, which depends on asset holdings  $a$ , capital holdings  $k$ , and land income  $y$ . Prove that:

$$c(a, k, y) = \underline{c} + r(a + k) + \frac{r}{1+r}y,$$

where  $\underline{c}$  is a constant. Find an expression for  $\underline{c}$  in terms of the parameters of the model.

**Hint:** If  $x \sim N(\mu, \sigma^2)$ , then  $E[e^x] = e^{\mu + \frac{1}{2}\sigma^2}$ .

c) Let  $C^F$  and  $K^F$  denote the aggregate consumption and capital holdings of farmers. Show that in an equilibrium with constant  $C^F$  and  $K^F$ , it must hold that  $\beta(1+r) < 1$ . **Hint:** Remember to check the budget constraint.

d) Let's turn to the workers. State their problem. **Hint:** Remember that workers cannot save in asset  $a$ ; they can only hold capital.

e) Let  $c^W$  and  $k^W$  denote the consumption and capital holdings of an individual worker. Show that in an equilibrium with constant  $c^W$ , it must be that  $k^W = 0$ . Conclude that an equilibrium with constant  $C^F$ ,  $K^F$ ,  $c^W$ , and  $k^W = 0$  exists **Hint:** Use the result that in equilibrium  $\beta(1+r) < 1$ .

f) Show that in such an equilibrium, an increase in  $\sigma^2$  increases the welfare of workers. Explain.