University of California, Davis
Department of Economics
Advanced Economic Theory

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Reading Time: 20 minutes

PRELIMINARY EXAMINATION FOR THE Ph.D. DEGREE

You need to answer four questions, at least one from each part.

Part 1

1. (Temptation and Self-Control)

a) Let X be a finite set of alternatives, and $\mathcal{M}$ the set of its non-empty subsets ("menus"). Consider a decision maker with menu preferences described by the functional form of Gul und Pesendorfer's (2000) model of "Temptation and Self-Control" that is based on a committment utility $u : X \to \mathbf{R}$ and temptation utility $t : X \to \mathbf{R}$.

- As in class, think of the $x \in X$ as abstract alternatives, disregarding the fact that GP study specifically menus of lotteries.

Write down the GP functional form, and explain the notion of "cost of self-control" in their model.

b) Describe the basic new behavioral phenomenon GP try to model?

c) How can one identify the commitment and temptation utility functions *ordinally* from choice behavior over menus? Assume for simplicity that $x \neq y$ implies $t(x) \neq t(y)$ and $u(x) \neq u(y)$.

d) Show that GP-style menu preferences satisfy their Set Betweenness axiom.

e) The Set Betweenness axiom can be decomposed in two that I called Upper and Lower Boundedness. Are these equally appropriate for a general behavioral theory of self-control? Discuss in some detail.

2. (The Von Neumann-Morgenstern Theorem)

Let X denote a finite set of prizes, and $\mathcal{L}(X)$ the set of all probability distributions on X.

Let $\succsim$ be a weak order on $\mathcal{L}(X)$.

a) State the von Neumann-Morgenstern Theorem.

b) As usual, it is more difficult to establish sufficiency of the axioms. In the first part of the sufficiency proof, it is established that, for any lotteries L, L', L'' such that $L \succ L'$ and $L \succsim L'' \succsim L'$, there is a unique $\alpha \in [0,1]$ such that $\alpha L + (1-\alpha) L' \sim L''$.

Which of the axioms are needed to establish this Lemma? (You don't need to prove the Lemma itself).

c) Complete the sufficiency proof, using this Lemma.

d) State and prove the uniqueness part of the Theorem.

Part 2

3. Consider a finance (one good) exchange economy with two periods ($t = 0, 1$) and uncertainty at date 1 represented by S possible states of nature. Suppose that there are J securities traded at date 0: security j has price q_j at date 0 and payoff $V^j \in \mathbb{R}^S$ at date 1. Let $q = (q_1, \ldots, q_J)$ the vector of security prices and V the $J \times S$ matrix of security payoffs.

(a) Explain what it means for the vector q to be a no-arbitrage price vector.

(b) Prove the following proposition: the price vector q is a no-arbitrage price vector if and only if there exist a vector $\pi \in \mathbb{R}^S_{++}$ such that $q = \pi V$. Give the economic interpretation of this proposition.

(c) Suppose that $S = 3$, that the first security (a stock) has payoff $V^1 = (64, 16, 4)$ and price $q_1 = 32$, that the second security (a bond) has payoff $V^2 = (1, 1, 1)$ and price $q_2 = 0.8$, and that the third security is a call option on security 1 with striking price 50 (an option to buy the stock at price 50), i.e. its payoff is the difference between the payoff of security 1 and 50 if this payoff is larger than 50, and zero otherwise. Find the set of no-arbitrage prices for security 3.

4. Consider a two-period ($t = 0, 1$) finance economy $\mathcal{E}(\mathbb{R}^{S+1}, u, \omega, V)$ with uncertainty at date 1, with security structure V, where V is the $S \times J$ matrix of date 1 payoffs of the J securities.

(a) Explain what it means for an allocation $x = (x^1, \ldots, x^I)$ to be *V-constrained feasible* and *V-constrained Pareto optimal.*

(b) Explain when the definitions in (a) reduce to the standard concepts of feasibility and Pareto optimality.

(c) Suppose agents utility functions u^i are smooth, strictly monotone, strictly quasiconcave and satisfy the Inada condition. Find the FOCs for an allocation to be V-constrained Pareto optimal.

(d) Let π^i_V denote the projection of π^i_1 (the present-value vector of agent i) onto the marketed subspace $\langle V \rangle$. Show that the condition in (c) implies $\pi^1_V = \ldots = \pi^I_V$.

(e) Interpret the conditions in (c) and (d).

(f) Explain how these conditions enable you to make precise the intuition that "trading securities tends to equalize agents' rates of substitution" by showing precislely how the extent of equalisation depends on the degree of completeness of the markets.

(g) Indicate briefly how you can show that if (x, z, q) is a financial market equilibrium, then the allocation x is V-constrained Pareto optimal.

(h) Suppose that instead of a finance (one good) economy we consider an economy with L goods, where the matrix V gives the payoffs of the financial securities in unit of a numeraire good. Define the V-feasible and V-constrained Pareto optimal allocations. Would the results of question (g) still hold? Justify your answer as precisely as you can.

Part 3

5. Sometimes seemingly simple games are a bit tricky. Consider a symmetric 2-player strategic game with action sets $A = (0, 1]$ for each player. Note that $0 \notin A$. The player's payoff functions are given by for $i \neq j$, $i, j \in \{1, 2\}$,

$$u_i(a_i, a_j) = \begin{cases} a_i & \text{if } a_i < a_j \\ \frac{a_i}{2} & \text{if } a_i = a_j \\ 0 & \text{if } a_i > a_j \end{cases}$$

(a) Show that $a_i = 1$ is strictly dominated.

(b) Are there any other strictly dominated strategies?

(c) Is there a Nash equilibrium in pure strategies? If yes, show the set of Nash equilibria. If no, show why there is no Nash equilibrium.

(d) (Ambitious) Is there a Nash equilibrium in mixed strategies? If yes, show the set of Nash equilibria. If no, show why there is no Nash equilibrium.

(e) Suppose now that we modify the game a little by adding a second period. In this second period the following stage game is being played:

	b_1	b_2
b_1	$\frac{1}{2}, \frac{1}{2}$	$-1, -1$
b_2	$-1, -1$	$0, 0$

The payoffs of the entire game are given by the sum of the payoffs from both periods. How would you specify the set of pure strategies for the entire game?

(f) Consider further the modified game in (e). What is the highest payoff for each player that can be achieved in a Nash equilibrium?

(g) Is the Nash equilibrium in (f) also subgame perfect?

6. Prove the following proposition:

Proposition 1 *Consider a large population of players who are randomly matched to play a finite 2-player symmetric strategic game. If a pure strategy is strictly dominated, then as time goes to infinity the share of population programmed to this pure strategy converges to zero in the continuous time replicator dynamics starting from any completely mixed population state.*

(Hint: Think about a "help" function with logarithms.)