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Reading Time: 20 minutes

PRELIMINARY EXAMINATION FOR THE Ph.D. DEGREE-

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Please answer four of the following equally weighted five questions

1. Variations on variable varieties

We consider a model of consumer preferences where the different commodities are viewed as different varieties of a good, and the number L of varieties is variable. Variations of this model have been extensively used in industrial organization and international trade.

Formally, we postulate that the preferences of our consumer are represented by the following family of utility functions, indexed by the number of varieties present in the market:

$$u_L(x_1,...,x_L) = L^{\gamma}\left(\sum_{i=1}^{L} x_i^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}, \gamma \in \Re, \sigma \in (-\infty,0) \cup (0,1) \cup (1,\infty). \qquad (1)$$

In words, $u_L(x_1,...,x_L)$ is the utility that our consumer gets when, in a L-variety world, she consumes x_j units of variety j, $j = 1,\ldots, L$.

Our consumer has wealth $w > 0$, takes prices $(p_1, \ldots, p_L)$ as given, and is always able to satisfy her Walrasian demand.

(a). Let $\gamma = 0$. We consider different worlds characterized by different numbers of varieties, but we assume that $p_1 = p_2 = \ldots = p_L = p$, for every L, i. e., every price is always equal to a given positive number p, or, in other words, prices are equal across worlds (or numbers of varieties) and across varieties.

Does a higher L benefit the consumer? Argue clearly, separately considering the following three cases.

Case 1: $\sigma > 1$ (this is the standard case);

Case 2: $\sigma \in (0,1)$;

Case 3: $\sigma < 0$.

(b). Now γ is unrestricted, but we let $\sigma > 1$ (as in Case 1 of part (a)). Compute the own-price elasticity of demand for variety j ($j = 1,\ldots, L$) for the utility function given in (1). (Hint.

Recall that the Walrasian demand function for good j for the CES function $\left(\sum_{i=1}^{L} x_i^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}$,

$\sigma > 1$, is $w\left(\sum_{i=1}^{L} p_i^{1-\sigma}\right)^{-1} p_j^{-\sigma}$.) What is the limit of this elasticity as $L \to \infty$ if every price is always equal to p? Interpret. Compare this limit elasticity with the elasticity of substitution of (1).

(c). Again, $\sigma > 1$ and γ is unrestricted. We define the love-of-variety function $\hat{\beta}(L,x) \equiv \frac{u_L(x,x,...,x)}{u_1(Lx)}$. Interpret it in words, and argue that, for the utility function (1), $\hat{\beta}(L,x)$ is constant with respect to x and, hence, it can be written as a function $\beta(L)$ of only L. Compute its elasticity $\eta(L) \equiv \beta'(L)\frac{L}{\beta(L)}$. What can you say about the sign of $\eta(L)$? Compare this sign with your answer in Case 1 of part (a) above.

(d). Once more, $\sigma > 1$, and γ is unrestricted. It is assumed in some applications that the love-of-variety elasticity $\eta(L)$ actually coincides with the ratio $\frac{\text{price - marginal cost}}{\text{marginal cost}}$ of a monopolist who supplies variety j to our consumer. Please compute this ratio as a function of the elasticity of the demand function faced by the monopolist, under the assumption that this elasticity is the one obtained in part (b) as the limit as $L \to \infty$ when every price is always equal to p. How does this ratio compare with the love-of-variety elasticity $\eta(L)$ computed in part (c)? Are they equal? Comment.

2. Consider an economy with two goods X and Y and two consumers, 1 and 2. The preferences of the consumers are represented by the utility functions

$$u_1(x_1, y_1) = \min(2x_1, y_1) \quad \text{for agent 1}$$

$$u_2(x_2, y_2) = x_2 y_2 \quad \text{for agent 2}$$

where (x_i, y_i) denotes the amounts of goods X and Y consumed by agent i ($i = 1, 2$). The aggregate resources of the economy consist of 1 unit of good X and 3 units of good Y.

(a) Find the Pareto optimal allocations for this economy. Represent the contract curve in an Edgeworth box.

(b) Find the Pareto optimal allocation such that the levels of utility of the two agents are the same. Locate the allocation in the Edgeworth box drawn in (a) (call it point A).

(c) Suppose that the resources of the economy are initially owned by the two agents with $\omega_1 = (1, 0)$ and $\omega_2 = (0, 3)$. Compute the competitive equilibrium allocation and locate it in the Edgeworth box drawn in (a) (call it point B).

(d) Suppose that an egalitarian planner who has only the power to redistribute income among the agents by using taxes and subsidies wants the two agents to have the same utility levels. Describe *precisely* how the planner can achieve her goal (i.e. indicate exactly the taxes and subsidies which should be used).

(e) Assuming that the planner knows the agents' preferences – an heroic assumption – do you think that equalization of utility levels is the right goal for an egalitarian planner? In this example? In general?

3. Consider a two-person exchange economy with two goods, good *x* and good *y*. Agent *a*'s preferences are represented by the utility function

$$u_a(x_a, y_a) = \ln(x_a) + \ln(y_a)$$

and his initial endowment is $\omega_a = (2, 2)$. Agent *b*'s preferences are represented by the utility function

$$u_b(x_b, y_b, x_a) = \ln(x_b) + \ln(y_b) + \ln(4 - x_a)$$

and her initial resources are also $\omega_b = (2, 2)$. As you can see from the form of u_b the consumption of good *x* by Mr *a* negatively affects Ms *b*.

(a) Find the Pareto optimal allocations of this economy.

(b) Does a competitive equilibrium allocation of this economy belong to the Pareto optimal allocations found in (a)? Give an intuitive explanation for your answer. (Calculation of the equilibrium is not required).

(c) In order to solve the problem, a social planner introduces the following rule: Instead of just the good prices (p_x, p_y) as in a competitive equilibrium, there are now six prices: the good prices (p_x, p_y) as before, but also four "transfer prices" (q_a, r_a, q_b, r_b). Each time Mr *a* consumes a unit of good *x* (good *y* respectively) he must transfer q_a (r_a respectively) units of account to Ms *b*. In the same way each time Ms. *b* consumes a unit of good *x* (good *y* respectively) she must transfer q_b (r_b respectively) units of account to Mr. *a*. The prices q_a, q_b, r_a and r_b may be positive, zero or even negative.

The economy functions as follows. Taking prices $(p_x, p_y, q_a, q_b, r_a, r_b)$ as given, Mr *a* chooses a complete allocation $(x_a, y_a, x_b, y_b) \geq 0$ for the economy, subject to a single budget constraint: the net inflow of units of account to *a* should equal or exceed the net outflow from him. That is,

$$p_x x_a + p_y y_a + q_a x_a + r_a y_a \leq 2p_x + 2p_y + q_b x_b + r_b y_b.$$

Mr *a* is in no obligation to make the chosen allocation feasible. In the same way Ms. *b* chooses a complete allocation, subject to the analogous budget constraint for her.

An equilibrium with transfer prices for the economy is a set of six prices and a single allocation such that

– the allocation maximizes the utility of each consumer given the prices, and
– the allocation is socially feasible.

(i) Show that, in an interior equilibrium with transfer prices, (r_a, q_b, r_b) must be equal to zero.

(ii) Compute the equilibrium with transfer prices. [Hint: use the fact that the utility functions are Cobb-Douglas. By comparing the market clearing equation for good *x* with the compatibility equation for the choice of x_a by agents *a* and *b*, find a simple relation between p_x and q_a. The system of equilibrium equations is then simple to solve.]

(iii) Show that the allocation is Pareto optimal and explain intuitively why it is so.

4. There are three types of workers, romantically called type 1, 2 and 3. The proportion of type i (i = 1, …, 4) in the population of workers is p_i , with $0 < p_i < 1$ and $p_1 + p_2 + p_3 = 1$. A newly set-up firm wishes to hire workers. The firm has the following information:

- The firm knows that a type i worker would be able to produce x_i units of output per year, with $0 < x_3 < x_2 < x_1$. The firm expects that it will be able to sell each unit of output for $\$R$. Labor costs are the only costs for the firm.
- Workers of type 1 will apply for a job with the new firm if and only if they expect to earn $\$w_H$ or more (per year), while workers of type 2 and 3 will apply if and only if they expect to earn $\$w_L$ or more (per year) where $0 < w_L < w_H < w_L \frac{x_1}{x_3}$.

The firm is considering two options: (1) offer to hire workers at a fixed yearly salary w, or (2) offer to hire workers at a piece rate b (that is, the worker will be paid $\$b$ for each unit of output she produces). When a worker applies for a job, the firm is unable to tell what type the applicant is. On the other hand, each applicant knows her own productivity (i.e. an applicant of type i knows that she will be able to produce x_i units of output in one year). The firm is risk neutral and is planning to hire N workers.

(a) If the firm decides to offer a fixed salary, what salary should it offer?

(b) If the firm decides to offer a piece-rate, what is the optimal piece rate? Carefully distinguish between the following cases: (b.1) $w_H < w_L \frac{x_1}{x_2}$ and (b.2) $w_L \frac{x_1}{x_2} < w_H < w_L \frac{x_1}{x_3}$.

For each of the following cases find the optimal policy for the firm (i.e. determine the optimal fixed salary - if a fixed salary is the best option - or the optimal piece rate - if a piece rate is the best option).

(c) $p_1 = \frac{1}{6}, p_2 = \frac{3}{6}, p_3 = \frac{2}{6};\ x_1 = 42, x_2 = 40, x_3 = 38;\ R = 54;\ w_L = 900, w_H = 924$

(d) $p_1 = \frac{2}{4}, p_2 = \frac{1}{4}, p_3 = \frac{1}{4};\ x_1 = 16, x_2 = 15, x_3 = 12;\ R = 40;\ w_L = 225, w_H = 252$

5. There is an ongoing conflict between Country 1 and Country 2. A superpower (Country 3) has put pressure on the two to reach a peaceful settlement. Consider the following game. Country 1 either makes a serious peace proposal to Country 2 or refuses to engage in serious talks with Country 2. If Country 1 makes a serious proposal, Country 2 can either accept, in which case peace is achieved (outcome $\mathbf{z}_1$), or reject the proposal. If peace is not achieved then Country 3 cannot tell whether it was because Country 1 refused to engage in serious talks or because Country 2 rejected the serious proposal of Country 1 (each would claim it was the other party's lack of cooperation). Then Country 3 has to decide whether to set an embargo against Country 1 or an embargo against Country 2. Let the outcomes be as follows:

$\mathbf{z}_2$: peace not achieved because 2 rejected the serious proposal of 1 and 3 sets an embargo on 1

$\mathbf{z}_3$: peace not achieved because 2 rejected the serious proposal of 1 and 3 sets an embargo on 2

$\mathbf{z}_4$: peace not achieved because 1 refused to talk about peace and 3 sets an embargo on 1

$\mathbf{z}_5$: peace not achieved because 1 refused to talk about peace and 3 sets an embargo on 2.

(a) Draw an extensive game to represent this situation. Don't worry about payoffs for the moment (just write the outcomes).

(b) Write the corresponding normal form (again, don't worry about payoffs, just write the corresponding outcomes). Assign the rows to Country 1, the columns to Country 2, etc.

The rankings of the outcomes are as follows (the top outcome is the best and the bottom outcome is the worst): For Country 1 $\begin{pmatrix} z_3 \\ z_5 \\ z_1 \\ z_4 \\ z_2 \end{pmatrix}$, for Country 2: $\begin{pmatrix} z_2, z_4 \\ z_1 \\ z_3, z_5 \end{pmatrix}$ and for Country 3:

$\begin{pmatrix} z_1 \\ z_2, z_5 \\ z_3, z_4 \end{pmatrix}$. Thus, for example, Country 2 is indifferent between z_2 and z_4, considers either of them better than z_1 and considers z_1 better than either z_3 or z_5 and is indifferent between z_3 and z_5. All three countries satisfy the axioms of expected utility.

Country 1 is indifferent between the following two lotteries: $\begin{pmatrix} z_1 & z_2 & z_3 & z_4 & z_5 \\ 0 & \frac{1}{3} & \frac{2}{3} & 0 & 0 \end{pmatrix}$ and $\begin{pmatrix} z_1 & z_2 & z_3 & z_4 & z_5 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$. It is also indifferent between the following two lotteries: $\begin{pmatrix} z_1 & z_2 & z_3 & z_4 & z_5 \\ 0 & \frac{5}{9} & \frac{4}{9} & 0 & 0 \end{pmatrix}$ and $\begin{pmatrix} z_1 & z_2 & z_3 & z_4 & z_5 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix}$. Furthermore, it is also indifferent between the following two lotteries: $\begin{pmatrix} z_1 & z_2 & z_3 & z_4 & z_5 \\ 0 & \frac{2}{3} & \frac{1}{3} & 0 & 0 \end{pmatrix}$ and $\begin{pmatrix} z_1 & z_2 & z_3 & z_4 & z_5 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}$.

Country 2 is indifferent between the following two lotteries: $\begin{pmatrix} z_1 & z_2 & z_3 & z_4 & z_5 \\ 0 & \frac{4}{9} & \frac{5}{9} & 0 & 0 \end{pmatrix}$ and $\begin{pmatrix} z_1 & z_2 & z_3 & z_4 & z_5 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix}$.

Country 3 is indifferent between the following two lotteries: $\begin{pmatrix} z_1 & z_2 & z_3 & z_4 & z_5 \\ \frac{1}{4} & 0 & \frac{3}{4} & 0 & 0 \end{pmatrix}$ and $\begin{pmatrix} z_1 & z_2 & z_3 & z_4 & z_5 \\ 0 & 1 & 0 & 0 & 0 \end{pmatrix}$.

(c) Write the reduced normal form corresponding to the extensive game (that is, the normal form of part (b) with the outcomes replaced by the corresponding von Neumann-Morgenstern payoffs for each player).

(d) Find the mixed-strategy Nash equilibrium of this game. [Hint: in equilibrium Country 2's strategy is to reject the proposal for sure.]

(e) What are the players' expected payoffs at the Nash equilibrium?