Answer Key Part A: Questions on ECN 200D (Rendahl)

1. (a) The employment duration is given by $1/(1-p)$.

   (b) The unemployment rate is given by $u = \frac{1-p}{1+(1-p)-s}$.

   (c) The unconditional probability of employment is $1-u$.

2. (a) A Cauchy sequence $\{x_t\}$ is such that for each ε there exist a N such that $|x_n - x_m| < \varepsilon$ for all $n, m \geq N$. We will show that $v_n(x)$ is Cauchy.

   (b) A complete metric space is a space in which every Cauchy sequence is a converging sequence. When we know that $v_n(x)$ is Cauchy, we will show that $v_n(x)$ exists in a complete metric space and therefore that $v_n(x) \to v(x)$.

   (c) The theorem of the maximum states, in particular, that if $v_n(x)$ is continuous, so is $v_{n+1}(x)$.

   (d) The contraction mapping theorem states that if T is a contraction mapping, then there exist a unique v such that $v = Tv$.

3. (a) A stationary competitive equilibrium is an interest-rate r and wage w such that,

   i. $g(a, \theta)$ solves the households problem.

   ii. The stationary distribution ψ satisfies

   $$\psi(a', \theta') = \sum_{\theta \in \{0,1\}} \sum_{a: a' = g(a,\theta)} \psi(a, \theta)\lambda(\theta', \theta)$$

   iii.

   $$A = \int_{a,\theta} a\psi(a, \theta) da$$

   iv. Markets clear, that is

   $$w = F_N(A, (1-u)) \qquad r = F_K(A, (1-u))$$

   (b) For the Hugget economy, we have to delete δ in the agents' budget constraints. A stationary competitive equilibrium is an interest-rate r such that,

   i. $g(a, \theta)$ solves the households problem.

ii. The stationary distribution ψ satisfies

$$\psi(a',\theta') = \sum_{\theta\in\{0,1\}} \sum_{a:a'=g(a,\theta)} \psi(a,\theta)\lambda(\theta',\theta)$$

iii.

$$0 = \int_{a,\theta} a\psi(a,\theta)da$$

4. (a) The SP is,

$$v^*(s_0) = \max_{\{h_t(s^t)\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \sum_{s^t\in S^{t+1}} \beta^t u(h_t(s^t)w(s_t) + (1-h_t(s^t))b)f(s^t)$$

$$\text{s.t.} \qquad h_t(s^t) \in \{0,1\} \quad \forall\, t = 0,1,\ldots, \forall\, s^t \in S^{t+1}$$

(b) There are a couple of ways to formulate the Bellman equation. I find the following the most elegant.

$$v(s) = \max\{u(w(s)), u(b)\} + \beta \sum_{s'\in S} v(s')\lambda(s',s)$$

(c) The SP is now

$$v^*(s_0) = \max_{\{h_t(s^t)\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \sum_{s^t\in S^{t+1}} \beta^t [u(h_t(s^t)w(s_t) + (1-h_t(s^t))b) - h_t(s^t)\delta] f(s^t)$$

$$\text{s.t.} \qquad h_t(s^t) \in \{0,1\} \quad \forall\, t = 0,1,\ldots, \forall\, s^t \in S^{t+1}$$

and the FE

$$v(s) = \max\{u(w(s)) - \delta, u(b)\} + \beta \sum_{s'\in S} v(s')\lambda(s',s)$$