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Date: June 27, 2005
Time: 4 hours
Reading Time: 20 minutes

PRELIMINARY EXAMINATION FOR THE Ph.D. DEGREE-

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Please answer **four** of the following five questions

1. *Choice with status-quo bias.*

A consumer's consumption set is X. Her choice depends on both her attainable (or budget) set $B \subset X$ and on her *status-quo* point $s \in B$. She is endowed with both a *utility function* $u: X \to \Re$ and a *moving-cost function* $c: X \to \Re_{++}$, where $c(s)$ is interpreted as the cost of leaving the *status quo* s. We restrict our attention to attainable sets B with the property that u has a unique maximizer on B, denoted $x^*(B)$.

At (B, s) (i. e., when facing the attainable set B from the *status-quo* point s), she chooses

(1) the *status-quo* point s if $u(s) > u(x^*(B)) - c(s)$,

(2) point $x^*(B)$ if $u(s) < u(x^*(B)) - c(s)$,

(3) either the *status-quo* point s or point $x^*(B)$ if $u(s) = u(x^*(B)) - c(s)$.
(Note that in this case her choice is not unique.)

(a) Show that her choices satisfy the following property: if $x^0 \in B^1 \subset B^0$ is chosen at (B^0, s), then x^0 is also chosen at (B^1, s). Interpret this property in terms of revealed preference.

(b) Show that there exists a function f that has both the consumption point x and the *status-quo* point s as vector arguments such that solving the problem $\max_x f(x, s)$ subject to $x \in B$ is equivalent to applying the choice rules (1) and (2). Interpret f. Is f continuous when $X = \Re^N_+$ and both u and c are continuous?

(c) Let $X = \Re^2_+$, $u(x_1, x_2) = (x_1)^{0.5} (x_2)^{0.5}$, and $(\omega_1, \omega_2) = (2, 2)$. Consider budget sets of the form $B[p] = \{(x_1, x_2) \in \Re^2_+ : x_1 + p\, x_2 \leq \omega_1 + p\omega_2\}$, parameterized by the positive number p (understood as the normalized price of good 2). In this section, graphs can be approximate, but they should clearly indicate the ranges over which the curves are increasing or decreasing.

(c.1) Assume that the consumer maximizes u in the usual manner, without *status-quo* bias, i.e., given $p > 0$, the consumer chooses (x_1, x_2) in order to maximize $u(x_1, x_2)$ subject to $(x_1, x_2) \in B[p]$. Compute the solution function $(x_1^*(p), x_2^*(p))$ and the value function $v(p)$ of this maximization problem. What is $(x_1^*(1), x_2^*(1))$? What is $v(1)$? Separately graph $x_2^*(p)$ and $v(p)$, with p on the horizontal axis.

(c.2) Assume now that the consumer has a *status-quo* bias, as defined above, with *status-quo* point $s = (\omega_1, \omega_2) = (2, 2)$, and with moving-cost function $c(x) = 0.2$. Obtain and graph the relationship between p and her choice of x_2 (again, with p on the horizontal axis). Comment.

2. *A two-firm, four-good economy.*

Consider a production economy with two firms and four goods, two factors of production that we will call labor and capital, and two produced goods that we will call bread and cars. Both are produced under constant returns to scale but the production of bread is more labor intensive than the production of cars. The production functions are $Y_b = F_b(L, K) = L^{\frac{2}{3}} K^{\frac{1}{3}}$ for bread and $Y_c = F_c(L, K) = L^{\frac{1}{3}} K^{\frac{2}{3}}$ for cars.

(a) Suppose that there a 64 units of labor and 225 units of capital in the economy. Draw a production Edgeworth box showing the feasible allocations of capital and labor among the bread and the car sectors of the economy. Draw in some isoquants of the production functions and explain the geometric condition that must be satisfied at an efficient allocation of labor and capital among the sectors (i.e. an allocation such that it is not possible to increase the production of one good without decreasing the production of the other).

(b) Derive analytically the set of efficient allocations of labor and capital and represent it in the Edgeworth box. Comment on its location relative to the diagonal of the box.

(c) Suppose that the consumption sector of the economy can be represented by a representative consumer owning the labor, the capital and the firms, and with the utility function $u(x_b, x_c) = x_b^{\alpha} x_c^{1-\alpha}$ with $0 < \alpha < 1$. That is, the consumer cannot use the capital goods for consumption and has no disutility for labor. Suppose the price of labor is normalized to 1. Compute the equilibrium price of capital and show that it is a decreasing function of α. Explain in words why this was to be expected.

3. *An excludable public good.*

Consider an economy with a private good and a public good. The economy consists of two consumers whose utility functions are

$$u_1(x_1, y) = \frac{1}{2}\ln(x_1) + \frac{1}{2}\ln(y) \quad \text{and} \quad u_2(x_2, y) = \frac{1}{3}\ln(x_2) + \frac{2}{3}\ln(y)$$

Agent 1 is endowed with 4 units of the private good and agent 2 with 2 units: $\omega_1 = 4$, $\omega_2 = 2$. Consumption of the public good is not required. If a quantity *y* is produced, each consumer may consume any non-negative amount less than or equal to *y*, i.e. it is possible to exclude an agent from consumption of the public good (tolls on highways for example). The production of public good uses the private good as an input, one unit of private good giving one unit of public good.

(a) The Pareto optimal allocations can be obtained by maximizing the social welfare function $\alpha u_1 + (1 - \alpha) u_2$ over the set of feasible allocations. Compute the Pareto optimal allocations as a function of α and show that the level of public good is always larger than 3.

(b) Compute the voluntary contribution equilibrium and show that it is not Pareto optimal.

(c) Since there are ways to exclude an agent from consuming the public good, using the market may lead to a better allocation, without relying on the questionable personalized prices of a Lindahl equilibrium. Suppose that the firm producing the public good (the public firm for short) produces $\bar{y}$ and that the agents buy the public good at price *p* per unit, choosing the quantity that they want as long as it is less than or equal to the produced level of public good $\bar{y}$.

(i) Normalizing the price of the private good to 1, compute the constrained demand $y_i(p, \bar{y})$ of each agent, i.e. the demand that maximizes agent *i*'s utility, subject to the budget constraint and to the additional constraint that $y \le \bar{y}$. For each agent write carefully the values of *p* for which the constraint on the maximum amount of public good available is, or is not, binding.

(ii) Note that if both agents buy $\bar{y}$, budget balance of the public firm requires that $p = 1/2$. What is the highest level of public good $\bar{y}$ for which there is an equilibrium with p = 1/2? Compute the equilibrium allocation.

(iii) Note that, at the equilibrium calculated in (ii), agent 1 is constrained: he would like to buy more public good than $\bar{y}$ at price p = 1/2. Find the unique equilibrium in which no agent is constrained and the public firm balances its budget. Show that the allocation found in the previous question Pareto dominates this equilibrium allocation. Intuitively explain why.

(iv) Do the market solutions considered in (ii) and (iii) give a better allocation than the voluntary contribution equilibrium of question (b)?

4. *A litigation game.*

The Shtinki Corporation operates a chemical plant, which is located on the banks of the Sacramento river. Downstream from the chemical plant is a group of fisheries. The Shtinki plant emits by-products that pollute the river, causing harm to the fisheries. The profit Shtinki obtains from operating the chemical plant is $\$X > 0$. The harm inflicted on the fisheries due to water pollution is equal to $\$Y > 0$ of lost profit [without pollution the fisheries' profits is $\$A$, while with pollution it is $\$(A - Y)$]. Suppose that the fisheries collectively sue the Shtinki Corporation. It is easily verified in court that Shtinki's plant pollutes the river. However, the values of X and Y cannot be verified by the court, although they are common knowledge among the litigants. Suppose that the court requires the Shtinki attorney (player 1) and the fisheries' attorney (player 2) to play the following litigation game. Lawyer 1 is supposed to announce a number $x \geq 0$, which the court interprets as a claim about the plant's profits. Lawyer 2 is supposed to announce a number $y \geq 0$, which the court interprets as the fisheries' claim about their profit *loss*. The announcements are made simultaneously and independently. Then the court uses *Posner's nuisance rule* to make its decision (R. Posner, *Economic analysis of Law*, 9th edition, 1997). According to the rule, if $y > x$, then Shtinki must shut down its chemical plant. If $x \geq y$, then the court allows Shtinki to operate the plant, but the court also requires Shtinki to pay the fisheries the amount y. Note that the court cannot force the attorneys to tell the truth (in fact, it would not be able to tell whether or not a specific amount stated by the lawyers were indeed the true amount). Assume that the attorneys want to maximize the payoff (profits) of their clients.

(a) Represent this situation as a normal-form game by describing the strategy set of each player and the payoff functions.

(b) For the case where $X > Y$ (recall that X and Y denote the *true* amounts), find **all** the pure-strategy Nash equilibria of the litigation game. [Prove that what you claim to be Nash equilibria are indeed Nash equilibria and that there are no other Nash equilibria.]

(c) For the case where $X < Y$ (recall that X and Y denote the *true* amounts), find **all** the pure-strategy Nash equilibria of the litigation game. [Prove that what you claim to be Nash equilibria are indeed Nash equilibria and that there are no other Nash equilibria.]

(d) Does the court rule give rise to a Pareto efficient outcome?

(e) Is it a dominant strategy for the Shtinky attorney to make a truthful announcement (i.e. to choose $x = X$)? [Prove your claim.]

(f) Is it a dominant strategy for the fisheries' attorney to make a truthful announcement (i.e. to choose $y = Y$)? [Prove your claim.]

5. *Second-degree price discrimination.*

A monopolist faces two consumers, one with demand function $D_H(P) = 12 - b_H P$ and the other with demand function $D_L(P) = 12 - b_L P$, where $0 < b_H < b_L$ and $b_L < 2b_H$. The monopolist cannot tell which consumer has the higher demand and which has the lower demand, although he knows the two demand functions. The monopolist decides to sell the good in bundles or packages. Denote a package as a pair (Q,V) where Q is the number of units of the product and V is the price of the entire package (not the price per unit). He considers three options.

Option 1: sell only one package, targeted to the consumer with high demand.

Option 2: sell two identical packages, designed in such a way that each consumer will buy one package.

Option 3: sell two different packages, one targeted to the high-demand consumer and the other to the low-demand consumer.

The monopolist has the following cost function: $C(Q) = Q$

(a) Determine the profit maximizing package for option 1 and calculate the corresponding profits.

(b) Determine the profit maximizing package for option 2 and calculate the corresponding profits.

(c) For option 3 write the constraints that must be satisfied in order for each consumer to end up buying the package which is designed for her.

(d) Determine the profit maximizing packages for option 3.

For the next two questions (and only for the next two questions) assume that $b_H = 2$ and $b_L = 3$.

(e) Rank the three options based on the profits they yield. Calculate total surplus with the best (in terms of profit-maximization) of the three options. Calculate also the effective price(s) per unit.

(f) What would the monopolist's profits be if he were able to use first-degree price discrimination?