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Department of Economics
Microeconomics

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PRELIMINARY EXAMINATION FOR THE Ph.D. DEGREE

Please answer four parts (out of five)

PART 1. WILLINGNESS TO PAY AND TO ACCEPT

Two goods. Good 2 is a consumption good that the consumer owns in the amount ω, whereas good 1 is an environmental good that the consumer enjoys. The consumer's preferences on (x_1, x_2) combinations of the two goods are represented by a utility function $u : \Re^2_{++} \to \Re$.

In what follows we consider two given levels of the environmental good, namely $\bar{x}$ and $\bar{\bar{x}}$, with $\bar{\bar{x}} > \bar{x} > 0$.

Define the consumer's *WTP* (for "willingness to pay for a move from $\bar{x}$ to $\bar{\bar{x}}$") as the maximal amount of good 2 that the consumer would be willing to part with in exchange for an <u>increase</u> in the amount of good 1 from $\bar{x}$ to $\bar{\bar{x}}$.

Define the consumer's *WTA* (for "willingness to accept for a move from $\bar{\bar{x}}$ to $\bar{x}$") as the minimal amount of good 2 that the consumer would be willing to receive (and add to ω) in exchange for a <u>decrease</u> in the amount of good 1 from $\bar{\bar{x}}$ to $\bar{x}$.

Question 1(a). Write the equations that implicitly define *WTP* and *WTA.* Graphically illustrate these two notions in a graph with axes x_1 and x_2, labeled Figure 1. Comment on the graph.

Question 1(b). How do *WTP* and *WTA* compare when u is quasilinear (with good 2 as numeraire)?

Question 1(c). Can you establish a conceptual parallel with the traditional (Mas-Colell *et al.*, 1995, Ch. 3) notions of Equivalent and Compensation Variations for a consumer with wealth w who buys L goods and takes as given the L-dimensional price vector?

For 1(d)-1(k) below, we specialize to the CES utility function

$$u(x_1, x_2) = \left[(x_1)^{\frac{\sigma-1}{\sigma}} + (x_2)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}, \qquad (1)$$

where σ is a positive parameter, $\sigma \in (0,1) \cup (1,\infty)$.

Question 1(d). Interpret the parameter σ. What is the limit case of this CES utility function as $\sigma \to 0$? What is the limit case of this CES utility function as $\sigma \to \infty$? Give verbal and graphical (Figure 2) interpretations of these limit cases.

Question 1(e). Explicitly compute *WTP* and *WTA* for the function (1).

For 1(f)-1(k) below, consider the simple case where $\overline{x} = \omega$ and $\overline{\overline{x}} = 2\omega$ (in addition to the CES assumption).

Question 1(f). Draw a graph, labeled Figure 3, that specializes Figure 1 to this case.

Question 1(g). Specialize to this case the expressions obtained in 1(e) above. Illustrate the *WTA* in Figure 3.

Question 1(h). What can you say about the magnitude of the ratio *WTA* / *WTP*? Argue in detail. Going back to 1(c) above, can you find a parallelism with the traditional Equivalent Variation and Compensating Variation measures?

Question 1(i). What can you say about the limit of *WTA* as $\sigma \to \infty$? What can you say about the limit of *WTP* as $\sigma \to \infty$? What does this imply for the limit of the ratio *WTA* / *WTP* as $\sigma \to \infty$? Comment.

Question 1(j). What can you say about the limit of *WTA* as $\sigma \to 0$? What can you say about the limit of *WTP* as $\sigma \to 0$? What does this imply for the limit of the ratio *WTA* / *WTP* as $\sigma \to 0$? Comment.

Question 1(k). Going back to 1(c) above once more, comment on the extent of the parallelism with the traditional Equivalent Variation and Compensating Variation measures.

PART 2. RISK, UTILITY AND PRUDENCE

Let f and g be real-valued functions defined on an open interval of the positive real line, with positive first order derivative, and negative second order derivative. We will be using the following lemma (which you do not have to prove: it is an adaptation of Proposition 6.C.2, definitions (i) and (ii), in Mas-Colell *et al.*, 1995).

Lemma. *The following two conditions are equivalent*:

(i) $-\dfrac{g''(x)}{g'(x)} \geq -\dfrac{h''(x)}{h'(x)}, \forall x$.

(ii) If $\int h(x)dF(x) = h(\bar{x})$, then $\int g(x)dF(x) \leq g(\bar{x})$, for all lotteries F, $\forall \bar{x}$.

If either of these conditions is satisfied, then we say that g is (weakly) *more concave* than h.

Question 2(a). Interpret condition (i) when g and h are understood as von Neumann-Morgenstern-Bernoulli (vNMB) utility functions.

In what follows we view x as the sum of ω and z, where ω is interpreted as the (nonrandom) wealth of the consumer, and z is distributed according to a cumulative distribution function F with mean zero (i. e., it is a pure risk). We consider a consumer with vNMB utility function u, defined on an open interval of the positive real line, thrice differentiable, with $u'(x) > 0$ and $u''(x) < 0$.

Definition. Given F and ω we implicitly define the *risk premium* ρ by the equation

$$\int u(\omega + z)dF(z) = u(\omega - \rho).$$

Definition. Given F and ω we define the *utility premium* ψ as $\psi = u(\omega) - \int u(\omega + z)dF(z)$.

Definition. The vNMB utility function u displays (weak) *prudence* if $u'''(x) \geq 0, \forall x$, and *strict prudence* if $u'''(x) > 0, \forall x$.

Question 2(b). Interpret the risk premium and the utility premium in words.

Question 2(c). We are interested in analyzing how the risk premium ρ varies with wealth ω. Show that $\dfrac{d\rho}{d\omega} \leq 0$ *if and only if u displays* (weakly) *decreasing absolute risk aversion.* [Hint: use the Lemma above, and consider the increasing functions $-u'$ and u.]

Question 2(d). Does decreasing absolute risk aversion imply prudence? Argue your answer.

Question 2(e). Again, we are interested in analyzing how the utility premium ψ varies with wealth ω. How is the sign of $\frac{d\psi}{d\omega}$ related to prudence?

Question 2(f). Summarize, according to your answers to 2(c), 2(d) and 2(e), the relations among the following four properties:

$\frac{d\rho}{d\omega} \leq 0$,

Decreasing absolute risk aversion,

Prudence,

$\frac{d\psi}{d\omega} \leq 0$.

Apply these relations to the vNMB utility functions: $-e^{-rx} \, (r > 0)$ and - $(a - x)^2$ (with domain $x < a$).

PART 3. PUNISHING POLLUTERS

Consider an economy with three goods and labor: good 1 is a composite good that we call "money;" good 2 is energy, and the third good is "spilled crude oil." There are n identical firms producing energy from labor, with possibility of an accident. A firm which invests a units of money in accident-prevention technology and uses L units of labor produces $f(L)$ units of energy with probability $1 - p(a)$ and, with probability $p(a)$, it does not produce anything and spills $\tilde{e}(L,a)$ units of crude oil in the ocean (the labor has been used, but instead of producing energy it produces pollution). We assume that f is increasing and concave, that p is decreasing and convex, and that $\tilde{e}(L,a)$ is increasing in L (the magnitude of the spill increases with the size of the firm) and decreasing in a (investment decreases both the probability and the magnitude of the spill).

We consider only symmetric allocations in which all firms choose the same combination (a, L). The risks of accident of the firms are independent and n is sufficiently large, so that we can use the Law of Large Numbers: if each firm invests a in accident-prevention technology, then $np(a)$ firms have an accident, and $n(1 - p(a))$ firms produce normally (we disregard the integer problem). Thus, although each firm faces risk, aggregate production is known with certainty (once a and L are known).

The consumer side is summarized by a representative consumer with initial resources m_0 in money and L_0 in time, which can be used for labor or leisure. The representative consumer' s utility function is $U(m,c,\ell,e)$, where m is her consumption of money, c is her consumption of energy, ℓ is her leisure, and e is the amount of crude oil in the ocean. The function U is strictly concave, increasing in the first three components, and decreasing in the fourth one.

In all the questions that follow we assume that the conditions of concavity/convexity needed for the first-order conditions to characterize the solutions to the constrained maximum problems that we consider are satisfied, and we also assume that the solutions are interior so that non-negativity constraints can be neglected.

Question 3(a). Write the constrained maximum problem whose solution is the Pareto optimal allocation of the economy. Be sure that the feasibility constraints you are writing make sense.

Question 3(b). Write the first-order conditions for the maximum problem in 3(a). Eliminate the multipliers and interpret the two relations that you obtain in terms of marginal benefit and marginal cost.

Question 3(c). Without government intervention, the competitive equilibrium in which money, energy and labor are traded on competitive markets and firms maximize expected profit is not Pareto optimal. Show that the Pareto optimality of the competitive equilibrium can be restored if the government imposes a fine on the firms spilling crude oil proportional to the quantity spilled (the revenue from the fine is given to the consumer as a lump sum). Find the optimal coefficient of proportionality-- i. e., the cost to a firm of one unit of crude oil spilled. Explain.

PART 4.

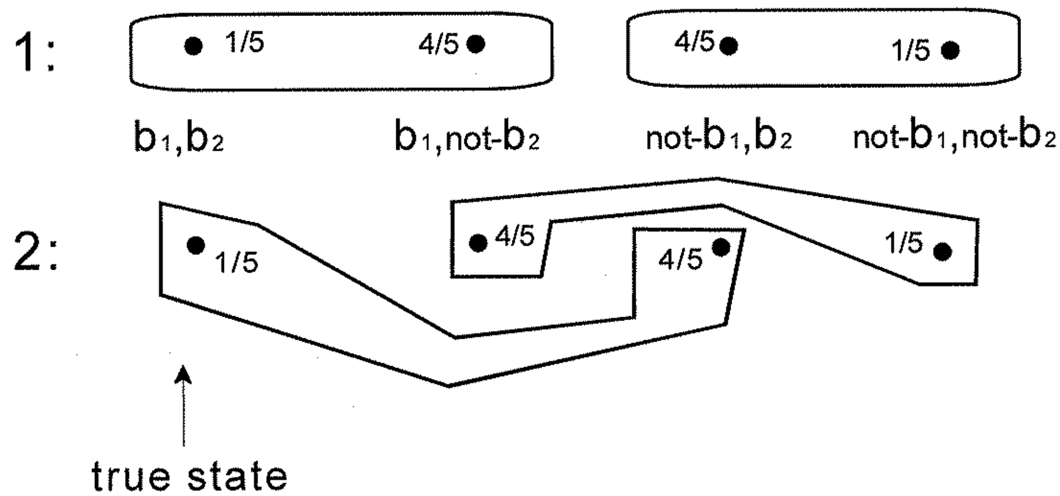
Many settings are naturally modeled as congestion games, including traffic, computer networks, and even the decision whether or not to go to a bar. In this problem we will consider a variant of the well-studied Santa Fe (or, El Farol) Bar Problem, in which a set of people are simultaneously deciding whether or not to go to a bar. We'll consider a version of the problem where two students are simultaneously deciding between going to the bar and going home. Each student has a preference for either going to the bar or going home; if student i prefers going to the bar to going home we write b_i whereas if she prefers going home to going to the bar we write not-b_i. Starting from a baseline utility of zero, a student gains 20 units of utility if she goes to the place that she prefers; however, the bar is extremely small and both students lose 40 units of utility if they both go to the bar. Thus for example if we are in the situation b_1 and b_2 (both prefer going to the bar) and they both go to the bar, then each gets a utility of $20 - 40 = -20$, whereas if player 1 goes to the bar and player 2 goes home, then player 1's utility is 20 and player 2's utility is 0. These payoffs are von Neumann-Morgenstern payoffs. Note that congestion occurs only at the bar, not at home (that is, there is no utility loss if both go home).

Question 4(a). Write the strategic-form games $G(b_1, b_2)$ (the game where both prefer going to the bar), $G(b_1, \text{not-}b_2)$, $G(\text{not-}b_1, b_2)$ and $G(\text{not-}b_1, \text{not-}b_2)$.

Question 4(b). (b.1) For each of the games of 4(a) find the pure-strategy Nash equilibria.

(b.2) For game $G(b_1, b_2)$ find also a mixed-strategy equilibrium where each choice is made with positive probability.

The students don't know each other's preferences and they are thus in a situation of incomplete information, as represented in the following figure.



Question 4(c). Use the Harsanyi transformation to represent the above situation of incomplete information as an extensive-form game.

Question 4(d). For the game of 4(c), pick one strategy of player 1 and explain in words what it means.

Question 4(e). For the game of 4(c), how many strategies do the players have?

Question 4(f). For the game of 4(c), give the strategic-form payoffs of both players for at least one row of the matrix.

Question 4(g). Find a pure-strategy Bayes-Nash equilibrium of the game of 4(c).

Question 4(h). For the Bayes-Nash equilibrium of 4(g), find:

(h.1) where the players are actually going, and

(h.2) the actual payoffs of the players in the game they are actually playing (that is, at the true state).

(h.3) Do their actual choices yield a Nash equilibrium of the game that they are actually playing?

Question 4(i). If you didn't know what the true state was but you knew the game of 4(c), what probability would you attach to the event that the players would end-up making actual choices that constitute a Nash equilibrium of the true game that they are playing?

PART 5

A firm wants to hire n workers ($n \geq 1$). There are 3 types or workers: *H*, *M* and *L*. A worker of type *H* would produce x_H units of output for the firm, a worker of type *M* would produce x_M units and a worker of type *L* would produce x_L units, with $x_H > x_M > x_L > 0$. Each worker knows his own type and his own productivity. The firm, however, cannot tell if an applicant is of type *H*, *M* or *L*. The firm can sell each unit for $\$R$ and has no other costs besides the labor costs. All the workers are currently employed elsewhere at a wage of w_0. Assume that every worker will apply to this firm if he expects a total compensation of at least w_0 (thus even if he expects to make the same amount of money as he is currently making). The firm is risk-neutral. There are $n_i > 0$ workers of type $i \in \{H,M,L\}$ in the population with $n_H + n_M \geq n$ (and thus $n_H + n_M + n_L > n$). Consider the following options for the firm.

OPTION 1. Offer a fixed salary w and hire n of the applicants.

OPTION 2. Offer a piece rate b (that is, a payment of $\$b$ for each unit of output produced by the worker) that would attract every type of worker, and hire n of the applicants.

OPTION 3. Offer a piece rate b that would attract only types *M* and *H*, and hire n of the applicants.

OPTION 4. Offer a piece rate b that would attract only type *H*, and hire n of the applicants.

Question 5(a). Show that the maximum profit the firm can make with Option 2 is less than the maximum profit it can make with Option 1.

Question 5(b). Show that the revenue the firm can make under Option 3 is higher than the revenue it can make under Option 1, and that minimum costs are higher under Option 3 than under Option 1.

Question 5(c). Suppose that $x_H = 115,\ x_M = 110,\ x_L = 100,\ R = 150,\ w_0 = 14{,}000$. Find values of the remaining parameters such that Option 1 gives higher profits than Option 3.

Question 5(d). Suppose that $x_H = 115,\ x_M = 110,\ x_L = 100,\ R = 150,\ w_0 = 14{,}000$. Find values of the remaining parameters such that Option 3 gives higher profits than Option 1.

Question 5(e). Now let us go back to the general case [that is, do not assume the parameter values given under 5(c) and 5(d)]. Show that if $n_H \geq n$, then the maximum profit the firm can make with Option 4 is greater than the maximum profit it can make with Option 3.

Question 5(f). Suppose that $x_H = 115,\ x_M = 110,\ x_L = 109,\ R = 150,\ w_0 = 14{,}000$. Find values of the remaining parameters such that Option 3 gives higher profits than Option 4.