Microeconomics Prelim August 31, 2006
Answer Keys

1. 1(a).

Case 1: $\sigma > 1$. In this case the utility function is strictly quasiconcave, and, because all prices are equal and the utility function is symmetric in $x_1, \ldots, x_L$, the consumer demands the same amount of every commodity or variety, i.e., her Walrasian demand for variety j is $\frac{w}{Lp}$, $j = 1,\ldots, L$. Hence she reaches the utility level

$$\left(\sum_{i=1}^{L}\left[\frac{w}{Lp}\right]^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} = \left(L\left[\frac{w}{Lp}\right]^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} = L^{\frac{\sigma}{\sigma-1}}\frac{w}{Lp} = L^{\frac{\sigma}{\sigma-1}-1}\frac{w}{p} = L^{\frac{1}{\sigma-1}}\frac{w}{p},$$

an expression that is increasing in L when $\sigma > 1$. Hence, in this case more variety is good for the consumer.

Case 2: $\sigma \in (0,1)$. As in Case 1, the utility function is strictly quasiconcave and the consumer reaches the utility level $L^{\frac{1}{\sigma-1}}\frac{w}{p}$, an expression that is now *decreasing* in L. Hence, in this case more variety hurts the consumer.

Case 3: $\sigma < 0$. Now the utility function is strictly quasiconvex, and all the solutions to the utility maximization problem are of the corner variety, with the consumer buying only one good. Thus, she reaches the utility level $\left(\left[\frac{w}{p}\right]^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} = \frac{w}{p}$, independent of L. Hence, in this case more variety neither benefits nor hurts the consumer.

1(b). The solution to the utility maximization problem for (1) is the same as the one for the usual CES function

$$\left(\sum_{i=1}^{L} x_i^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}, \qquad (2)$$

because (1) is an increasing transformation of (2). Hence,

$$\tilde{x}_j(p_1,\ldots,p_L,w) = w\left(\sum_{i=1}^{L} p_i^{1-\sigma}\right)^{-1} p_j^{-\sigma},$$

with elasticity

$$\frac{\partial \tilde{x}_j}{\partial p_j}\frac{p_j}{\tilde{x}_j} = w\left((-1)\left(\sum_{i=1}^{L} p_i^{\,1-\sigma}\right)^{-2}(1-\sigma)\,p_j^{\,1-\sigma-1}\,p_j^{\,-\sigma} + \left(\sum_{i=1}^{L} p_i^{\,1-\sigma}\right)^{-1}(-\sigma)\,p_j^{\,-\sigma-1}\right)\frac{p_j}{w\left(\sum_{i=1}^{L} p_i^{\,1-\sigma}\right)^{-1} p_j^{\,-\sigma}}$$

$$= -\frac{\left(\sum_{i=1}^{L} p_i^{\,1-\sigma}\right)^{-2}(1-\sigma)\,p_j^{\,-\sigma-\sigma}\,p_j}{\left(\sum_{i=1}^{L} p_i^{\,1-\sigma}\right)^{-1} p_j^{\,-\sigma}} - \frac{\sigma\left(\sum_{i=1}^{L} p_i^{\,1-\sigma}\right)^{-1} p_j^{\,-\sigma-1}\,p_j}{\left(\sum_{i=1}^{L} p_i^{\,1-\sigma}\right)^{-1} p_j^{\,-\sigma}} = -(1-\sigma)\,p_j^{\,1-\sigma}\left(\sum_{i=1}^{L} p_i^{\,1-\sigma}\right)^{-1} - \sigma\,.$$

If $p_1 = p_2 = \ldots = p_L = p$, then this expression becomes

$$-(1-\sigma)\,p^{1-\sigma}\left(Lp^{1-\sigma}\right)^{-1} - \sigma = -(1-\sigma)\,p^{1-\sigma-1+\sigma}L^{-1} - \sigma = -(1-\sigma)L^{-1} - \sigma\,,$$

which tends to $-\sigma$ as $L \to \infty$.

Intuitively, when L is large, the term $\left(\sum_{i=1}^{L} p_i^{\,1-\sigma}\right)^{-1}$ varies very little with changes in p_j, so that $\tilde{x}_j(p_1,\ldots,p_L,w) \approx CONSTANT \times p_j^{\,-\sigma}$, a demand curve of constant elasticity equal to $-\sigma$. This limit demand elasticity equals (in absolute value) the elasticity of substitution of (1).

1(c). The quotient $\hat{\beta}(L,x) \equiv \dfrac{u_L(x,x,\ldots,x)}{u_1(Lx)}$ is the ratio of the utility that our consumer gets in a L-variety world where she spreads a total amount of Lx units of the good equally among the L varieties, to the utility that she gets in a one-variety world where she consumes Lx units of the only variety available. If $\hat{\beta}(L,x) > 1$, she displays love for variety, and higher values of $\hat{\beta}(L,x)$ indicate a stronger preference for variety.

Because (1) is homogeneous of degree one in $(x_1,\ldots,x_L)$, we can write

$$\hat{\beta}(L,x) \equiv \frac{u_L(x,x,\ldots,x)}{u_1(Lx)} = \frac{x u_L(1,1,\ldots,1)}{Lxu_1(1)} = \frac{u_L(1,1,\ldots,1)}{Lu_1(1)} = \frac{u_L(1,1,\ldots,1)}{L} \equiv \beta(L)\,,$$

and we can compute

$$\beta(L) = \frac{L^{\gamma}\left(\sum_{i=1}^{L} 1^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}}{L} = \frac{L^{\gamma}(L)^{\frac{\sigma}{\sigma-1}}}{L} = L^{\gamma-1+\frac{\sigma}{\sigma-1}} = L^{\gamma+\frac{1}{\sigma-1}}\,.$$

We can compute its elasticity as

$$\eta(L) \equiv \beta'(L)\frac{L}{\beta(L)} = \gamma + \frac{1}{\sigma-1}\,. \qquad (3)$$

In Case 1 of 1(a) above we found that, for $\gamma = 0$ and $\sigma > 1$, a higher number of varieties benefits the consumer. Expression (3) agrees with this finding, since $\eta(L) > 0$ when $\gamma =$

0 and σ > 1. In general, the term $\frac{1}{\sigma - 1}$ is positive for σ > 1, but γ can in principle be of any sign. A negative γ of large absolute value could make $\eta(L)$ negative even with σ > 1.

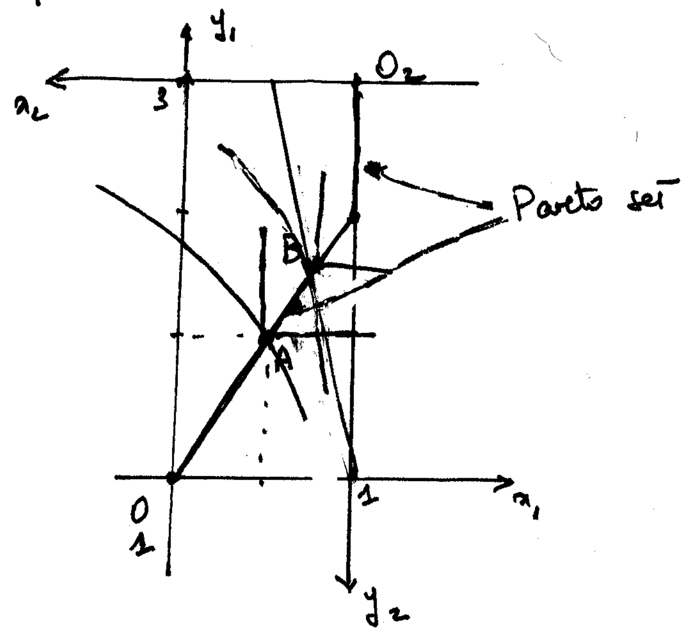
1(d). The monopolist faces a constant-elasticity demand curve of the form $\tilde{y}_j(p_j) = ap_j^{-\sigma}$, or, in indirect form, $\tilde{p}_j(y_j) = a^{\frac{1}{\sigma}} y_j^{-\frac{1}{\sigma}}$, and maximizes $\tilde{p}_j(y_j)y_j - c_j(y_j) = a^{\frac{1}{\sigma}} y_j^{-\frac{1}{\sigma}} y_j - c_j(y_j)$, where $c_j(y_j)$ is the monopolist's cost function. The FOC is $(1 - \frac{1}{\sigma})a^{\frac{1}{\sigma}} y_j^{-\frac{1}{\sigma}} = c_j'(y_j)$, or $\frac{\sigma - 1}{\sigma} p_j = c_j'(y_j)$, which yields $p_j = \frac{\sigma}{\sigma - 1} c_j'(y_j)$. We can then compute

$$\frac{p_j - c_j'}{c_j'} = \frac{\frac{\sigma}{\sigma - 1} c_j' - c_j'}{c_j'} = \frac{\sigma}{\sigma - 1} - 1 = \frac{\sigma - \sigma + 1}{\sigma - 1} = \frac{1}{\sigma - 1}. \qquad (4)$$

Comparing (3) and (4), we observe that the elasticity of the love-of-variety function coincides with the $\frac{\text{price - marginal cost}}{\text{marginal cost}}$ ratio if and only if γ = 0, which is the case often postulated in the literature (in other words, (1) is often replaced by (2)). But these two magnitudes will in general differ unless the utility function $u_L(x_1, ..., x_L)$ is the one given by (2).

2 (a) P.O. allocations:

$$\begin{cases} 2x_1 = y_1 \ , \quad 0 \le x_1 \le 1 \\ x_2 = 1 - x_1 \\ y_2 = 3 - 2x_1 \end{cases} \quad \text{and} \quad \begin{cases} x_1 = 1 \quad 2 \le y_1 \le 3 \\ x_2 = 0 \quad y_2 = 3 - y_1 \end{cases}$$



(b) equal utilities: $2x_1 = (1-x_1)(3-2x_1)$

$\Leftrightarrow 2x_1^2 - 7x_1 + 3 = 0 \Rightarrow x_1 = \frac{1}{2}$. Thus $x_1 = \frac{1}{2}$ $y_1 = 1$,

$x_2 = \frac{1}{2}$, $y_2 = 2$.

(c) Competitive equilibrium with $\omega_1 = (1,0)$ $\omega_2 = (0,2)$

demand of agent 1

$$\begin{cases} y_1 = 2x_1 \\ p_x x_1 + p_y y_1 = p_x \end{cases}$$

$$x_1 = \frac{p_x}{p_x + 2p_y} \qquad y_1 = \frac{2p_x}{p_x + 2p_y}$$

demand of agent 2: $x_2 = \frac{1}{2} \frac{3p_y}{p_x}$ $y_2 = \frac{1}{2} \frac{3p_y}{p_y} = \frac{3}{2}$

market clearing: (for good y, it is sufficient by Walras Law)

$$\frac{2p_x}{p_x + 2p_y} + \frac{3}{2} = 3 \iff 2p_x = \frac{3}{2}(p_x + 2p_y) \Rightarrow p_x = 6p_y.$$

Normalizing $p_y = 1$, the equilibrium is

$$p = (6, 1) \qquad (x_1, y_1) = \left(\frac{3}{4}, \frac{3}{2}\right) \qquad (x_2, y_2) = \left(\frac{1}{4}, \frac{3}{2}\right)$$

(c) In the equilibrium, agent 1 who owns the precious good 1 is more advantaged than the planner would like: to reach the goal of the planner, agent 1 must transfer income to agent 2. However, transferring income changes the aggregate demand and the prices of the goods. To compute the transfer this must be taken into account. The price vector $\bar{p}$ which supports the Pareto allocation A is given by the MRS of agent 2:

$$\frac{\bar{p}_x}{\bar{p}_y} = \frac{MU^2_x(\frac{1}{2}, 2)}{MU^2_y(1/2, 2)} = \frac{2}{1/2} = 4 \Rightarrow \bar{p}_x = 4\bar{p}_y$$

In order to consume $(1/2, 2)$ agent 2 needs an income of $4 \times 1/2 + 2 = 4$ (keeping the normalization $p_y = 1$) With 3 units of good Y as endowment he has an

income of 3, so he must receive a transfer of 1 unit of account (3)
from agent 1. Assuming that the planner can choose the price normalization, ($p_y = 1$) he can impose a tax of one unit of account on agent 1, give it as a subsidy to agent 2, and let agents exchange on the basis of a price system.

(d) Given that many utility functions represent the same preferences, equalization of the utility levels is generally ill-defined and does not guarantee that the allocation is "fair." The notion of fairness generally accepted is that of an "envy-free" allocation: it turns out that in this example the allocation A is envy free since

$$u_1\left(\tfrac{1}{2}, 2\right) = u_1\left(\tfrac{1}{2}, 1\right)$$

$$u_2\left(\tfrac{1}{2}, 1\right) < u_2\left(\tfrac{1}{2}, 2\right)$$

so that, with this choice of utility functions, equalization of utilities leads to a fair allocation. But this is chance rather than a general property.

3. (a) Pareto optimal allocations:

$$\max \; \alpha\left(\ln x_a + \ln(y_a)\right) + (1-\alpha)\left(\ln(x_b) + \ln(y_b) + \ln(4 - x_a)\right)$$

subject to $x_a + x_b = 4$, $y_a + y_b = 4$ + non negativity constraints which are not important given the form of the utility function.

FOCs

$$\frac{\alpha}{x_a} - \frac{1-\alpha}{4-x_a} - \frac{1-\alpha}{4-x_a} = 0 \Leftrightarrow \alpha(4 - x_a) = 2(1-\alpha)x_a$$

$$\frac{\alpha}{y_a} - \frac{1-\alpha}{4-y_a} = 0. \quad \Leftrightarrow \quad \alpha(4 - y_a) = (1-\alpha)\, y_a$$

Pareto optimal allocations: $\begin{cases} x_a = \dfrac{4\alpha}{2-\alpha}, & y_a = 4\alpha, \quad 0 \le \alpha \le 1 \\ x_b = 4 - x_a & y_b = 4 - y_a \end{cases}$

(b) Competitive equilibrium: since agent b take x_a as given, agents have the same preferences and the same income. The equilibrium is symmetric in goods and agents

$p = (1, 1)$ $\quad x_a = y_a = x_b = y_b = 2$.

(c) If $y_a = 2$ and $y_a = 4\alpha$ then $\alpha = 1/2$ (same weight on the agents) But then the optimal consumption in good x would be $x_a = \frac{4}{3}$ instead of 2: the equilibrium prices do not transmit information on the negative externality, which is thus not taken into account by agent a. Because of the negative externality, agent a should consume less good x than good y.

(d) (i) $\max_{x_a, y_a, x_b, y_b} u_a(x_a, y_a)$

subject to $p_x x_a + p_y y_a + q_a x_a + r_a y_b \leq 2p_x + 2p_y + q_b x_b + r_b x_b$

FOCs: $\frac{\partial u_a}{\partial x_b} = -\lambda_a q_b$ $\quad \frac{\partial u_a}{\partial y_b} = -\lambda_a r_b$ where λ_a is the multiplier of the b.c. of agent a. Since $\frac{\partial u_a}{\partial y_b} = 0$ $\frac{\partial u_a}{\partial x_b} = 0$ and since $\lambda_a > 0$, $q_b = r_b = 0$. In the same way since agent b is not affected by agent a's consumption of good y $a = 0$. Only the goods which create an externality have a non-zero transfer price

(ii) demand of agent a

$$\max u_a(x_a, y_a) = \ln(x_a) + \ln(y_a)$$

subject to: $p_x x_a + p_y y_a + q_a x_a \leq 2p_x + 2p_y$

$$x_a = \frac{1}{2} \frac{2p_x + 2p_y}{p_x + q_a} \qquad y_a = \frac{2p_x + 2p_y}{2p_y}$$

$$x_b \in [0 + \infty[\qquad y_b \in [0 + \infty[$$

demand of agent b

$$\max \; u_b(x_b, y_b, x_a) = \ln(x_b) + \ln(y_b) + \ln(4 - x_a)$$

subject to $\quad p_x x_b + p_y y_b \le 2p_x + 2p_y + q_a x_a$

$$\Leftrightarrow \quad p_x x_b + p_y y_b + q_a(4 - x_a) \le 2p_x + 2p_y + 4q_a$$

$$x_b = \frac{1}{3}\frac{2p_x + 2p_y + 4q_a}{p_x} \qquad y_b = \frac{1}{3}\frac{2p_x + 2p_y + 4q_a}{p_y}$$

$$4 - x_a = \frac{1}{3}\frac{2p_x + 2p_y + 4q_a}{q_a}. \qquad y_a \in [0 \; +\infty[$$

equilibrium:

$$\begin{cases} \frac{1}{2}\frac{2p_x + 2p_y}{p_x + q_a} = 4 - \frac{1}{3}\frac{2p_x + 2p_y + 4q_a}{q_a} & \text{(same choice for } x_a\text{)} \\ \frac{1}{2}\frac{2p_x + 2p_y}{p_x + q_a} + \frac{1}{3}\frac{2p_x + 2p_y + 4q_a}{p_x} = 4 & \text{(market clearing in } x\text{)} \\ \frac{1}{2}\frac{2p_x + 2p_y}{p_y} + \frac{1}{3}\frac{2p_x + 2p_y + 4q_a}{p_y} = 4 & \text{(market clearing in } y\text{)} \end{cases}$$

Comparing the two first equations, the system has a solution only if $q_a = p_x$, in which case the two equations are the same. Inserting in the second equation, this gives

(7)

$P_x = \frac{7}{9} P_y$, and the third equation is then satisfied.

The equilibrium with transfer prices is thus (normalizing $P_y = 1$)

$$p = \left(7/9,\ 1,\ \frac{7}{9},\ 0,\ 0;\ 0\right),\quad (x_a, y_a) = \left(\frac{8}{7}, \frac{16}{9}\right),\ (x_b, y_b) = \left(\frac{20}{7}, \frac{20}{9}\right)$$

which is the Pareto optimal allocation corresponding to $\alpha = \frac{4}{3}$.

Intuitively the system works because the transfer price q_a internalizes the externality. It is equivalent to a market for pollution rights whereby agent a buys from agent b the rights to "pollute" at price q_a. Adding q_a to the social value P_x of good x in the choice of agent a makes him take into account the marginal disutility of agent b.

4.

(a) The firm would only consider offering a wage of w_L or a wage of w_H. If the firm offers w_H then all types apply and the expected profit per worker is

$$\pi_{all} = (p_1x_1 + p_2x_2 + p_3x_3)R - w_H \qquad \text{(all types)} \qquad (1)$$

while if the firm offers w_L then only types 2 and 3 apply and the profit per worker is (using Bayes' rule)

$$\pi_{12} = \left(\frac{p_2}{p_2+p_3}x_2 + \frac{p_3}{p_2+p_3}x_3\right)R - w_L \qquad \text{(types 1 and 2)} \qquad (2)$$

Thus the firm should offer w_H if $\pi_{\text{all}} > \pi_{12}$ and w_L if $\pi_{12} > \pi_{\text{all}}$.

(b) Since $x_3 < x_2$, $\frac{w_L}{x_2} < \frac{w_L}{x_3}$.

(b.1) In this case we have $\frac{w_H}{x_1} < \frac{w_L}{x_2} < \frac{w_L}{x_3}$. The only piece rates that will be considered by the firm are $b = \frac{w_L}{x_3}$, $b = \frac{w_L}{x_2}$ and $b = \frac{w_H}{x_1}$. If the firm offers $b = \frac{w_L}{x_3}$ then everybody will apply and the profit per worker will be given by

$$\pi_3 = (p_1x_1 + p_2x_2 + p_3x_3)\left(R - \frac{w_L}{x_3}\right). \qquad \text{(all types)} \qquad (3)$$

If the firm offers $b = \frac{w_L}{x_2}$ then only types 1 and 2 will apply and profit per worker will be

$$\pi_4 = \left(\frac{p_1}{p_1+p_2}x_1 + \frac{p_2}{p_1+p_2}x_2\right)\left(R - \frac{w_L}{x_2}\right). \qquad \text{(types 1 and 2)} \qquad (4)$$

If the firm offers $b = \frac{w_H}{x_1}$ then only type 1 will apply and profit per worker will be

$$\pi_5 = x_1\left(R - \frac{w_H}{x_1}\right). \qquad \text{(only type 1)} \qquad (5)$$

Since $x_1 > x_2 > x_3$, $x_1 > \left(\frac{p_1}{p_1+p_2}x_1 + \frac{p_2}{p_1+p_2}x_2 \right)$ and $x_1 > (p_1x_1 + p_2x_2 + p_3x_3)$. Furthermore, since $\frac{w_H}{x_1} < \frac{w_L}{x_2} < \frac{w_L}{x_3}$, $\left(R - \frac{w_H}{x_1} \right) > \left(R - \frac{w_L}{x_2} \right) > \left(R - \frac{w_L}{x_3} \right)$. Thus $\pi_5 > \pi_4 > \pi_3$ and therefore **the optimal piece rate is** $b = \frac{w_H}{x_1}$.

(b.2) In this case we have $\frac{w_L}{x_2} < \frac{w_H}{x_1} < \frac{w_L}{x_3}$. If the firm offers $b = \frac{w_L}{x_3}$ then everybody will apply and the profit per worker will be given by $\pi_3 = (p_1x_1 + p_2x_2 + p_3x_3)\left(R - \frac{w_L}{x_3} \right)$ (see (3) above). If the firm offers $b = \frac{w_H}{x_1}$ then types 1 and 2 will apply and the profit per worker will be

$$\pi_6 = \left(\frac{p_1}{p_1+p_2}x_1 + \frac{p_2}{p_1+p_2}x_2 \right)\left(R - \frac{w_H}{x_1} \right). \qquad \text{(types 1 and 2)} \qquad (6)$$

If the firm offers $b = \frac{w_L}{x_2}$ then only type 2 workers will apply and profit per worker will be

$$\pi_7 = x_2\left(R - \frac{w_L}{x_2} \right). \qquad \text{(only type 2)} \qquad (7)$$

Since $\left(\frac{p_1}{p_1+p_2}x_1 + \frac{p_2}{p_1+p_2}x_2 \right) > (p_1x_1 + p_2x_2 + p_3x_3)$ and $\left(R - \frac{w_H}{x_1} \right) > \left(R - \frac{w_L}{x_3} \right)$, $\pi_6 > \pi_3$.

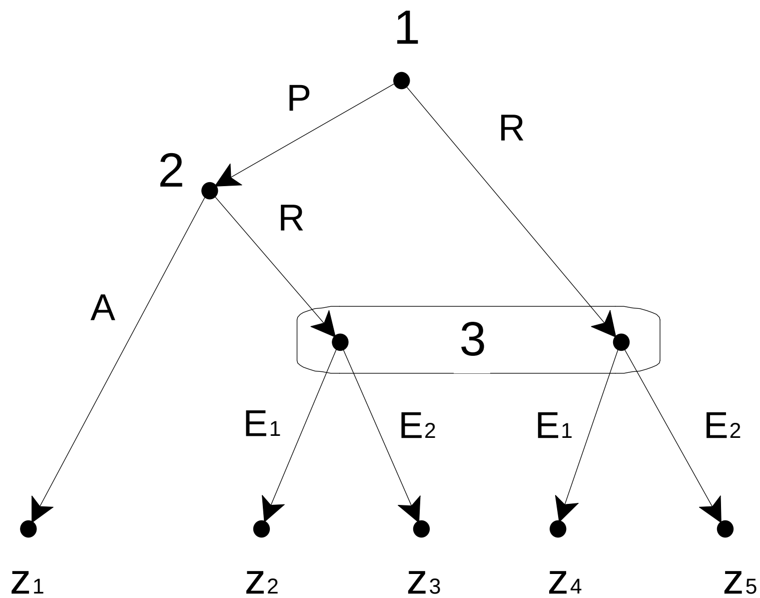
Hence **the optimal piece rate is** $b = \frac{w_H}{x_1}$ **if** $\pi_6 > \pi_7$ **and** $b = \frac{w_L}{x_2}$ **if** $\pi_7 > \pi_6$.

(c) If $p_1 = \frac{1}{6}, p_2 = \frac{3}{6}, p_3 = \frac{2}{6}$; $x_1 = 42, x_2 = 40, x_3 = 38$; $R = 54$; $w_L = 900, w_H = 924$ then we are in case (b.1) above $\left(w_H = 924 < w_L\frac{x_1}{x_2} = 900\frac{42}{40} = 945 \right)$. Thus the only options considered by the firm are w_H, w_L and $b = \frac{w_H}{x_1} = 22$. The first yields a profit of $\pi_{all} = 1{,}218$ (see (1) above), the second a profit of $\pi_{12} = 1{,}216.8$ (see (2) above) and the third a profit of $\pi_5 = 1{,}344$ (see (5) above). Thus π_5 is the largest profit and **the optimal choice is a piece rate of *b* =** $\frac{w_H}{x_1} = 22$**.**

(d) If $p_1 = \frac{2}{4}, p_2 = \frac{1}{4}, p_3 = \frac{1}{4}$; $x_1 = 16, x_2 = 15, x_3 = 12$; $R = 40$; $w_L = 225, w_H = 252$ then we are in case (b.2) above $\left(\frac{w_L}{x_2} = 15 < \frac{w_H}{x_1} = 15.75 < \frac{w_L}{x_3} = 18.75 \right)$. Thus the firm will only consider w_H , w_L , $b = \frac{w_H}{x_1}$ and $b = \frac{w_L}{x_2}$. The first gives a profit of π_{all} = 338 (see (1) above), the second gives π_{12} = 315 (see (2) above), the third gives π_6 = 379.92 (see (6) above) and the fourth gives π_7 = 375 (see (6) above). Thus **the best option is a piece rate of** $b = \frac{w_H}{x_1}$ **=** 15.75.

5.

(a) The extensive game is as follows (P means propose, R means refuse or reject, A means accept, E_1 means embargo on country 1, E_2 means embargo on country 2):



(a) The corresponding normal form is:

Country 3: E1

Country 1 \ Country 2	A	R
P	z1	z2
R	z4	z4

Country 3: E2

Country 1 \ Country 2	A	R
P	z1	z3
R	z5	z5

(b) The von Neumann-Morgenstern payoffs are as follows (assign utility 1 to the best outcome, 0 to the worst, etc.):

$$\text{Country 1: } \begin{pmatrix} z_3 & 1 \\ z_5 & \frac{2}{3} \\ z_1 & \frac{4}{9} \\ z_4 & \frac{1}{3} \\ z_2 & 0 \end{pmatrix}, \qquad \text{Country 2: } \begin{pmatrix} z_2, z_4 & 1 \\ z_1 & \frac{4}{9} \\ z_3, z_5 & 0 \end{pmatrix}, \qquad \text{Country 3: } \begin{pmatrix} z_1 & 1 \\ z_2, z_5 & \frac{1}{4} \\ z_3, z_4 & 0 \end{pmatrix}$$

Country 3: E1

Country 1 \ Country 2	A	R
P	$\frac{4}{9}, \frac{4}{9}, 1$	$0, 1, \frac{1}{4}$
R	$\frac{1}{3}, 1, 0$	$\frac{1}{3}, 1, 0$

Country 3: E2

Country 1 \ Country 2	A	R
P	$\frac{4}{9}, \frac{4}{9}, 1$	$1, 0, 0$
R	$\frac{2}{3}, 0, \frac{1}{4}$	$\frac{2}{3}, 0, \frac{1}{4}$

(c) Suppose Country 2 chooses R for sure. Let p be the probability that Country 1 chooses P and let q be the probability that Country 3 chooses E_1. Then the following must be true:

(1) Country 1 must be indifferent between choosing P and choosing R: $1-q=\frac{1}{3}q+\frac{2}{3}(1-q)$, which gives $q=\frac{1}{2}$; and

(2) Country 3 must be indifferent between choosing E_1 and choosing E_2: $\frac{1}{4}p=\frac{1}{4}(1-p)$, which gives $p=\frac{1}{2}$.

Given these probabilities, Country 2 strictly prefers R to A. Thus the mixed-strategy Nash equilibrium is $\left(\begin{array}{cc|cc|cc} P & R & A & R & E_1 & E_2 \\ \frac{1}{2} & \frac{1}{2} & 0 & 1 & \frac{1}{2} & \frac{1}{2} \end{array}\right)$

(d) The expected payoffs at the Nash equilibrium are as follows. For Country 1: $\frac{1}{2}$, for Country 2: $\frac{1}{2}$, for Country 3: $\frac{1}{8}$.